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THEORY OF MOTION OF THE PLANET PLUTO

Part Two: Plutonian Perturbations of the Second Order With Respect To Perturbing Masses

Part Three: Recalculation of First-Order Plutonian Perturbations With Respect to Perturbing Masses

Part Four: New Elements of Pluto

by Sh. G. Sharaf and N. A. Budnikova

Transactions of the Institute of Theoretical Astronomy, Vol. 10
"Nauka" Press, Moscow-Leningrad, 1964

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Translation of "Teoriya Dvizheniya Plutona. Chast' Vtoraya. Vozmushcheniya Plutona Vtorogo Poryadka Otnositel'no Vozmushchayushchikh Mass. Chast' Tret'ya. Perevychisleniye Vozmushcheniy Plutona Pervogo Poryadka Otnositel'no Vozmushchayushchikh Mass. Chast' Chetvertaya. Novyye Elementy Plutona."

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ANNOTATION

The present work, which is a direct continuation of the first part of "Theory of Motion of the Planet Pluto", published in the Transactions of the Institute of Theoretical Astronomy, vol. 4*, consists of three parts. The references, the abstract, and the Table of Contents pertain to all three parts. The enumeration of the chapters, sections, formulas, and tables in each part is independent. At all stages of the work, the authors utilized the valuable advice of Prof. M. F. Subbotin, O. A. Sizova, Ye. N. Lemekhova; M. V. Fridlyand helped in the calculations, and T. V. Rodionova worked on the punched-card machinery. The authors express their sincere gratitude to all these persons. The authors also wish to thank V. V. Lavdovskiy, senior scientific associate of the Main Astronomical Observatory of the Academy of Sciences of the USSR, for contributing his observations of Pluto.

M. F. Subbotin, Editor-in-Chief
Director of the Institute of
Theoretical Astronomy,
Academy of Sciences of the USSR

*Translated as NASA TT F-490.

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PART TWO.

PLUTONIAN PERTURBATIONS OF THE SECOND ORDER WITH RESPECT TO PERTURBING MASSES.

Sh. G. Sharaf and N. A. Budnikova

ABSTRACT. The first part of this monograph is dedicated to the development of the complete theory of Plutonian motion and was published previously. The second part deals with the determination of the Plutonian perturbations of the second order, in relation to the masses of Jupiter, Saturn, and Uranus. The Laplace-Newcomb method is utilized for the perturbation calculations. Newcomb operators, as related to mean anomalies, are employed in the calculation of the perturbation function development. The equations of motion are obtained, the second members evaluated, and formulas presented for the determination of long-period terms by a more convenient method. In the third part, the Plutonian perturbations of the first order are determined once again, in relation to the masses of the large planets, using the new elements of Pluto, while increasing the accuracy of the development of the perturbing function, and while using formulas transformed by the long-period perturbations of Pluto, due to Uranus. In the supplement, there are tables of coefficients of Newcomb operators, completing those of the first part. The Plutonian elements are corrected in the fourth part, according to observations made between 1914 and 1958.

INTRODUCTION

Part Two of "Theory of Motion of the Planet Pluto" is devoted to determining second-order perturbations of Pluto's coordinates, in relation to the disturbing masses of Jupiter, Saturn, and Uranus. As in Part One of "Theory of Motion of the Planet Pluto" of which this work is the direct continuation, we used the Laplace-Newcomb method· development of the perturbing function and its derivatives is performed by the method of Newcomb operators (relative to mean anomalies). /5*

In this part of the paper, considerable attention is devoted to deriving the basic equations of motion, evaluating the second members, and transforming formulas in the presence of pronounced covariances of mean motions. All of these can also be employed when the indicated method is applied to other celestial bodies.

Part Two of "Theory of Motion of the Planet Pluto" consists of four chapters.

*- Numbers in the margin indicate pagination in foreign text.

Chapter I presents theoretical problems of determining perturbations of the second order by the Laplace-Newcomb method.

Chapter II contains perturbations of the coordinates of Jupiter, Saturn, and Uranus in a form convenient for use in the given method.

Chapter III examines in detail the second members of equations of motion; the initial series entering the second members are given.

Chapter IV gives the final expressions of perturbations.

CHAPTER I

DETERMINATION OF SECOND-ORDER PERTURBATIONS

1. Perturbations of the Logarithm of the Radius Vector and Longitude in Orbit

We determined second-order perturbations by the same Laplace-Newcomb method which was used for finding perturbations of the first order (Sharaf, 1955).** In deriving the basic equations in Part One, we limited ourselves to values of the first order. Now we will obtain these equations with an accuracy to values of the second order. /6*

The equations of motion in polar coordinates, if the plane of the osculating orbit of the perturbed planet is taken as the fundamental plane, can be reduced to the form:

$$\begin{gathered} r' \frac{d^2 r'}{dt^2} - r'^2 \left(\frac{dw'}{dt}\right)^2 + \frac{k^2 (1 + m')}{r'} = k^2 m \frac{\partial R}{\partial r'}, \\ \frac{d}{dt}\left(r'^2 \frac{dw'}{dt}\right) = k^2 m \frac{\partial R}{\partial W'}. \end{gathered} \tag{1}$$

Here r' is the radius vector of the perturbed planet; w' the longitude in the osculating orbit; m' and m the masses of the perturbed and perturbing planets; k^2 the coefficient of attraction; $R = k^2 mR$ (ρ', ρ, W', W, and σ) the perturbing function; W' and W the longitudes of planets calculated from the common node of the planets' orbits; ρ' and ρ the logarithms of the radius vectors;

$\sigma = \sin \frac{J}{2}$, where J is the mutual inclination.

As usual, the primes pertain to the superior planet.

We multiply the first equality in Equation (1) by $2\frac{d\rho'}{dt}$, the second equality by $2\frac{dw'}{dt}$, add, and after integration obtain

$$\left(\frac{dr'}{dt}\right)^2 + \left(r' \frac{dw'}{dt}\right)^2 - \frac{2k^2 (1 + m')}{r'} = 2k^2 m \int D_t' R dt + C_0, \tag{2}$$

where C_0 is the constant of integration, and

$$D_t' R = \frac{\partial R}{\partial \rho'} \frac{d\rho'}{dt} + \frac{\partial R}{\partial W'} \frac{dw'}{dt}.$$

We add to Equation (2) the first equality in Equation (1) and after certain transformations, we find:

*Numbers in the margin indicate pagination in the foreign text.
**This work will be referred to simply as Part One.

$$\frac{1}{2}\frac{d^2r'^2}{dt^2}-\frac{k^2(1+m')}{r'}=2k^2m\int D'_tRdt+k^2m\frac{\partial R}{\partial\varrho'}+C_0. \tag{3}$$

In the case of unperturbed motion the radius vector r'_0 satisfies Jacobi's equation:

$$\frac{1}{2}\frac{d^2r_0'^2}{dt^2}-\frac{k^2(1+m')}{r'_0}=-\frac{k^2(1+m')}{a'}. \tag{4}$$

If we subtract termwise Equation (4) from Equation (3), designate $\rho'_0 = \log r'_0$ and $\delta\rho = \rho' - \rho'_0$, take into account that

$$r'^2-r_0'^2=r_0'^2(2\delta\varrho'+2\delta\varrho'^2+\ldots),$$

$$r'^{-1}-r_0'^{-1}=r_0'^{-1}\left(-\delta\varrho'+\frac{1}{2}\delta\varrho'^2+\ldots\right),$$

and limit ourselves to terms of the second order $\delta\rho'$, then we arrive at the equation:

$$\frac{d^2}{dt^2}\left(r_0'^2\delta\varrho'\right)+\frac{k^2(1+m')}{r_0'^3}\left(r_0'^2\delta\varrho'\right)=k^2m\left[2\int D'_tRdt+\frac{\partial R}{\partial\varrho'}\right]+C_0+$$
$$+\frac{k^2(1+m')}{a'}-\frac{d^2}{dt^2}(r'_0\delta\varrho')^2+\frac{1}{2}k^2(1+m)\frac{(\delta\varrho')^2}{r'_0}. \tag{5}$$

Since

$$k^2=\frac{n'^2a'^3}{1+m'},$$

then, having designated

$$2k^2mC_1=C_0+\frac{k^2(1+m')}{a'},$$

$$a'Q=2\int a'D'_tRdt+a'\frac{\partial R}{\partial\varrho'}+\frac{1+m'}{m}\left[-\frac{d^2}{(dn't)^2}\left(\frac{r'_0}{a'}\delta\varrho'\right)^2+\frac{1}{2}\left(\frac{r'_0}{a'}\right)^{-1}(\delta\varrho')^2\right],$$

Equation (5) can be written in the following manner:

$$\frac{d^2}{dt^2}\left(r_0'^2\delta\varrho'\right)+\frac{k^2(1+m')}{r_0'^3}\left(r_0'^2\delta\varrho'\right)=\frac{m}{1+m'}a'^2n'^2(a'Q+2a'C_1). \tag{6}$$

As is known, Equation (6) has a solution in the form:

$$\delta\varrho'=\frac{m}{1+m'}\left(\frac{a'}{r'_0}\right)^2\left[n'q\int p(a'Q+2a'C_1)\,dt-n'p\int q(a'Q+2a'C_1)\,dt+a'K_1p+a'K_2q\right], \tag{7}$$

where

$$p = \cos E' - e',$$
$$q = \sin E',$$

E' is the eccentric anomaly, e' = sin ϕ ' is the eccentricity.

We continue to a determination of perturbations of longitude in the orbit. Integration of the second equality in Equations (1) gives

$$\frac{dw'}{dt} = \frac{k^2 m}{r'^2}\left(\int \frac{\partial R}{\partial W'}\, dt + C\right). \tag{7'}$$

For unperturbed motion the integral of areas in polar coordinates is written as

$$\frac{dw'_0}{dt} = \frac{a'^2 n'}{r'^2_0}\cos\varphi', \tag{8}$$

where w'$_0$ is the unperturbed value of longitude in the orbit.

If we subtract Equation (8) termwise from Equation (7'), take into account that

$$r'^{-2} = r'^{-2}_0 (1 - 2\delta\rho' + 2\delta\rho'^2 + \ldots),$$
$$w' = w'_0 + \delta w',$$

assume

$$k^2 m C = a'^2 n' \cos\varphi' + a'^2 n' m C_2$$

and limit ourselves, as above, to terms of the second order, then we obtain /8

$$\frac{d\delta w'}{dt} = \left(\frac{a'}{r'_0}\right)^2 n' \left\{(1 - 2\delta\rho')\frac{m}{1+m'}\left[n'\int a'\frac{\partial R}{\partial W'}\, dt + C_2\right] - 2[\delta\rho' - (\delta\rho')^2]\cos\varphi'\right\}, \tag{9}$$

from which, after integration, we have

$$\delta w' = n'\int\left(\frac{a'}{r'_0}\right)^2\left\{(1 - 2\delta\rho')\frac{m}{1+m'}\left[n'\int a'\frac{\partial R}{\partial W'}\, dt + C_2\right] - 2\cos\varphi'[\delta\rho' - (\delta\rho')^2]\right\} dt + C_3, \tag{10}$$

C_2 and C_3 being constants of integration.

Expressions (7) and (10) make it possible to determine perturbations

of the logarithm of the radius vector and longitude in orbit, with an accuracy to the second order of the perturbing mass, provided that the derivatives of the perturbing function entering the second members of the equations are obtained with the same accuracy.

The perturbing function, just as its derivatives, is a function of ρ', ρ, W', W, and σ:

$$R = R(\rho', \rho, W', W, \sigma).$$

In turn, ρ', ρ, ... can be represented as the sums

$$\rho' = \rho'_0 + \delta\rho', \quad \rho = \rho_0 + \delta\rho, \ldots \sigma = \sigma_0 + \delta\sigma,$$

where the quantities with the zero subscripts pertain to unperturbed motion, and $\delta\rho'$, $\delta\rho$, ... $\delta\sigma$ are perturbations of the corresponding quantities.

Then we can write

$$R = R(\rho'_0 + \delta\rho', \ \rho_0 + \delta\rho, \ \ W'_0 + \delta W', \ \ W_0 + \delta W, \ \sigma_0 + \delta\sigma). \tag{11}$$

If we expand the perturbing function and its derivatives in powers of perturbations and limit them to first powers, we obtain

$$\left.\begin{aligned}
R &= R_0 + \delta R, \\
\frac{\partial R}{\partial \rho'} &= \left(\frac{\partial R}{\partial \rho'}\right)_0 + \delta\frac{\partial R}{\partial \rho'}, \\
\frac{\partial R}{\partial \rho} &= \left(\frac{\partial R}{\partial \rho}\right)_0 + \delta\frac{\partial R}{\partial \rho}, \\
\frac{\partial R}{\partial W'} &= \left(\frac{\partial R}{\partial W'}\right)_0 + \delta\frac{\partial R}{\partial W'}, \\
\frac{\partial R}{\partial W} &= \left(\frac{\partial R}{\partial W}\right)_0 + \delta\frac{\partial R}{\partial W}, \\
\frac{\partial R}{\partial \sigma} &= \left(\frac{\partial R}{\partial \sigma}\right)_0 + \delta\frac{\partial R}{\partial \sigma},
\end{aligned}\right\} \tag{12}$$

where

$$
\left.
\begin{aligned}
\delta R &= \frac{\partial R}{\partial \rho'}\delta\rho' + \frac{\partial R}{\partial \rho}\delta\rho + \frac{\partial R}{\partial W'}\delta W' + \frac{\partial R}{\partial W}\delta W + \frac{\partial R}{\partial \sigma}\delta\sigma,\\
\delta\frac{\partial R}{\partial \rho'} &= \frac{\partial^2 R}{\partial \rho'^2}\delta\rho' + \frac{\partial^2 R}{\partial \rho'\partial \rho}\delta\rho + \frac{\partial^2 R}{\partial \rho'\partial W'}\delta W' + \frac{\partial^2 R}{\partial \rho'\partial W}\delta W + \frac{\partial^2 R}{\partial \rho'\partial \sigma}\delta\sigma,\\
\delta\frac{\partial R}{\partial \rho} &= \frac{\partial^2 R}{\partial \rho\partial \rho'}\delta\rho' + \frac{\partial^2 R}{\partial \rho^2}\delta\rho + \frac{\partial^2 R}{\partial \rho\partial W'}\delta W' + \frac{\partial^2 R}{\partial \rho\partial W}\delta W + \frac{\partial^2 R}{\partial \rho\partial \sigma}\delta\sigma,\\
\delta\frac{\partial R}{\partial W'} &= \frac{\partial^2 R}{\partial W'\partial \rho'}\delta\rho' + \frac{\partial^2 R}{\partial W'\partial \rho}\delta\rho + \frac{\partial^2 R}{\partial W'^2}\delta W' + \frac{\partial^2 R}{\partial W'\partial W}\delta W + \frac{\partial^2 R}{\partial W'\partial \sigma}\delta\sigma,\\
\delta\frac{\partial R}{\partial W} &= \frac{\partial^2 R}{\partial W\partial \rho'}\delta\rho' + \frac{\partial^2 R}{\partial W\partial \rho}\delta\rho + \frac{\partial^2 R}{\partial W\partial W'}\delta W' + \frac{\partial^2 R}{\partial W^2}\delta W + \frac{\partial^2 R}{\partial W\partial \sigma}\delta\sigma,\\
\delta\frac{\partial R}{\partial \sigma} &= \frac{\partial^2 R}{\partial \sigma\partial \rho'}\delta\rho' + \frac{\partial^2 R}{\partial \sigma\partial \rho}\delta\rho + \frac{\partial^2 R}{\partial \sigma\partial W'}\delta W' + \frac{\partial^2 R}{\partial \sigma\partial W}\delta W + \frac{\partial^2 R}{\partial \sigma^2}\delta\sigma.
\end{aligned}
\right\} \qquad (13)
$$

The quantity $D'_t R$ is obtained by taking the derivative with respect to time of $R = R_0 + \delta R$; furthermore, only the time entering the coordinates of the perturbed planet is considered: /9

$$
D'_t R = \frac{\partial}{\partial \rho'}(R_0 + \delta R)\frac{d\rho'}{dt} + \frac{\partial}{\partial W'}(R_0 + \delta R)\frac{dW'}{dt} = D'_t R_0 + \delta D'_t R, \qquad (14)
$$

where

$$
\begin{aligned}
\delta D'_t R = &\left(D'_t \frac{\partial R}{\partial \rho'}\right)\delta\rho' + \left(D'_t \frac{\partial R}{\partial \rho}\right)\delta\rho + \left(D'_t \frac{\partial R}{\partial W'}\right)\delta W' + \left(D'_t \frac{\partial R}{\partial W}\right)\delta W +\\
&+ \left(D'_t \frac{\partial R}{\partial \sigma}\right)\delta\sigma + \frac{\partial R}{\partial \rho'}\frac{d\delta\rho'}{dt} + \frac{\partial R}{\partial W'}\frac{d\delta W'}{dt}.
\end{aligned}
$$

When determining perturbations of the first order, we examined the perturbing function and its derivatives, as functions of the unperturbed values of coordinates and mutual inclination: in other words, in Expressions (12) and (14) we took into account only the first terms. In the right-hand members of Equations (7) and (10), we can separate terms containing first and second orders of perturbing masses, and isolate perturbations of the second order.

Taking into account Formulas (12) and (14), we can write:

$$
\left.
\begin{aligned}
a'Q &= a'Q_0 + \delta a'Q,\\
\frac{\partial R}{\partial \omega'} &= \frac{\partial R}{\partial W'} = \left(\frac{\partial R}{\partial W'}\right)_0 + \delta\frac{\partial R}{\partial W'},\\
a'C_1 &= (a'C_1)_0 + \delta a'C_1,\\
a'K_1 &= (a'K_1)_0 + \delta a'K_1,\\
a'K_2 &= (a'K_2)_0 + \delta a'K_2,\\
C_2 &= (C_2)_0 + \delta C_2,\\
C_3 &= (C_3)_0 + \delta C_3,
\end{aligned}
\right\} \qquad (15)
$$

where

$$a'Q_0 = 2\int (a'D_t'R)_0\,dt + \left(a'\frac{\partial R}{\partial \rho'}\right)_0,$$

$$a'\delta Q = 2\int a'\delta D_t'R\,dt + a'\delta\frac{\partial R}{\partial \rho'} + \frac{1+m'}{m}\left[-\frac{d^2}{(dn't)^2}\left(\frac{r_0'}{a'}\delta\rho'\right)^2 + \frac{1}{2}\left(\frac{r_0'}{a'}\right)^{-1}(\delta\rho')^2\right],$$

$(a'C_1)_0$, $(a'K_1)_0$, $(a'K_2)_0$, $(C_2)_0$, and $(C_3)_0$ are parts of the constants of integration depending on magnitudes of the first order, and $a'\delta C_1$, $a'\delta K_1$, $a'\delta K_2$, δC_2, and δC_3 are parts of the constants of integration depending on magnitudes of the second order.

Equations (7) and (10) are rewritten as

$$\left.\begin{aligned}
&\delta\rho' = \delta_1\rho' + \delta_2\rho' = \frac{m}{1+m'}\left(\frac{a'}{r_0'}\right)^2\left\{n'q\int p\,[a'Q_0 + 2(a'C_1)_0]\,dt - n'p\int q\,[a'Q + 2(a'C_1)_0]\,dt + \right.\\
&\left. + (a'K_1)_0 p + (a'K_2)_0 q\right\} + \frac{m}{1+m'}\left(\frac{a'}{r_0'}\right)^2\left\{n'q\int p\,[a'\delta Q + 2a'\delta C_1]\,dt - \right.\\
&\left. - n'p\int q\,[a'\delta Q + 2a'\delta C_1]\,dt + a'\delta K_1 p + a'\delta K_2 q\right\},\\
&\delta w' = \delta_1 w' + \delta_2 w' = n'\int\left(\frac{a'}{r_0'}\right)^2\left\{\frac{m}{1+m'}\left[n'\int a'\frac{\partial R}{\partial W'}\right)_0 dt + (C_2)_0\right] - 2\cos\varphi'\delta_1\rho'\right\}dt + (C_3)_0 + \\
&+ n'\int\left(\frac{a'}{r_0'}\right)^2\left\{\frac{m}{1+m'}\left[n'\int a'\delta\frac{\partial R}{\partial W'}\,dt + \delta C_2\right] - 2\delta\rho'\frac{m}{1+m'}\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt + (C_2)_0\right] - \right.\\
&\left. - 2\cos\varphi'\,[\delta_2\rho' - (\delta_1\rho')^2]\right\}dt + \delta C_3.
\end{aligned}\right\} \quad (16)$$

The equations given in the first part of this paper for determining perturbations of the first order will be written as /10

$$\left.\begin{aligned}
&\delta_1\rho' = \frac{m}{1+m'}\left(\frac{a'}{r_0'}\right)^2\left\{qn'\int p\,[a'Q_0 + 2(a'C_1)_0]\,dt - pn'\int q\,[a'Q_0 + 2(a'C_1)_0]\,dt + \right.\\
&\left. + (a'K_1)_0 p + (a'K_2)_0 q\right\},\\
&\delta_1 w' = n'\int\left(\frac{a'}{r_0'}\right)^2\left\{\frac{m}{1+m'}\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt + (C_2)_0\right] - 2\cos\varphi'\delta_1\rho'\right\}dt + (C_3)_0.
\end{aligned}\right\} \quad (17)$$

Subtracting termwise each of these equations from the corresponding Equation (16), we obtain the final expressions for determining perturbations of the second order of the logarithm of the radius vector and longitude in orbit:

$$\left.\begin{aligned}
\delta_2 p' &= \frac{m}{1+m'}\left(\frac{a'}{r'_0}\right)^2\left\{qn'\int p\,[a'\delta Q + 2a'\delta C_1]\,dt - pn'\int q\,[a'\delta Q + 2a'\delta C_1]\,dt + a'\delta K_1 p + a'\delta K_2 q\right\},\\
\delta_2 w' &= n'\int\left(\frac{a'}{r'_0}\right)^2\left\{\frac{m}{1+m'}\left[n'\int a'\delta\frac{\partial R}{\partial W'}\,dt + \delta C_2\right] - \right.\\
&\left. - 2\delta_1 p'\frac{m}{1+m'}\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt + (C_2)_0\right] - 2\cos\varphi'\,[\delta_2 p' - (\delta_1 p')^2]\right\}dt + \delta C_3.
\end{aligned}\right\} \qquad (18)$$

2. Perturbations of the Longitude of the Node and Inclination

In the first part of this paper, we derived formulas for determining perturbations of the longitude of the node and inclination. We will represent them in the following form:

$$\left.\begin{aligned}
\sin i'\frac{d\delta\Omega'}{dt} &= -\frac{m}{\sqrt{1+m'}}\frac{1}{\sqrt{p'}}k\left[\left(\frac{\partial R}{\partial W'}\operatorname{ctn}J + \frac{\partial R}{\partial W}\frac{1}{\sin J}\right)\sin N' + \frac{1}{2}\frac{\partial R}{\partial z}\cos\frac{J}{2}\cos N'\right],\\
\frac{d\delta i'}{dt} &= -\frac{m}{\sqrt{1+m'}}\frac{1}{\sqrt{p'}}k\left[\left(\frac{\partial R}{\partial W'}\operatorname{ctn}J + \frac{\partial R}{\partial W}\frac{1}{\sin J}\right)\cos N' - \frac{1}{2}\frac{\partial R}{\partial z}\cos\frac{J}{2}\sin N'\right]
\end{aligned}\right\} \qquad (19)$$

As usual, we denote: i and i' are the inclinations of the planets to the ecliptic; N and N' the distances of the common node of the planets from the ascending nodes of the inferior and superior planets; p' is a parameter.

We regard $\delta\Omega'$ and $\delta i'$ as the sums of perturbations of the first and second orders. To determine perturbations of longitude of the node and inclination, within an accuracy to magnitudes of the second order, we substitute into Equation (19) instead of p', J, N', and i' their perturbed values:

$$\begin{aligned}
p' &= p'_0 + \delta_1 p',\\
J &= J_0 + \delta_1 J,\\
N' &= N'_0 + \delta_1 N',\\
i' &= i'_0 + \delta_1 i',
\end{aligned}$$

and in the derivatives of the perturbing function we restrict ourselves to the terms:

Note: ctn = cot.

$$\frac{\partial R}{\partial W'}=\left(\frac{\partial R}{\partial W'}\right)_0+\delta\frac{\partial R}{\partial W'},$$

$$\frac{\partial R}{\partial W}=\left(\frac{\partial R}{\partial W}\right)_0+\delta\frac{\partial R}{\partial W},$$

$$\frac{\partial R}{\partial \sigma}=\left(\frac{\partial R}{\partial \sigma}\right)_0+\delta\frac{\partial R}{\partial \sigma},$$

then,

$$\left.\begin{aligned}
&\sin i'_0\delta\Omega'=\sin i'_0(\delta_1\Omega'+\delta_2\Omega')=-\cos i'_0\int\delta_1 i'\frac{d\delta_1\Omega'}{dt}\,dt-\\
&-\frac{m}{1+m'}\sec\varphi' n'\left[\int\left\{\left[\left(a'\frac{\partial R}{\partial W'}\right)_0\operatorname{ctn}J_0+\left(a'\frac{\partial R}{\partial W}\right)_0\frac{1}{\sin J_0}\right]\sin N'_0+\right.\\
&+\frac{1}{2}\left(a'\frac{\partial R}{\partial z}\right)_0\cos\frac{J_0}{2}\cos N'_0\Big\}dt+\int\Big\{\Big[\delta\left(a'\frac{\partial R}{\partial W'}\right)\operatorname{ctn}J_0+\\
&+\delta\left(a'\frac{\partial R}{\partial W}\right)\frac{1}{\sin J_0}\Big]\sin N'_0+\frac{1}{2}\delta\left(a'\frac{\partial R}{\partial z}\right)\cos\frac{J_0}{2}\cos N'_0\Big\}dt-\\
&-\int\Big\{\Big[\left(a'\frac{\partial R}{\partial W'}\right)_0\frac{1}{\sin^2 J_0}+\left(a'\frac{\partial R}{\partial W}\right)_0\frac{\cos J_0}{\sin^2 J_0}\Big]\sin N'_0+\\
&+\frac{1}{4}\left(a'\frac{\partial R}{\partial z}\right)_0\sin\frac{J_0}{2}\cos N'_0\Big\}\delta_1 J dt\Big]+\int\delta_1 N'\frac{d\delta_1 i'}{dt}dt\\
&-\frac{1}{2}\frac{1}{p'_0}\int\delta_1 p'\sin i'_0\frac{d\delta_1\Omega'}{dt}\,dt+(C_3)_0+\delta C_3,\\
&\delta i'=\delta_1 i'+\delta_2 i'=-\frac{m}{1+m'}\sec\varphi' n'\Big[\int\Big\{\Big[\left(a'\frac{\partial R}{\partial W'}\right)_0\operatorname{ctn}J_0+\\
&+\left(a'\frac{\partial R}{\partial W}\right)_0\frac{1}{\sin J_0}\Big]\cos N'_0-\frac{1}{2}\left(a'\frac{\partial R}{\partial z}\right)_0\cos\frac{J_0}{2}\sin N'_0\Big\}dt+\\
&+\int\Big\{\Big[\delta\left(a'\frac{\partial R}{\partial W'}\right)\operatorname{ctn}J_0+\delta\left(a'\frac{\partial R}{\partial W}\right)\frac{1}{\sin J_0}\Big]\cos N'_0-\frac{1}{2}\delta\left(a'\frac{\partial R}{\partial z}\right)\times\\
&\times\cos\frac{J_0}{2}\sin N'_0\Big\}dt-\int\Big\{\Big[\left(a'\frac{\partial R}{\partial W'}\right)_0\frac{1}{\sin^2 J_0}+\left(a'\frac{\partial R}{\partial W}\right)_0\frac{\cos J_0}{\sin^2 J_0}\Big]\times\\
&\times\cos N'_0-\frac{1}{4}\left(a'\frac{\partial R}{\partial z}\right)_0\sin\frac{J_0}{2}\sin N'_0\Big\}\delta_1 J dt\Big]-\int\delta_1 N'\sin i'_0\frac{d\delta_1\Omega'}{dt}dt-\\
&-\frac{1}{2}\frac{1}{p'_0}\int\delta_1 p'\frac{d\delta_1 i'}{dt}dt+(C_4)_0+\delta C_4.
\end{aligned}\right\}\quad(20)\ /11$$

From the Lagrange equations we have the formula for determining perturbations of parameter p':

$$\frac{dp'}{dt}=\frac{2m}{1+m'}\frac{p'}{\sqrt{1-e'^2}}n'\left(a'\frac{\partial R}{\partial \pi'}+a'\frac{\partial R}{\partial \varepsilon'}\right).$$

Hence, taking into account (see Part One, Chapter II) that

$$a' \frac{\partial R}{\partial \pi'} + a' \frac{\partial R}{\partial \varepsilon'} = a' \frac{\partial R}{\partial W'},$$

we obtain

$$\frac{d\delta p'}{dt} = \frac{2m}{1+m'} \frac{p'}{\sqrt{1-e'^2}} n' a' \frac{\partial R}{\partial W'}. \tag{21}$$

Perturbations $\delta_1 J$, $\delta_1 N$, and $\delta_1 N'$ are determined from Equations (43) of Part One:

$$\left.\begin{aligned}
\delta_1 J &= \cos N_0 \delta_1 i - \cos N_0' \delta_1 i' - \sin i_0' \sin N_0' (\delta_1 \Omega' - \delta_1 \Omega),\\
\sin J_0 \delta_1 N &= -\cos J_0 \sin N_0 \delta_1 i + \sin N_0' \delta_1 i - \sin i_0' \cos N_0' (\delta_1 \Omega' - \delta_1 \Omega),\\
\sin J_0 \delta_1 N' &= -\sin N_0 \delta_1 i + \cos J_0 \sin N_0' \delta_1 i' - \cos J_0 \sin i_0' \cos N_0' (\delta_1 \Omega' - \delta_1 \Omega) -\\
&\quad - \sin J_0 \cos i_0' (\delta_1 \Omega' - \delta_1 \Omega).
\end{aligned}\right\} \tag{22}$$

Since

$$\left.\begin{aligned}
\sin i_0' \delta_1 \Omega' &= -\frac{m}{1+m'} \sec \varphi' n' \int \left\{ \left[\left(a' \frac{\partial R}{\partial W'} \right)_0 \operatorname{ctn} J_0 + \left(a' \frac{\partial R}{\partial W} \right)_0 \frac{1}{\sin J_0} \right] \sin N_0' + \right.\\
&\quad \left. + \frac{1}{2} \left(a' \frac{\partial R}{\partial \sigma} \right)_0 \cos \frac{J_0}{2} \cos N_0' \right\} dt + (C_3)_0,\\
\delta_1 i' &= -\frac{m}{1+m'} \sec \varphi' n' \int \left\{ \left[\left(a' \frac{\partial R}{\partial W'} \right)_0 \operatorname{ctn} J_0 + \left(a' \frac{\partial R}{\partial W} \right)_0 \frac{1}{\sin J_0} \right] \cos N_0' - \right.\\
&\quad \left. - \frac{1}{2} \left(a' \frac{\partial R}{\partial \sigma} \right)_0 \cos \frac{J_0}{2} \sin N_0' \right\} dt + (C_4)_0,
\end{aligned}\right\} \tag{23}$$

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then, subtracting termwise Equation (23) from Equation (20), we arrive at formulas determining perturbations of the second order of the longitude of the node and inclination:

$$\left.\begin{aligned}
\sin i_0' \delta_2 \Omega' &= -\cos i_0' \int \delta_1 i' \frac{d\delta_1 \Omega'}{dt} dt - \frac{m}{1+m'} \sec \varphi' n' \left[\int \left\{ \left[\delta \left(a' \frac{\partial R}{\partial W'} \right) \operatorname{ctn} J_0 + \right. \right. \right.\\
&\quad \left. \left. \left. + \delta \left(a' \frac{\partial R}{\partial W} \right) \frac{1}{\sin J_0} \right] \sin N_0' + \frac{1}{2} \delta \left(a' \frac{\partial R}{\partial \sigma} \right) \cos \frac{J_0}{2} \cos N_0' \right\} dt - \right.\\
&\quad - \int \left\{ \left[\left(a' \frac{\partial R}{\partial W'} \right)_0 \frac{1}{\sin^2 J_0} + \left(a' \frac{\partial R}{\partial W} \right)_0 \frac{\cos J_0}{\sin^2 J_0} \right] \sin N_0' + \right.\\
&\quad \left. + \frac{1}{4} \left(a' \frac{\partial R}{\partial \sigma} \right)_0 \sin \frac{J_0}{2} \cos N_0' \right\} \delta_1 J dt \Big] + \int \delta_1 N' \frac{d\delta_1 i'}{dt} dt -\\
&\quad - \frac{1}{2} \frac{1}{p_0'} \int \delta_1 p' \sin i_0' \frac{d\delta_1 \Omega'}{dt} dt + \delta C_3,\\
\delta_2 i' &= -\frac{m}{1+m'} \sec \varphi' n' \left[\int \left\{ \left[\delta \left(a' \frac{\partial R}{\partial W'} \right) \operatorname{ctn} J_0 + \delta \left(a' \frac{\partial R}{\partial W} \right) \frac{1}{\sin J_0} \right] \cos N_0' - \right.\right.\\
&\quad \left. - \frac{1}{2} \delta \left(a' \frac{\partial R}{\partial \sigma} \right) \cos \frac{J_0}{2} \sin N_0' \right\} dt - \int \left\{ \left[\left(a' \frac{\partial R}{\partial W'} \right)_0 \frac{1}{\sin^2 J_0} + \right.\\
&\quad \left. + \left(a' \frac{\partial R}{\partial W} \right)_0 \frac{\cos J_0}{\sin^2 J_0} \right] \cos N_0' - \frac{1}{4} \left(a' \frac{\partial R}{\partial \sigma} \right)_0 \sin \frac{J_0}{2} \sin N_0' \Big\} \delta_1 J dt \Big] -\\
&\quad - \int \delta_1 N' \sin i_0' \frac{d\delta_1 \Omega'}{dt} dt - \frac{1}{2} \frac{1}{p_0'} \int \delta_1 p' \frac{d\delta_1 i'}{dt} dt + \delta C_4.
\end{aligned}\right\} \tag{24}$$

3. Perturbations of Latitude and Longitude in the Ecliptic

In Figure 1, PQ = b' is the latitude of the perturbed planet; $\gamma Q = l'$ is the longitude in the ecliptic; $x\Omega' = \theta'$ is the distance from a fixed point to the ascending node of the planet. In this case:

$$\theta'_0 = \Omega'_0$$

and

$$\frac{d\theta'}{dt} = \cos i' \frac{d\Omega'}{dt}.$$

From triangle $\Omega'PQ$ we have:

$$\begin{aligned} \sin b' &= \sin i' \sin (w' - \theta'), \\ \cos b' \sin (l' - \Omega') &= \cos i' \sin (w' - \theta'), \\ \cos b' \cos (l' - \Omega') &= \cos (w' - \theta'). \end{aligned} \tag{25}$$

If we substitute into these formulas

$$\begin{aligned} b' &= b'_0 + \delta b', \\ i' &= i'_0 + \delta i', \\ w' &= w'_0 + \delta w', \\ \theta' &= \Omega'_0 + \int \cos i' \frac{d\delta\Omega'}{dt} dt, \\ w'_0 - \theta'_0 &= w'_0 - \Omega'_0 = u'_0 \end{aligned}$$

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and restrict ourselves to terms of the second order, then we arrive at formulas determining perturbations of latitude and longitude in the ecliptic.

The first equation of (25) gives

$$\begin{aligned} \cos b'_0 \delta b' = {} & \frac{1}{2} \sin b'_0 (\delta_1 b')^2 + \sin u'_0 \cos i'_0 \delta i' + \cos u'_0 \sin i'_0 \delta w' - \cos u'_0 \cos i'_0 \sin i'_0 \delta\Omega' + \\ & + \cos u'_0 \sin^2 i'_0 \int \delta_1 i' \frac{d\delta_1\Omega'}{dt} dt - \frac{1}{2} \sin u'_0 \sin i'_0 (\delta_1 i')^2 - \frac{1}{2} \sin u'_0 \sin i'_0 (\delta_1 w')^2 - \\ & - \frac{1}{2} \sin u'_0 \sin i'_0 \cos^2 i'_0 (\delta_1\Omega')^2 + \cos u'_0 \cos i'_0 \delta_1 i' \delta_1 w' + \\ & + \sin u'_0 \sin i'_0 \cos i'_0 \delta_1\Omega' \delta_1 w' - \cos u'_0 \cos^2 i'_0 \delta_1 i' \delta_1\Omega'. \end{aligned} \tag{26}$$

From the last two equations of (25) we can obtain

$$\cos^2 b_0'\delta l' = \sin^2 i_0' \cos^2 u_0'\delta\Omega' + \cos i_0'\delta w' -$$
$$- \sin i_0' \sin u_0' \cos u_0' \delta i' + \sin b_0' \cos b_0'\delta_1 b' (\delta_1 l' - \delta_1\Omega') +$$
$$+ \cos i_0' \sin i_0' \int \delta_1 i' \frac{d\delta_1\Omega'}{dt} dt - \sin i_0' \cos^2 u_0'\delta_1 i'\delta_1 w' + \qquad (27)$$
$$+ \sin i_0' \cos i_0' \cos^2 u_0'\delta_1 i'\delta_1\Omega' -$$
$$- \frac{1}{2} \cos i_0' \sin u_0' \cos u_0' (\delta_1 i')^2.$$

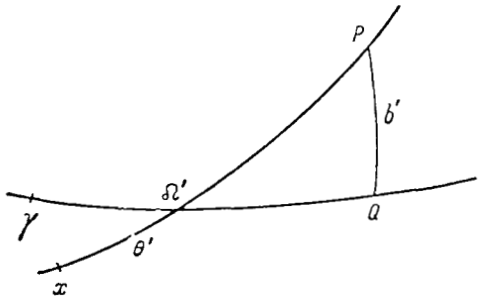

Figure 1.

Taking into account that

$$\cos b_0'\delta_1 b' = \sin u_0' \cos i_0'\delta_1 i' + \cos u_0' \sin i_0'\delta_1 w' -$$
$$\cos i_0' \cos u_0' \sin i_0'\delta_1\Omega',$$
$$\cos^2 b_0'\delta_1 l' = \sin^2 i_0' \cos^2 u_0'\delta_1\Omega' + \cos i_0'\delta_1 w' -$$
$$\sin i_0' \sin u_0' \cos u_0'\delta_1 i',$$

for determining second-order perturbations of latitude and longitude in the ecliptic, we obtain the following formulas:

$$\left.\begin{aligned}
\cos b_0'\delta_2 b' &= \frac{1}{2} \sin b_0' (\delta_1 b')^2 + \sin u_0' \cos i_0'\delta_2 i' + \cos u_0' \sin i_0'\delta_2 w' - \cos u_0' \cos i_0' \sin i_0'\delta_2\Omega' + \\
&+ \cos u_0' \sin^2 i_0' \int \delta_1 i' \frac{d\delta_1\Omega'}{dt} dt - \frac{1}{2} \sin b_0' (\delta_1 i')^2 - \frac{1}{2} \sin b_0' (\delta_1 w')^2 - \\
&- \frac{1}{2} \sin b_0' \cos^2 i_0' (\delta_1\Omega')^2 + \cos u_0' \cos i_0' \delta_1 i'\delta_1 w' + \sin u_0' \cos i_0' \sin i_0'\delta_1\Omega'\delta_1 w' - \\
&- \cos u_0' \cos^2 i_0'\delta_1 i'\delta_1\Omega', \\
\cos^2 b_0'\delta_2 l' &= \sin^2 i_0' \cos^2 u_0'\delta_2\Omega' + \cos i_0'\delta_2 w' - \sin i_0' \sin u_0' \cos u_0'\delta_2 i' + \\
&+ \sin b_0' \cos b_0'\delta_1 b' (\delta_1 l' - \delta_1\Omega') + \cos i_0' \sin i_0' \int \delta_1 i' \frac{d\delta_1\Omega'}{dt} dt - \\
&- \sin i_0' \cos^2 u_0'\delta_1 i'\delta_1 w' + \sin i_0' \cos i_0' \cos^2 u_0'\delta_1 i'\delta_1\Omega' - \frac{1}{2} \cos i_0' \sin u_0' \cos u_0' (\delta_1 i')^2.
\end{aligned}\right\} \qquad (28)$$

4. Right-hand Members of the Equations

Right-hand members of Equations (18), (24), and (28), determining perturbations of the second order, primarily contain terms of the derivatives of the perturbing function which depend on magnitudes of the second order of the perturbing

masses. Let us examine these expressions in greater detail. For an example we will take δR:

$$\delta R = \left(\frac{\partial R}{\partial \rho'}\right)_0 \delta\rho' + \left(\frac{\partial R}{\partial \rho}\right)_0 \delta\rho + \left(\frac{\partial R}{\partial W'}\right)_0 \delta W' + \left(\frac{\partial R}{\partial W}\right)_0 \delta W + \left(\frac{\partial R}{\partial \sigma}\right)_0 \delta\sigma. \tag{29}$$

Perturbations of longitudes W' and W enter Formula (29). As is known, W' and W are longitudes of planets calculated from the common node of the perturbed and perturbing planets. At the same time theories of planetary motions give us perturbations of longitudes in orbit δw' and δw; w' and w are calculated from a fixed point in the orbit. /14

In Figure 2

$$w' = x'P', \quad W' = JP,$$
$$w = xP, \quad W = JP.$$

It is known that

$$\frac{d\theta'}{dt} = \cos i' \frac{d\Omega'}{dt}, \quad \frac{d\theta}{dt} = \cos i \frac{d\Omega}{dt}.$$

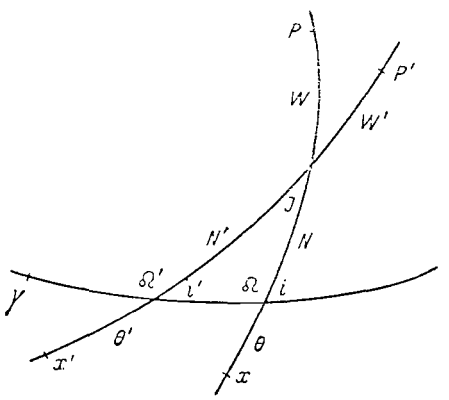

Figure 2.

With an accuracy to magnitudes of the first order, considering that for unperturbed motion,

$$\theta'_0 = \Omega'_0, \quad \theta_0 = \Omega_0,$$

we have

$$\theta' = \Omega'_0 + \cos i'_0 \delta_1 \Omega',$$
$$\theta = \Omega_0 + \cos i_0 \delta_1 \Omega,$$

then,

$$W' = w' - N' - \Omega'_0 - \cos i'_0 \delta\Omega',$$
$$W = w - N - \Omega_0 - \cos i_0 \delta\Omega,$$

Whence

$$\delta W' = \delta w' - \delta N' - \cos i'_0 \delta\Omega',$$
$$\delta W = \delta w - \delta N - \cos i_0 \delta\Omega.$$

After substituting these values of $\delta W'$ and δW in Equation (29), we obtain

$$\delta R = \left(\frac{\partial R}{\partial p'}\right)_0 \delta p' + \left(\frac{\partial R}{\partial p}\right)_0 \delta p + \left(\frac{\partial R}{\partial W'}\right)_0 \delta w' + \left(\frac{\partial R}{\partial W}\right)_0 \delta w - \left(\frac{\partial R}{\partial W'}\right)_0 (\delta N' + \cos i' \delta \Omega') - \\ - \left(\frac{\partial R}{\partial W}\right)_0 (\delta N + \cos i \delta \Omega) + \left(\frac{\partial R}{\partial \sigma}\right)_0 \delta \sigma. \tag{30}$$

We denote the last three addends of Equation (30) by I and substitute into I instead of $\delta N'$, δN, and $\delta\sigma = \frac{1}{2}\cos\frac{J}{2}\delta J$, their expressions from Equations (22).

Then,

$$I = -\frac{\partial R}{\partial W'}\left[-\csc J_0 \sin N_0 \delta i + \operatorname{ctn} J_0 \sin N_0' \delta i' + \left(-\frac{\sin i_0 \cos N_0}{\sin J_0} + \cos i_0'\right)\delta\Omega' + \right.$$
$$\left. + \frac{\sin i_0 \cos N_0}{\sin J_0}\delta\Omega\right] - \frac{\partial R}{\partial W}\left[-\operatorname{ctn} J_0 \sin N_0 \delta i + \csc J_0 \sin N_0' \delta i' - \frac{\sin i_0' \cos N_0'}{\sin J_0}\delta\Omega' + \right.$$
$$\left. + \left(\cos i_0 + \frac{\sin i_0' \cos N_0'}{\sin J_0}\right)\delta\Omega\right] + \frac{1}{2}\cos\frac{J}{2}\frac{\partial R}{\partial \sigma}\left[\cos N_0 \delta i - \cos N_0' \delta i' - \right.$$
$$\left. - \sin i_0' \sin N_0' \delta\Omega' + \sin i_0' \sin N_0' \delta\Omega\right]$$

or, after certain transformations,

$$a' I = \left[\left(a'\frac{\partial R}{\partial W'}\csc J_0 + a'\frac{\partial R}{\partial W}\operatorname{ctn} J_0\right)\sin N_0 + \frac{1}{2}\cos\frac{J_0}{2}a'\frac{\partial R}{\partial \sigma}\cos N_0\right]\delta i + \tag{31}$$
$$+ \left[-\left(a'\frac{\partial R}{\partial W'}\csc J_0 + a'\frac{\partial R}{\partial W}\operatorname{ctn} J_0\right)\cos N_0 + \frac{1}{2}\cos\frac{J_0}{2}a'\frac{\partial R}{\partial \sigma}\sin N_0\right]\sin i_0 \delta\Omega -$$
$$- \left[\left(a'\frac{\partial R}{\partial W'}\operatorname{ctn} J_0 + a'\frac{\partial R}{\partial W}\csc J_0\right)\sin N_0' + \frac{1}{2}\cos\frac{J_0}{2}a'\frac{\partial R}{\partial \sigma}\cos N_0'\right]\delta i' +$$
$$+ \left[\left(a'\frac{\partial R}{\partial W'}\operatorname{ctn} J_0 + a'\frac{\partial R}{\partial W}\csc J_0\right)\cos N_0' - \frac{1}{2}\cos\frac{J_0}{2}a'\frac{\partial R}{\partial \sigma}\sin N_0'\right]\sin i_0' \delta\Omega'.$$

We denote /15

$$A = a'\frac{\partial R}{\partial W'}\operatorname{ctn} J_0 + a'\frac{\partial R}{\partial W}\csc J_0,$$
$$A' = a'\frac{\partial R}{\partial W'}\csc J_0 + a'\frac{\partial R}{\partial W}\operatorname{ctn} J_0,$$
$$B = \frac{1}{2}\cos\frac{J_0}{2}a'\frac{\partial R}{\partial \sigma}$$

and proceed to determine the order of magnitude of a'I.

Assuming, for simplicity, e = e' = 0, and limiting ourselves in the expansion of the main part of the perturbation function to terms of the second order relative to σ, we have:

$$a'R = \frac{1}{2}\sum a'A_i \cos(iL' - iL) + \sigma^2 \sum a'B_i \cos[(i+1)L' - (i-1)L],$$

where

$$a'A_i = c_1^i - \frac{1}{2}\sigma^2 (c_3^{i+1} + c_3^{i-1}),$$
$$a'B_i = \frac{1}{2} c_3^i.$$

From this,

$$a' \frac{\partial R}{\partial W'} = a' \frac{\partial R}{\partial L'} = -\frac{1}{2}\sum \left[ic_1^i - \frac{1}{2} i\sigma^2 (c_3^{i+1} + c_3^{i-1}) \right] \sin(iL' - iL) -$$
$$- \frac{1}{2}\sum \sigma^2 (i+1) c_3^i \sin[(i+1)L' - (i-1)L],$$
$$a' \frac{\partial R}{\partial W} = a' \frac{\partial R}{\partial L} = \frac{1}{2}\sum \left[ic_1^i - \frac{1}{2} i\sigma^2 (c_3^{i+1} + c_3^{i-1}) \right] \sin(iL' - iL) +$$
$$+ \frac{1}{2}\sum \sigma^2 (i-1) c_3^i \sin[(i+1)L' - (i-1)L],$$
$$a' \frac{\partial R}{\partial \sigma} = -\frac{1}{2}\sum \sigma (c_3^{i+1} + c_3^{i-1}) \cos(iL' - iL) +$$
$$+ \sum \sigma c_3^i \cos[(i+1)L' - (i-1)L].$$

Then,

$$A = -\frac{\sigma^2}{2\sigma\sqrt{1-\sigma^2}} \sum c_3^i \sin[(i+1)L' - (i-1)L] + \sigma^2 \sum ic_1^i \sin(iL' - iL),$$
$$A' = -\frac{\sigma^2}{2\sigma\sqrt{1-\sigma^2}} \sum c_3^i \sin[(i+1)L' - (i-1)L] - \sigma^2 \sum ic_1^i \sin(iL' - iL),$$
$$B = \frac{1}{2}\sqrt{1-\sigma^2}\,\sigma \left\{ -\frac{1}{2}\sum (c_3^{i+1} + c_3^{i-1}) \cos(iL' - iL) + \right.$$
$$\left. + \sum c_3^i \cos[(i+1)L' - (i-1)L] \right\}.$$

The Laplace coefficients are determined by the following series:

$$\frac{1}{2} c_n^i = \frac{n(n+2)\ldots(n+2i-2)}{2\cdot 4\ldots 2i} \alpha^i \alpha^{\frac{n-1}{2}} \left(1 + \frac{n}{2}\frac{n+2i}{2i+2}\alpha^2 + \ldots\right),$$
$$\frac{1}{2} c_n^0 = \alpha^{\frac{n-1}{2}} \left[1 + \left(\frac{n}{2}\right)^2 \alpha^2 + \ldots\right].$$

Consequently:

1) The first term in A and A' is a quantity of order $\sigma\alpha$;

2) The second term in A and A' is of order $\sigma^2\alpha$ (in this case i cannot be equal to zero);

3) The first and second terms in B are quantities of order $\sigma\alpha$.

Thus, A, A', and B are quantities of order $\sigma\alpha$.

Equations (23) can be rewritten (omitting the constants of integration) as: /16

$$\delta i' = -\frac{m}{1+m'}\sec\varphi' n' \int (A\cos N' - B\sin N')\,dt,$$

$$\sin i'\delta\Omega' = -\frac{m}{1+m'}\sec\varphi' n' \int (A\sin N' + B\cos N')\,dt.$$

We have:

$$n'\int A dt = \frac{\sigma}{2\sqrt{1-\sigma^2}} \sum \frac{n'}{(i+1)n' - (i-1)n} c_3^i \cos[(i+1)L' - (i-1)L] +$$
$$+ \sigma^2 \sum_{i\neq 0} \frac{in'}{in' - in} c_1^i \cos(iL' - iL),$$

$$n'\int B dt = \frac{1}{4}\sigma\sqrt{1-\sigma^2}\left\{-\sum_{i\neq 0}\frac{n'}{in'-in}(c_3^{i+1}+c_3^{i-1})\sin(iL'-iL) + \right.$$
$$\left. + \sum \frac{n'}{(i+1)n' - (i-1)n} c_3^i \sin[(i+1)L' - (i-1)L] - 2c_3^1 n' t\right\}.$$

Hence we conclude:

1) The first term of $n'\int$ Adt for i = 0 will be of order $\sigma\alpha^{5/2}$, and for $i \neq 0$, $\sigma\alpha^2$;

2) The second term of $n'\int$ Adt will be of order $\sigma^2\alpha^{5/2}$;

3) The first term of $n'\int$ Bdt will be of order $\sigma\alpha^{7/2}$;

4) The second term of $n'\int$ Bdt for i = 0 will be of order $\sigma\alpha^{5/2}$ and for $i \neq 0$, $\sigma\alpha^2$;

5) The third term of $n'\int$ Bdt will be of order $\sigma\alpha^2$. We see that $n'\int$ Adt and $n'\int$ Bdt, and hence $\delta i'$ and $\sin i'\delta\Omega'$ will be of order $\sigma\alpha^2$. A similar analysis shows that δi and $\sin i\delta\Omega$ will also be of order $\sigma_1\alpha_1^2$, where α_1 are the ratios of the semimajor axes of planet P and of planet P_1 perturbing it, and $\sigma_1 = \sin\frac{J_1}{2}$, where J_1 is the inclination of the orbit of P_1 to the orbit of P.

From the above we have reasons to believe that in $a'\delta R$ the term a'I will be a quantity at least of the order $\sigma^2\alpha^3$.

In the derivatives $a'\delta\frac{\partial R}{\partial W'}$, $a'\delta\frac{\partial R}{\partial W}$, $a'\delta\frac{\partial R}{\partial\varphi'}$, $a'\delta\frac{\partial R}{\partial\varphi}$ this term also gives a

quantity of order $\sigma^2\alpha^3$; in derivative $a'\delta\frac{\partial R}{\partial\sigma}$ this term gives a quantity of order $\sigma\alpha^3$ and, when determining perturbations of the second order in $a'\delta R$, $\frac{\partial R}{\partial\rho'}$, and other derivatives, we can limit ourselves to terms:

$$
\left.\begin{aligned}
a'\delta R &= a'\frac{\partial R}{\partial\rho'}\delta\rho' + a'\frac{\partial R}{\partial\rho}\delta\rho + a'\frac{\partial R}{\partial W'}\delta w' + a'\frac{\partial R}{\partial W}\delta w,\\
a'\delta\frac{\partial R}{\partial\rho'} &= a'\frac{\partial^2 R}{\partial\rho'^2}\delta\rho' + a'\frac{\partial^2 R}{\partial\rho'\partial\rho}\delta\rho + a'\frac{\partial^2 R}{\partial\rho\partial W'}\delta w' + a'\frac{\partial^2 R}{\partial\rho'\partial W}\delta w,\\
a'\delta\frac{\partial R}{\partial\rho} &= a'\frac{\partial^2 R}{\partial\rho\partial\rho'}\delta\rho' + a'\frac{\partial^2 R}{\partial^2\rho}\delta\rho + a'\frac{\partial^2 R}{\partial\rho\partial W'}\delta w' + a'\frac{\partial^2 R}{\partial\rho\partial W}\delta w,\\
a'\delta\frac{\partial R}{\partial W'} &= a'\frac{\partial^2 R}{\partial W'\partial\rho'}\delta\rho' + a'\frac{\partial^2 R}{\partial W'\partial\rho}\delta\rho + a'\frac{\partial^2 R}{\partial W'^2}\delta w' + a'\frac{\partial^2 R}{\partial W'\partial W}\delta w,\\
a'\delta\frac{\partial R}{\partial W} &= a'\frac{\partial^2 R}{\partial W\partial\rho'}\delta\rho' + a'\frac{\partial^2 R}{\partial W\partial\rho}\delta\rho + a'\frac{\partial^2 R}{\partial W\partial W'}\delta w' + a'\frac{\partial^2 R}{\partial W^2}\delta w,\\
a'\delta\frac{\partial R}{\partial\sigma} &= a'\frac{\partial^2 R}{\partial\sigma\partial\rho'}\delta\rho' + a'\frac{\partial^2 R}{\partial\sigma\partial\rho}\delta\rho + a'\frac{\partial^2 R}{\partial\sigma\partial W'}\delta w' + a'\frac{\partial^2 R}{\partial\sigma\partial W}\delta w,\\
a'\delta D_t' R &= D_t'\left[a'\frac{\partial R}{\partial\rho'}\delta\rho' + a'\frac{\partial R}{\partial W'}\delta w'\right] + a'\frac{\partial R}{\partial\rho'}\frac{d\delta\rho'}{dM}\frac{dM}{dt} +\\
&+ a'\frac{\partial R}{\partial W'}\frac{d\delta w'}{dM}\frac{dM}{dt} + \left(D_t' a'\frac{\partial R}{\partial\rho}\right)\delta\rho + \left(D_t' a'\frac{\partial R}{\partial W}\right)\delta w.
\end{aligned}\right\} \qquad (32)
$$

Determination of the second derivatives of the perturbing function entering these formulas is not particularly difficult, and in Section 9 we will discuss it in greater detail. /17

Perturbations of the logarithms of the radius vectors, and longitudes in the orbit of the perturbed and perturbing planets, will be taken from the corresponding theories of determining perturbations of the first order. Thus, determination of the corrections to the perturbing function and to its derivatives reduces to multiplying series with several arguments.

Right-hand members of the equations for determining perturbations of the second order of the logarithm of the radius vector and longitude in orbit also contain the expressions $\frac{d^2}{(dn't)^2}\left(\frac{r_0'}{a'}\delta_1\rho'\right)^2$, $\frac{1}{2}\left(\frac{r_0'}{a'}\right)^{-1}\delta_1\rho'^2$, $2\delta_1\rho'\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt + C^2\right]$; with expansions of $\left(\frac{r_0'}{a'}\right)^2$ and $\left(\frac{r_0'}{a'}\right)^{-1}$ with respect to mean anomalies, the determination of these quantities also reduces to multiplication of series.

An analysis of Equations (24) shows that $\delta_2 i'$ and $\sin i'\delta_2\Omega'$ will be quantities at least of order $m^2\sigma\alpha$. Consequently, when determining perturbations of latitude and longitude in the ecliptic, we can disregard second-order perturbations of the longitude of the node and inclination.

After comparison with observations and calculations of the ephemerides, the perturbed values of latitude and longitude in the ecliptic can be obtained

from the formulas:

$$\begin{aligned} \sin b' &= \sin(i'_0 + \delta_1 i')\sin(w'_0 + \delta_1 w' + \delta_2 w' - \Omega'_0 - \cos i'_0 \delta_1 \Omega'), \\ \tan(l' - \Omega'_0 - \delta_1 \Omega') &= \cos(i'_0 + \delta_1 i')\tan(w'_0 + \delta_1 w' + \delta_2 w' - \Omega'_0 - \cos i'_0 \delta_1 \Omega'). \end{aligned} \quad (33)$$

5. Determination of the Constants of Integration

When determining perturbations of the second order of the logarithm of the radius vector and longitude in orbit by Formulas (18), we have five constants of integrations $a'\delta C_1$, $a'\delta K_1$, $a'\delta K_2$, δC_2, and δC_3. As in the case of perturbations of the first order, between the constants there is a relation which follows from the energy integral. Unlike the first order, this relation is not so simple.

From the energy integral of perturbed motion:

$$\left(\frac{dr'}{dt}\right)^2 + r'^2\left(\frac{dw'}{dt}\right)^2 - \frac{2k^2(1+m')}{r'} = 2k^2 m \int D'_t R dt + C_0$$

we subtract the integral for unperturbed motion:

$$\left(\frac{dr'_0}{dt}\right)^2 + r'^2_0\left(\frac{dw'_0}{ds}\right)^2 - \frac{2k^2(1+m')}{r'_0} = -\frac{k^2(1+m')}{a'}.$$

Then after certain transformations, restricting ourselves to terms of the second order, we arrive at the relation:

$$\begin{aligned} &\left(\frac{3a'}{r'_0} - 1\right)\delta\rho' + \frac{e'}{n'}\sin E'\frac{d\delta\rho'}{dt} + \frac{\cos\varphi'}{n'}\frac{d\delta w'}{ds} + \frac{1}{2}\left(\frac{a'}{r'_0} - 1\right)\delta\rho'^2 + \frac{e'}{n'}\sin E'\frac{d\delta\rho'}{dt}\delta\rho' + \\ &+ \frac{2\cos\varphi'}{n'}\frac{d\delta w'}{dt}\delta\rho' + \frac{1}{2n'^2}\left(\frac{r'_0}{a'}\right)^2\left[\left(\frac{d\delta\rho'}{dt}\right)^2 + \left(\frac{d\delta w'}{dt}\right)^2\right] = \\ &= \frac{m}{1+m'}\left[\int (a'D'_t R_0 + a'\delta D'_t R)\,dt + (a'C_1)_0 + \delta a' C_1\right]. \end{aligned} \quad (34)$$

In Part One (Section 8) the following relation was obtained for perturbations of the first order:

$$\left(\frac{3a'}{r_0'}-1\right)\delta_1\rho'+\frac{e'}{n'}\sin E'\frac{d\delta_1\rho'}{dt}+\frac{\cos\varphi'}{n'}\frac{d\delta_1 w'}{dt}=\frac{m}{1+m'}\left[\int a'D_t'R_0dt+(a'C_1)_0\right]. \tag{35}$$

Substituting in Equations (34), as usual $\delta\rho' = \delta_1\rho' + \delta_2\rho'$ and $\delta w' = \delta_1 w' + \delta_2 w'$, and subtracting from Equations (34) termwise Equation (35), we obtain an equation which relates the constants of integration of second-order perturbations: /18

$$\begin{aligned}\left(\frac{3a'}{r_0'}-1\right)\delta_2\rho'+\frac{e'}{n'}\sin E'\frac{d\delta_2\rho'}{dt}+\frac{\cos\varphi'}{n'}\frac{d\delta_2 w'}{dt}+\frac{1}{2}\left(\frac{a'}{r_0'}-1\right)(\delta_1\rho')^2+\\ +\frac{e'}{n'}\sin E'\frac{d\delta_1\rho'}{dt}\delta_1\rho'+\frac{2\cos\varphi'}{n'}\frac{d\delta_1 w'}{dt}\delta_1\rho'+\\ +\frac{1}{2n'^2}\left(\frac{r_0'}{a'}\right)^2\left[\left(\frac{d\delta_1\rho'}{dt}\right)^2+\left(\frac{d\delta_1 w'}{dt}\right)^2\right]=\frac{m}{1+m'}\left[\int a'\delta D_t'Rdt+\delta a'C_1\right].\end{aligned} \tag{36}$$

As in the case of first-order perturbations, the constants of integration are determined from the condition that at moment $t = t_0$ (moment of osculation), the perturbations and derivatives of perturbations are equal to zero.

From Equations (34), (18), and (22), it is easy to obtain the following values of the constants:

$$\left.\begin{aligned}a'\delta C_1&=-\left[\int a'\delta D_t'Rdt\right]_{t=t_0},\\ a'\delta K_1&=\left[n'\int q\,(a'\delta Q+2\delta a'C_1)\,dt\right]_{t=t_0},\\ a'\delta K_2&=-\left[n'\int p\,(a'\delta Q+2\delta a'C_1)\,dt\right]_{t=t_0},\\ \delta C_2&=-\left[n'\int a'\delta\frac{\partial R}{\partial W'}dt\right]_{t=t_0},\\ \delta C_3&=-\left[n'\int\left(\frac{a'}{r_0'}\right)^2\left\{\frac{m}{1+m'}\left[n'\int a'\delta\frac{\partial R}{\partial W'}dt+\delta C_2\right]-\right.\right.\\ &\left.\left.-2\delta_1\rho'\frac{m}{1+m'}\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0dt+(C_2)_0\right]-2\cos\varphi'\left[\delta_2\rho'-(\delta_1\rho')^2\right]\right\}dt\right]_{t=t_0}.\end{aligned}\right\} \tag{37}$$

6. Long-period Terms

Additional difficulties in constructing theories of planetary motion are caused by pronounced covariances of mean motions. Their presence leads to a

great loss of accuracy in integration. In order to maintain a high accuracy in the final series of perturbations, it is necessary either to take a larger number of terms in the expansion of the perturbing function, or resort to a somewhat artificial method. For example, with pronounced covariances, the periods of which are not less than several thousands of years, the long-period perturbations can be represented as secular. This can be done in the following manner.

Let us assume that a pronounced covariance arises in the combination $j'n' + jn = \nu$. We will select in $a'D_t'R = a'D_t'R_0 + a'\delta D_t'R$ a term depending on ν:

$$\left(\frac{1}{n'}a'D_t'R\right)_\nu = A\sin(\nu t + C_0) + B\cos(\nu t + C_0) + n't C\cos(\nu t + C_0) + n't D\cos(\nu t + C_0),$$

where $C_0 = j'M_0' + jM_0$, A, B, C, and D are constants depending on the coefficients of development of the perturbing function.

We expand $\left(\frac{1}{n'}a'D_t'R\right)_\nu$ in a series in powers of νt, and after certain transformations we obtain

$$\begin{aligned}\frac{1}{n'}a'D_t'R = {} & A\sin C_0 + B\cos C_0 + \\ & + \left[\left(C - \frac{\nu}{n'}B\right)\sin C_0 + \left(D + \frac{\nu}{n'}A\right)\cos C_0\right]n't + \\ & + \left\{-\left[\frac{\nu}{n'}D + \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 A\right]\sin C_0 + \left[\frac{\nu}{n'}C - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 B\right]\cos C_0\right\}(n't)^2 + \\ & + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\end{aligned}$$

Integration gives /19

$$\begin{aligned}2\int a'D_t'R\,dt = {} & 2\left[A\sin C_0 + B\cos C_0\right]n't + \left[\left(C - \frac{\nu}{n'}B\right)\sin C_0 + \left(D + \frac{\nu}{n'}A\right)\cos C_0\right](n't)^2 + \\ & + \left\{-\frac{1}{3}\left[2\frac{\nu}{n'}D + \left(\frac{\nu}{n'}\right)^2 A\right]\sin C_0 + \frac{1}{3}\left[2\frac{\nu}{n'}C - \left(\frac{\nu}{n'}\right)^2 B\right]\cos C_0\right\}(n't)^3 + \\ & + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (*)\end{aligned}$$

An analogous term depending on ν in

$$\left\{\left(a'\frac{\partial R}{\partial \rho'}\right)_0 + a'\delta\frac{\partial R}{\partial \rho'} + \frac{1+m'}{m}\left[-\frac{d^2}{(dn't)^2}\left(\frac{r_0'}{a'}\delta\rho'\right)^2 + \frac{1}{2}\left(\frac{r_0'}{a'}\right)^{-1}(\delta_1\rho')^2\right]\right\}$$

will be

$$\{\dots\dots\}_\nu = A'\sin C_0 + B'\cos C_0 +$$
$$+ \left[\left(C' - \frac{\nu}{n'}B'\right)\sin C_0 + \left(D' + \frac{\nu}{n'}A'\right)\cos C_0\right] n't +$$
$$+ \left\{-\left[\frac{\nu}{n'}D' + \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 A'\right]\sin C_0 + \left[\frac{\nu}{n'}C' - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 B'\right]\cos C_0\right\}(n't)^2 +$$
$$+ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

Then,

$$a'Q = a'Q_0 + a'\delta Q = a_0 + a_1 n't + a_2 (n't)^2 + a_3 (n't)^3 + \dots,$$

while

$$\left.\begin{aligned}
a_0 &= A'\sin C_0 + B'\cos C_0,\\
a_1 &= \left(2A + C' - \tfrac{\nu}{n'}B'\right)\sin C_0 + \left(2B + D' + \tfrac{\nu}{n'}A'\right)\cos C_0,\\
a_2 &= \left[C - \tfrac{\nu}{n'}B - \tfrac{\nu}{n'}D' - \tfrac{1}{2}\left(\tfrac{\nu}{n'}\right)^2 A'\right]\sin C_0 + \left[D + \tfrac{\nu}{n'}A + \tfrac{\nu}{n'}C' - \tfrac{1}{2}\left(\tfrac{\nu}{n'}\right)^2 B'\right]\cos C_0,\\
a_3 &= \left[-\tfrac{2}{3}\tfrac{\nu}{n'}D - \tfrac{1}{3}\left(\tfrac{\nu}{n'}\right)^2 A - \tfrac{1}{2}\left(\tfrac{\nu}{n'}\right)^2 C' + \tfrac{1}{6}\left(\tfrac{\nu}{n'}\right)^3 B'\right]\sin C_0 +\\
&+ \left[\tfrac{2}{3}\tfrac{\nu}{n'}C - \tfrac{1}{3}\left(\tfrac{\nu}{n'}\right)^2 B - \tfrac{1}{2}\left(\tfrac{\nu}{n'}\right)^2 D' - \tfrac{1}{6}\left(\tfrac{\nu}{n'}\right)^3 A'\right]\cos C_0,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots
\end{aligned}\right\} \quad (38)$$

Thus, the terms depending on ν isolated from a'Q give the following correction for $\delta\rho'$:

$$(\Delta\delta\rho') = \frac{m}{1+m'}\left(\frac{a'}{r_0'}\right)^2\left[a_0\left(qn'\int pdt - pn'\int qdt\right) + a_1\left(qn'\int n'tpdt - pn'\int n'tqdt\right) +\right.$$
$$+ a_2\left(qn'\int (n't)^2 pdt - pn'\int (n't)^2 qdt\right) + a_3\left(qn'\int (n't)^3 pdt - pn'\int (n't)^3 qdt\right) +$$
$$+ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\left. + a'\Delta K_1 p + a'\Delta K_2 q\right]. \quad (39)$$

But this correction to $\delta\rho'$ does not contain all long-period perturbations of the logarithm of the radius vector which depend on ν. The long-period terms appear again, after multiplying the periodic part of a'Q (terms of a'Q from which we select terms depending on ν) by series p and q. They will have

the following form:

$$(pa'Q)_\nu = A_1 \sin(\nu t + C_0) + B_1 \cos(\nu t + C_0) + C_1 n' t \sin(\nu t + C_0) + D_1 n' t \cos(\nu t + C_0),$$
$$(qa'Q)_\nu = A_2 \sin(\nu t + C_0) + B_2 \cos(\nu t + C_0) + C_2 n' t \sin(\nu t + C_0) + D_2 n' t \cos(\nu t + C_0),$$

or, after transformations,

$$(pa'Q)_\nu = b_0 + b_1 n' t + b_2 (n't)^2 + \ldots,$$
$$(qa'Q)_\nu = c_0 + c_1 n' t + c_2 (n't)^2 + \ldots$$

Here /20

$$\left.\begin{aligned}
b_0 &= A_1 \sin C_0 + B_1 \cos C_0,\\
b_1 &= \left(C_1 - \frac{\nu}{n'} B_1\right)\sin C_0 + \left(D_1 + \frac{\nu}{n'} A_1\right)\cos C_0,\\
b_2 &= -\left[\frac{\nu}{n'} D_1 + \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 A_1\right]\sin C_0 + \left[\frac{\nu}{n'} C_1 - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 B_1\right]\cos C_0,\\
b_3 &= \left[-\frac{1}{2}\left(\frac{\nu}{n'}\right)^2 C_1 + \frac{1}{6}\left(\frac{\nu}{n'}\right)^3 B_1\right]\sin C_0 - \left[\frac{1}{2}\left(\frac{\nu}{n'}\right)^2 D_1 + \frac{1}{6}\left(\frac{\nu}{n'}\right)^3 A_1\right]\cos C_0,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
c_0 &= A_2 \sin C_0 + B_2 \cos C_0,\\
c_1 &= \left(C_2 - \frac{\nu}{n'} B_2\right)\sin C_0 + \left(D_2 + \frac{\nu}{n'} A_2\right)\cos C_0,\\
c_2 &= -\left[\frac{\nu}{n'} D_2 + \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 A_2\right]\sin C_0 + \left[\frac{\nu}{n'} C_2 - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 B_2\right]\cos C_0,\\
c_3 &= \left[-\frac{1}{2}\left(\frac{\nu}{n'}\right)^2 C_2 + \frac{1}{6}\left(\frac{\nu}{n'}\right)^3 B_2\right]\sin C_0 - \left[\frac{1}{2}\left(\frac{\nu}{n'}\right)^2 D_2 + \frac{1}{6}\left(\frac{\nu}{n'}\right)^3 A_2\right]\cos C_0,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}\right\} \quad (40)$$

Then the correction to $\delta\rho'$, depending on $(pa'Q)_\nu$ and $(qa'Q)_\nu$ will have the form:

$$\Delta'\delta\rho' = \frac{m}{1+m'}\left(\frac{a}{r_0'}\right)^2\left[(b_0 q - c_0 p)\, n' t + (b_1 q - c_1 p)\frac{(n't)^2}{2} + (b_2 q - c_2 p)\frac{(n't)^3}{3} + \ldots\right]. \quad (41)$$

And finally, after combining the corrections $\Delta\,\delta\rho'$ and $\Delta'\,\delta\rho'$ and integrating, taking into account that

$$p = p_0 + \sum_1 p_k \cos kM',$$
$$q = \sum_1 q_k \sin kM',$$

we obtain

$$\Delta_1.\mathrm{p}\delta p' = \frac{m}{1+m'}\left(\frac{a'}{r_0'}\right)^2\left\{a_0\left[qn'\int(p-p_0)\,dt - pn'\int q\,dt\right] - a_1\left[qn'^2\iint(p-p_0)(dt)^2 - pn'^2\iint q(dt)^2\right] + \right.$$
$$+ 2a_2\left[qn'^3\iiint(p-p_0)(dt)^3 - pn'^3\iiint q(dt)^3\right] - 6a_3\left[qn'^4\iiiint(p-p_0)(dt)^4 - pn'^4\iiiint q(dt)^4\right] +$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ (p_0a_0q + b_0q - c_0p)\,n't + a_1n't\left[qn'\int(p-p_0)\,dt - pn'\int q\,dt\right] -$$
$$- 2a_2n't\left[qn'^2\iint(p-p_0)(dt)^2 - pn'^2\iint q(dt)^2\right] + 6a_3n't\left[qn'^3\iiint(p-p_0)(dt)^3 - pn'^3\iiint q(dt)^3\right] +$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ \left(\frac{1}{2}a_1p_0q + \frac{1}{2}b_1q - \frac{1}{2}c_1p\right)(n't)^2 + a_2(n't)^2\left[qn'\int(p-p_0)\,dt - pn'\int q\,dt\right] -$$
$$- 3a_3(n't)^2\left[qn'^2\iint(p-p_0)(dt)^2 - pn'^2\iint q(dt)^2\right] +$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ \left(\frac{1}{3}a_2p_0q + \frac{1}{3}b_2q - \frac{1}{3}c_2p\right)(n't)^3 + a_3(n't)^3\left[qn'\int(p-p_0)\,dt - pn'\int q\,dt\right] +$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\left. + a'\Delta_1.\mathrm{p}K_1p + a'\Delta_1.\mathrm{p}K_2q\right\}. \qquad (42)$$

The corrections to the constants of integration will be determined by formulas /21

$$a'\Delta_1.\mathrm{p}C_1 = 0,$$
$$a'\Delta_1.\mathrm{p}K_1 = \left[a_0n'\int q\,dt - a_1n'^2\iint q(dt)^2 + 2a_2n'^3\iiint q(dt)^3 - 6a_3n'^4\iiiint q(dt)^4 + \ldots\right]_{t=t_0},$$
$$a'\Delta_1.\mathrm{p}K_2 = -\left[a_0n'\int p\,dt - a_1n'^2\iint p(dt)^2 + 2a_2n'^3\iiint p(dt)^3 - 6a_3n'^4\iiiint p(dt)^4 + \ldots\right]_{t=t_0}. \qquad (43)$$

In the same way we determine the long-period perturbations of longitude in the orbit. We select $a'\frac{\delta R}{\partial W'} = \left(a'\frac{\partial R}{\partial W'}\right)_0 + a'\delta\frac{\partial R}{\partial W'}$ terms depending on ν:

$$\frac{m}{1+m'}a'\frac{\partial R}{\partial W'} = A_\nu\sin(\nu t + C_0) + B_\nu\cos(\nu t + C_0) + C_\nu n't\sin(\nu t + C_0) + D_\nu n't\cos(\nu t + C_0).$$

After expansion in series, and integration, we obtain

$$\frac{m}{1+m'} n' \int a' \frac{\partial R}{\partial W'} dt = (A_W \sin C_0 + B_W \cos C_0)\, n't + \\ + \left[\left(C_W - \frac{\nu}{n'} B_W\right) \sin C_0 + \left(D_W + \frac{\nu}{n'} A_W\right) \cos C_0\right] \frac{(n't)^2}{2} + \\ + \left\{-\left[\frac{\nu}{n'} D_W + \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 A_W\right] \sin C_0 + \left[\frac{\nu}{n'} C_W - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 B_W\right] \cos C_0\right\} \frac{(n't)^3}{3} + \\ + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

The term depending on ν in expression $-2[\delta\rho' - (\delta\rho')^2] \cos\varphi' - 2\delta\rho' \frac{m}{1+m'} \left[n' \int \left(a' \frac{\partial R}{\partial W'}\right)_0 dt + C_2\right]$ will be designated as I, then:

$$\left.\begin{aligned} \mathrm{I} &= A'_W \sin(\nu t + C_0) + B'_W \cos(\nu t + C_0) + C'_W n't \sin(\nu t + C_0) + D'_W n't \cos(\nu t + C_0), \\ \text{and} \\ \frac{d\delta w'}{dt} &= \left(\frac{a'}{r'_0}\right)^2 [a'_0 + a'_1 n't + a'_2 (n't)^2 + \ldots], \end{aligned}\right\} \quad (44)$$

if we designate

$$\left.\begin{aligned} a'_0 &= A'_W \sin C_0 + B'_W \cos C_0, \\ a'_1 &= \left[A_W + C'_W - \frac{\nu}{n'} B'_W\right] \sin C_0 + \left[B_W + D'_W + \frac{\nu}{n'} A'_W\right] \cos C_0, \\ a'_2 &= \left[\frac{1}{2} C_W - \frac{\nu}{n'} B_W - \frac{\nu}{n'} D'_W - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 A'_W\right] \sin C_0 + \\ &+ \left[\frac{1}{2} D_W + \frac{1}{2} \frac{\nu}{n'} A_W + \frac{\nu}{n'} C'_W - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 B'_W\right] \cos C_0, \\ a'_3 &= \left[-\frac{1}{3} \frac{\nu}{n'} D_W - \frac{1}{6}\left(\frac{\nu}{n'}\right)^2 A_W - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 C'_W + \frac{1}{6}\left(\frac{\nu}{n'}\right)^3 B'_W\right] \sin C_0 + \\ &+ \left[\frac{1}{3} \frac{\nu}{n'} C_W - \frac{1}{6}\left(\frac{\nu}{n'}\right)^2 B_W - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 D'_W - \frac{1}{6}\left(\frac{\nu}{n'}\right)^3 A'_W\right] \cos C_0, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad (45)$$

Furthermore, terms depending on ν in $\frac{d\delta w'}{dt}$ still appear after multiplying the periodic part $\left(\frac{m}{1+m'} n' \int a' \frac{\partial R}{\partial W'} dt + \mathrm{I}\right)$ by $\left(\frac{a'}{r'_0}\right)^2$. We denote this product by II:

$$\mathrm{II} = A''_W \sin(\nu t + C_0) + B''_W \cos(\nu t + C_0) + C''_W n't \sin(\nu t + C_0) + D''_W n't \cos(\nu t + C_0),$$

then after appropriate transformations we obtain:

$$\mathrm{II} = a''_0 + a''_1 n't + a''_2 (n't)^2 + \ldots, \quad (46)$$

while

$$\left.\begin{aligned}
a_0'' &= A_W'' \sin C_0 + B_W'' \cos C_0,\\
a_1'' &= \left(C_W'' - \frac{\nu}{n'} B_W''\right)\sin C_0 + \left(D_W'' + \frac{\nu}{n'} A_W''\right)\cos C_0,\\
a_2'' &= -\left[\frac{\nu}{n'} D_W'' + \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 A_W''\right]\sin C_0 + \left[\frac{\nu}{n'} C_W'' - \frac{1}{2}\left(\frac{\nu}{n'}\right)^2 B_W''\right]\cos C_0,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}\right\} \qquad (47)$$

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Combining Equations (44) and Equation (46) and taking into account in integration that

$$\left(\frac{a'}{r_0'}\right)^2 = C_0^{-2,0} + \sum_1 C_k^{-2,0}\cos kM',$$

we obtain the long-period perturbations of longitude in orbit $\Delta_{1.p.}\delta w'$, which depend on ν:

$$\begin{aligned}
\Delta_{1.p.}\delta w' &= a_0' n' \int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right]dt - a_1' n'^2 \int\int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right](dt)^2 + \\
&+ 2a_2' n'^3 \int\int\int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right](dt)^3 - 6a_3' n'^4 \int\int\int\int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right](dt)^4 + \ldots\\
&+ n't\left\{a_0'' + a_0' C_0^{-2,0} + a_1' n' \int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right]dt - 2a_2' n'^2 \int\int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right](dt)^2 + \right.\\
&\left. + 6a_3' n'^3 \int\int\int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right](dt)^3 + \ldots\right\} +\\
&+ (n't)^2\left\{\frac{a_1''}{2} + \frac{1}{2} a_1' C_0^{-2,0} + a_2' n' \int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right]dt - 3a_3' n'^2 \int\int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right](dt)^2 + \right.\\
&\left. + \ldots\right\} + (n't)^3\left\{\frac{a_2''}{3} + \frac{1}{3} a_2' C_0^{-2,0} + a_3' n' \int\left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right]dt + \ldots\right\} +\\
&+ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&- 2\cos\varphi' \int \Delta_{1.p.}\delta\rho'\left(\frac{a'}{r_0'}\right)^2 dt + \Delta_{1.p.} C_3.
\end{aligned} \qquad (48)$$

We should add to the constants of integration corrections determined by the formulas:

$$\Delta_{1.\,p.}C_2 = 0,$$

$$\Delta_{1.\,p.}C_3 = -\left\{a_0' n' \int \left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right] dt - \right.$$

$$\left. - a_1' n'^2 \int\int \left[\left(\frac{a'}{r_0'}\right)^2 - C_0^{-2,0}\right] (dt)^2 + \ldots - 2\cos\varphi' \int \Delta_{1.\,p.}\delta\rho' \left(\frac{a'}{r_0'}\right)^2 dt\right\}_{t=t_0} \quad (49)$$

Integration of long-period terms in the longitude of the node and inclination is carried out by formulas analogous to (*).

CHAPTER II

PERTURBATIONS OF THE COORDINATES OF JUPITER, SATURN, AND URANUS

When using the method presented above for determining perturbations of coordinates of the second order, relative to the masses of Jupiter, Saturn, and Uranus, we must have the perturbations of the logarithm of the radius vector and longitude in the orbit of these planets. The mean anomalies should serve as the arguments in the series of these perturbations. In the following sections we will reduce the available perturbations to a convenient form. /23

7. Perturbations of the Logarithms of the Radius Vectors and Longitudes in the Orbit of Jupiter and Saturn

When determining perturbations of Plutonian coordinates of the first order, relative to the perturbing masses of Jupiter and Saturn, we considered the elements of these planets given by Hill. Therefore, $\delta\rho_1$, δw_1 and $\delta\rho_2$, δw_2 (quantities with the subscript 1 pertain to Jupiter and with the subscript 2 to Saturn) are also extracted from Hill's theory of motion of these planets (Hill, 1890). When constructing the theory of motion of Jupiter and Saturn, Hill employed Hansen's method and gave series for the Hansen unknown values $n\delta z$ and ν, arranged according to mean anomalies. To convert from $n\delta z$ and ν to $\delta\rho$ and δw, it suffices to use the relations:

$$\begin{aligned}\delta\rho &= \nu + \left(\frac{a_0}{r_0}\right)^2 e_0 \sin E_0 n\delta z,\\ \delta w &= \left(\frac{a_0}{r_0}\right)^2 \sqrt{1-e_0^2}\, n\delta z,\end{aligned} \tag{50}$$

where the quantities a_0, r_0, e_0, and $\sin E_0$ are taken from the unperturbed motion of the planet.

The expansions of $\left(\frac{a_0}{r_0}\right)^2$ and $\sin E_0$, in multiples of the mean anomaly, were obtained with an accuracy to the ninth power of eccentricity in the author's work (1953).

Then, multiplying the series

$$\left(\frac{a}{r}\right)^2 = \sum C_k^{-2,0} \cos kM,$$

$$\sin E = \sum q_k \sin kM,$$

we obtain

$$\left(\frac{a}{r}\right)^2 e \sin E = \sum S_k \sin kM$$

and

$$\sqrt{1-e^2}\left(\frac{a}{r}\right)^2 = \sum C_k \cos kM. \tag{51}$$

Substituting the appropriate values of the eccentricities of Jupiter and Saturn we have:

	Jupiter $\sin kM_1$	Saturn $\sin kM_2$		Jupiter $\cos kM_1$	Saturn $\cos kM_2$
S_0	—	—	C_0	1.0000000	1.0000000
S_1	0.0482118	0.0559946	C_1	0.0964796	0.1120775
S_2	0.0034877	0.0047052	C_2	0.0058160	0.0078479
S_3	0.0002382	0.0003734	C_3	0.0003648	0.0005715
S_4	0.0000160	0.0000291	C_4	0.0000232	0.0000423
S_5	0.0000010	0.0000023	C_5	0.0000015	0.0000031
S_6	—	0.0000002	C_6	0.0000001	0.0000002

Having series $n_1\delta z_1$, ν_1, $n_2\delta z_2$, ν_2 and series (51), after necessary multi- /24
plications, we obtain $\delta\rho_1$, δw_1, $\delta\rho_2$, and δw_2 in the form of series with arguments $i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4$ (indexes 3 and 4 pertain, respectively, to Uranus and Neptune). The mixed and secular terms in $\delta\rho_1$, δw_1, $\delta\rho_2$, and δw_2 will be somewhat transformed; from factors $n_1(t - t_{1850.0})$ and $n_2(t - t_{1850.0})$ we change to $n_1(t - t_0)$ and $n_2(t - t_0)$, where t_0 = 1930 Sept. 20.0, the instant of osculation of Pluto's elements.

The series of perturbations $\delta\rho_1$, δw_1, $\delta\rho_2$, and δw_2, obtained in this manner are given in Tables 1 and 2.

8. Perturbations of the Logarithm of the Radius Vector and Longitude in Uranus' Orbit

The determination of perturbations of Plutonian coordinates of the first order, relative to the mass of Uranus, was supported by elements of Uranus obtained by Newcomb. It is natural that, in this case, we use his series of perturbations of the logarithm of the radius vector and longitude in Uranus' orbit (Newcomb, 1899), reducing the mixed and secular terms to a convenient form. The series of perturbations $\delta\rho_3$ and δw_3 are given in Table 3, where T is calculated in centuries from t_0 = 1930 Sept. 20.0.

Table 1

i_4	i_3	i_2	i_1	(40) $= \delta\varphi_1$		(41) $= \delta w_1$	
				sin	cos	sin	cos
					"		"
0	0	0	0	—	+0.0154n_1t	—	−0.0672n_1t
			1	−0.″6988n_1t	−0.6357n_1t	+1.″2734n_1t	−0.3986n_1t
			2	−0.0505n_1t	−0.0460n_1t	0.0768n_1t	−0.0843n_1t
			3	−0.0035n_1t	−0.0032n_1t	0.0048n_1t	−0.0052n_1t
			4	−0.0002n_1t	−0.0002n_1t	+0.0002n_1t	−0.0003n_1t
0	0	1	−2		−0.0001n_1t	−0.0003n_1t	+0.0003n_1t
			−1		0.0000	+0.0001n_1t	0.0001n_1t
			0	+0.0005n_1t	−0.0005n_1t	0.0018n_1t	0.0021n_1t
			1	+0.0002n_1t		0.0001n_1t	0.0002n_1t
0	0	2	−3	−0.0002n_1t	+0.0005n_1t	0.0012n_1t	+0.0004n_1t
			−2	+0.0005n_1t	0.0004n_1t	0.0010n_1t	−0.0010n_1t
			−1	+0.0020n_1t	+0.0013n_1t	0.0128n_1t	−0.0203n_1t
			0	−0.0005n_1t	−0.0008n_1t	0.0015n_1t	−0.0010n_1t
0	0	3	−4				+0.0002n_1t
			−3	−0.0002n_1t	+0.0004n_1t	0.0007n_1t	0.0006n_1t
			−2	−0.0044n_1t	0.0077n_1t	+0.0187n_1t	+0.0105n_1t
			−1	−0.0002n_1t		−0.0016n_1t	−0.0032n_1t
			0	−0.0001n_1t		+0.0002n_1t	−0.0002n_1t
0	0	4	−4	+0.0001n_1t		0.0001n_1t	−0.0002n_1t
			−3	+0.0023n_1t	0.0010n_1t	0.0016n_1t	−0.0037n_1t
			−2	−0.0001n_1t	+0.0020n_1t	0.0086n_1t	−0.0002n_1t
			−1		−0.0002n_1t	0.0003n_1t	
0	0	5	−5		+0.0001n_1t	0.0002n_1t	+0.0002n_1t
			−4	−0.0021n_1t	0.0022n_1t	0.0032n_1t	0.0032n_1t
			−3	−0.0309n_1t	+0.0260n_1t	0.0525n_1t	+0.0619n_1t
			−2	−0.0012n_1t	−0.0042n_1t	0.7866n_1t	−0.4728n_1t
			−1	−0.0106n_1t	−0.0216n_1t	0.0430n_1t	−0.0214n_1t
			0	−0.0008n_1t	−0.0016n_1t	0.0026n_1t	−0.0014n_1t
			1		−0.0001n_1t	0.0002n_1t	−0.0001n_1t
0	0	6	−5	−0.0002n_1t			+0.0002n_1t
			−4	+0.0002n_1t	+0.0005n_1t	0.0008n_1t	−0.0004n_1t
			−3	−0.0003n_1t	0.0003n_1t	0.0010n_1t	−0.0012n_1t
0	0	7	−5			0.0002n_1t	+0.0002n_1t
			−4	+0.0003n_1t	0.0002n_1t	0.0008n_1t	−0.0009n_1t
			−3		0.0001n_1t	+0.0014n_1t	+0.0004n_1t
0	0	8	−4				0.0004n_1t
0	0	10	−5			−0.0003n_1t	0.0004n_1t
			−4			+0.0017n_1t	0.0013n_1t
			−3			0.0001n_1t	

Table 1 (continued)

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i_4	i_3	i_2	i_1	(40) = δg_1		(41) = δw_1	
				sin	cos	sin	cos
				"	"		"
0	0	0	0	—	+86.22	—	— 2.68
			1	—30.36	—26.50	+ 54.44	—59.79
			2	— 2.07	— 1.82	3.11	— 3.48
			3	— 0.12	— 0.16	0.23	— 0.19
			4	— 0.01	— 0.01	0.01	— 0.01
0	0	1	—4	— 0.01			+ 0.02
			—3	— 0.26	— 0.01	0.01	0.39
			—2	— 3.02	+ 0.39	0.66	5.31
			—1	—25.51	4.63	+ 14.52	78.76
			0	+ 2.63	1.67	— 8.91	+ 9.23
			1	— 0.13	0.83	— 1.39	— 0.17
			2	+ 0.08	0.10	— 0.13	+ 0.10
			3			— 0.01	+ 0.01
0	0	2	—5	0.01	0.03	+ 0.04	— 0.02
			—4	0.23	0.47	0.69	— 0.33
			—3	3.17	7.19	11.52	— 5.02
			—2	+42.93	103.53	185.09	—76.51
			—1	— 0.69	+13.81	131.98	— 1.30
			0	+ 2.17	— 1.73	+ 2.89	+ 5.24
			1	0.11	+ 0.18	— 0.22	0.22
			2	+ 0.01		— 0.01	0.01
0	0	3	—5	— 0.04	0.04	+ 0.04	0.05
			—4	— 0.77	0.44	0.60	1.11
			—3	—11.35	+ 4.83	+ 6.33	16.90
			—2	—27.80	—20.55	— 48.48	+66.40
			—1	— 0.25	— 0.67	+ 7.15	— 7.29
			0	— 2.03	— 0.18	0.40	— 3.61
			1	— 0.15	— 0.02	+ 0.03	— 0.23
			2	— 0.01			— 0.01
0	0	4	—6	— 0.01		— 0.01	+ 0.02
			—5	— 0.24	— 0.08	— 0.10	0.31
			—4	— 2.51	— 1.41	— 1.64	3.15
			—3	— 4.40	+ 8.22	+ 13.33	7.35
			—2	— 3.88	— 0.54	— 1.48	16.99
			—1	+ 0.31	+ 0.21	— 0.66	0.66
			0	0.03	+ 0.02	— 0.03	+ 0.04
0	0	5	—7		— 0.01	— 0.01	
			—6		— 0.12	— 0.15	
			—5	+ 0.19	— 1.17	— 1.50	— 0.14
			—4	— 4.24	— 5.34	— 8.80	+ 6.45
			—3	—33.14	—66.96	—135.53	66.48
			—2	+ 9.02	— 1.90	+499.62	1091.35
			—1	26.51	—12.28	24.65	53.05
			0	1.91	— 0.88	1.48	3.19
			1	0.13	— 0.06	0.09	0.20
			2	0.01		+ 0.01	+ 0.01
0	0	6	—7	0.04	— 0.01	— 0.01	— 0.05
			—6	+ 0.32	+ 0.06	+ 0.06	— 0.36
			—5	— 0.05	— 0.63	— 0.79	+ 0.09
			—4	— 0.96	+ 0.23	+ 0.35	1.50
			—3	— 0.22	— 0.36	— 1.03	0.65
			—2	+ 0.04	— 0.03	+ 0.09	0.12
			—1				+ 0.01
0	0	7	—8	0.01	+ 0.02	+ 0.02	— 0.01
			—7	0.01	+ 0.13	+ 0.14	— 0.02
			—6	+ 0.25	— 0.06	— 0.07	— 0.30

Table 1 (continued)

i_4	i_3	i_2	i_1	(40) $= \delta\rho_1$		(41) $= \delta w_1$	
				sin	cos	sin	cos
				"	"	"	"
0	0	7	−5	−0.02	−0.29	−0.36	0.00
			−4	+1.00	−1.08	−1.85	−1.78
			−3	+0.08	−0.15	−1.54	−0.93
			−2	−0.02	+0.04	−0.05	−0.05
0	0	8	−8	−0.05	0.01	+0.02	+0.05
			−7	+0.04	+0.10	+0.11	−0.05
			−6	0.10	−0.05	−0.06	−0.12
			−5	+0.16	+0.16	+0.22	−0.21
			−4	−0.15	0.67	+1.59	+0.39
			−3	−0.01	+0.01	−0.16	−0.06
			−2			−0.01	
0	0	9	−9	−0.01	−0.02	−0.02	+0.01
			−8	−0.04	+0.03	+0.03	+0.04
			−7	+0.04	0.04	0.05	−0.05
			−6	0.00	0.07	0.08	0.00
			−5	0.28	0.02	0.05	−0.46
			−4	+0.02	+0.10	0.45	−0.15
			−3		−0.01	+0.02	−0.01
0	0	10	−9	−0.02	−0.02	−0.02	+0.02
			−8	−0.01	+0.02	+0.03	0.01
			−7	−0.03	0.00	0.01	0.04
			−6	−0.08	0.11	0.17	0.13
			−5	−1.21	+0.86	1.69	+2.43
			−4	−0.04	−0.10	6.08	−6.17
			−3	−0.15	−0.15	0.29	−0.30
			−2	−0.01	−0.01	0.02	−0.02
0	0	11	−7	−0.02	+0.01	0.02	+0.03
			−6	+0.03	0.03	0.05	−0.04
			−5	−0.02	0.03	+0.07	+0.05
0	0	12	−8			−0.02	0.01
			−7			+0.02	+0.01
			−6	+0.04	−0.02	0.04	−0.09
			−5			+0.07	
0	1	0	−2	−0.01	−0.02	−0.05	+0.02
			−1	−0.16	−0.38	−0.85	0.36
			0	+0.01	+0.02	−0.31	0.05
			1		+0.01	−0.02	
0	2	0	−3	−0.01	−0.01	−0.02	0.02
			−2	−0.24	−0.23	−0.34	0.36
			−1	−0.14	−0.15	−0.40	0.38
			0	+0.01	+0.01	−0.03	+0.02
0	3	0	−3	+0.03	+0.01	+0.01	−0.02
			−2	−0.07	−0.06	−0.09	+0.12
			−1	−0.03	−0.02	−0.06	0.08
0	4	0	−2	−0.02	−0.01	−0.02	0.03
1	0	0	−2		+0.01	+0.01	0.01
			−1	−0.07	+0.11	0.24	0.15
			0		−0.01	+0.01	+0.02
2	0	0	−3				−0.01
			−2	+0.06	−0.03	−0.05	−0.10
			−1	0.07	−0.04	−0.08	−0.16
			0				−0.01

Table 2

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i_4	i_3	i_1	i_2	(42) = $\delta\rho_2$ sin	(42) = $\delta\rho_2$ cos	(43) = δw_2 sin	(43) = δw_2 cos
					″		″
0	0	0	0	—	$-0.0939n_2t$	—	$-0.6000n_2t$
			1	$-5.3662n_2t$	$+3.3231n_2t$	$-6.6652n_2t$	$-10.7458n_2t$
			2	$-0.4504n_2t$	$0.2767n_2t$	$-0.4636n_2t$	$-0.7512n_2t$
			3	$-0.0360n_2t$	$0.0205n_2t$	$-0.0326n_2t$	$-0.0547n_2t$
			4	$-0.0027n_2t$	$0.0016n_2t$	$-0.0024n_2t$	$-0.0041n_2t$
			5	$-0.0002n_2t$	$+0.0001n_2t$	$-0.0002n_2t$	$-0.0001n_2t$
0	0	1	— 5			$-0.0002n_2t$	$+0.0002n_2t$
			— 4	$-0.0002n_2t$	$-0.0016n_2t$	$-0.0028n_2t$	$0.0020n_2t$
			— 3	$-0.0110n_2t$	$-0.0092n_2t$	$-0.0276n_2t$	$0.0292n_2t$
			— 2	$+0.0418n_2t$	$-0.0243n_2t$	$+0.1025n_2t$	$+0.1654n_2t$
			— 1	$-0.0010n_2t$	$+0.0014n_2t$	$-0.0018n_2t$	$-0.0022n_2t$
			0	$-0.0013n_2t$	$-0.0017n_2t$	$+0.0058n_2t$	$-0.0030n_2t$
			1	$-0.0006n_2t$	$-0.0003n_2t$	$0.0006n_2t$	$-0.0008n_2t$
0	0	2	— 8	$-0.0008n_2t$	$+0.0010n_2t$	$0.0016n_2t$	$+0.0012n_2t$
			— 7	$-0.0106n_2t$	$0.0140n_2t$	$0.0230n_2t$	$0.0174n_2t$
			— 6	$-0.1128n_2t$	$0.1577n_2t$	$0.3209n_2t$	$0.2343n_2t$
			— 5	$-0.0597n_2t$	$+0.1147n_2t$	$4.7069n_2t$	$2.7868n_2t$
			— 4	$+0.0800n_2t$	$-0.5111n_2t$	$1.0344n_2t$	$0.1641n_2t$
			— 3	$0.0250n_2t$	$-0.0154n_2t$	$0.0312n_2t$	$0.0394n_2t$
			— 2	$+0.0016n_2t$	$-0.0011n_2t$	$+0.0022n_2t$	$+0.0023n_2t$
			— 1	$-0.0004n_2t$	$-0.0001n_2t$	$-0.0003n_2t$	$-0.0014n_2t$
0	0	3	— 8	$+0.0008n_2t$	$-0.0008n_2t$	$-0.0014n_2t$	$-0.0026n_2t$
			— 7	$-0.0020n_2t$	$+0.0050n_2t$	$-0.0042n_2t$	$-0.0110n_2t$
			— 6	$+0.0046n_2t$	$0.0040n_2t$	$-0.0036n_2t$	$+0.0051n_2t$
			— 5	$0.0040n_2t$	$+0.0008n_2t$	$-0.0010n_2t$	$0.0046n_2t$
			— 4	$0.0030n_2t$	$-0.0009n_2t$	$+0.0008n_2t$	$0.0032n_2t$
			— 3				$+0.0002n_2t$
0	0	4	—11	$+0.0001n_2t$	$+0.0003n_2t$	$-0.0004n_2t$	$-0.0004n_2t$
			—10	$-0.0029n_2t$	$-0.0035n_2t$	$+0.0096n_2t$	$-0.0066n_2t$
			— 9	$+0.0174n_2t$	$+0.0071n_2t$	$-0.0090n_2t$	$+0.0222n_2t$
			— 8	$-0.0014n_2t$	$0.0008n_2t$	$-0.0010n_2t$	$-0.0024n_2t$
			— 7	$-0.0010n_2t$	$+0.0006n_2t$	$-0.0003n_2t$	$-0.0006n_2t$
			— 6	$+0.0004n_2t$	$-0.0008n_2t$	$+0.0008n_2t$	$+0.0004n_2t$
			— 5		$-0.0009n_2t$	$+0.0010n_2t$	
0	0	5	—12				$+0.0008n_2t$
			—11	$-0.0005n_2t$	$+0.0003n_2t$	$-0.0004n_2t$	$-0.0008n_2t$
			—10		$0.0004n_2t$	$-0.0003n_2t$	
			— 9		$0.0006n_2t$	$-0.0004n_2t$	
			— 8		$+0.0005n_2t$	$-0.0004n_2t$	
			— 7				
			— 6	$-0.0003n_2t$			$+0.0003n_2t$
0	1	0	— 2			$-0.0003n_2t$	
			— 1			$0.0000n_2t$	
			0			$+0.0004n_2t$	$-0.0003n_2t$
0	2	0	— 3				$+0.0005n_2t$
			— 2			$-0.0002n_2t$	$-0.0001n_2t$
			— 1	$+0.0004n_2t$	$-0.0005n_2t$	$-0.0037n_2t$	$-0.0027n_2t$
			0			$-0.0002n_2t$	$-0.0002n_2t$
0	3	0	— 3	$0.0004n_2t$		$-0.0002n_2t$	$-0.0011n_2t$
			— 2	$0.0025n_2t$	$+0.0015n_2t$	$+0.0030n_2t$	$-0.0054n_2t$
			— 1		$0.0003n_2t$	$-0.0181n_2t$	$-0.0012n_2t$
			0		$0.0009n_2t$	$-0.0018n_2t$	$-0.0004n_2t$
0	4	0	— 2				$-0.0006n_2t$
0	5	0	— 3			$-0.0003n_2t$	

Table 2 (continued)

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i_4	i_3	i_1	i_2	(42) = δp_z		(43) = δw_z	
				sin	cos	sin	cos
					"		"
0	0	0	0	—	— 3.23	—	— 10.32
			1	— 84.41	+ 61.44	— 114.84	— 184.89
			2	— 5.59	5.69	— 9.34	— 10.67
			3	+ 0.14	0.41	— 0.65	— 0.17
			4	— 0.01	0.05	— 0.02	— 0.02
0	0*	1	— 6	— 0.01	0.01	+ 0.02	+ 0.01
			— 5	— 0.10	0.14	0.23	0.13
			— 4	— 1.26	3.04	2.60	1.73
			— 3	— 9.18	+ 18.36	40.58	+ 28.57
			— 2	+ 9.61	—116.35	424.80	— 5.75
			— 1	176.12	+ 20.18	30.76	— 1.04
			0	3.55	— 2.60	1.51	+ 11.90
			1	0.63	— 0.44	0.46	0.91
			2	0.08	— 0.23	0.22	0.10
			3		— 0.01	0.01	+ 0.01
0	0	2	— 9	0.04	+ 0.02	0.03	— 0.06
			— 8	0.50	0.23	0.35	— 0.77
			— 7	6.35	2.91	4.85	— 10.56
			— 6	75.85	34.71	69.69	— 151.10
			— 5	+ 66.29	24.52	+1219.45	—2681.44
			— 4	—404.25	+ 1.49	— 4.36	— 824.54
			— 3	— 12.89	— 15.42	+ 20.33	— 29.01
			— 2	+ 10.70	— 28.56	30.56	+ 10.04
			— 1	+ 2.81	— 1.27	2.66	— 2.09
			0	— 0.14	— 0.25	0.23	+ 0.04
			1	— 0.07	— 0.04	+ 0.05	— 0.06
0	0	3	—10				+ 0.01
			— 9	— 0.05	— 0.02	— 0.04	0.08
			— 8	— 0.20	— 0.35	— 0.93	0.36
			— 7	+ 0.83	+ 1.58	— 5.25	3.15
			— 6	0.19	— 0.50	+ 2.91	1.57
			— 5	+ 0.28	— 2.76	3.62	+ 0.35
			— 4	— 1.94	— 4.33	4.65	— 2.04
			— 3	— 5.69	— 4.24	4.06	— 5.38
			— 2	— 0.15	— 0.38	0.38	— 0.20
			— 1	+ 0.03	+ 0.02	+ 0.03	— 0.16
			0	— 0.02	0.02	— 0.02	— 0.02
0	0	4	—12	— 0.04	0.04	+ 0.06	+ 0.06
			—11	— 0.43	0.43	0.83	0.86
			—10	— 0.53	+ 0.48	14.98	15.37
			— 9	+ 1.97	— 4.58	+ 9.03	3.77
			— 8	0.35	+ 0.72	— 0.94	0.54
			— 7	+ 0.24	+ 0.34	— 0.31	+ 0.23
			— 6	— 0.63	— 0.14	+ 0.21	— 0.70
			— 5	— 1.43	+ 0.33	— 0.30	— 1.36
			— 4	— 1.63	+ 1.41	— 1.29	— 1.50
			— 3	— 0.14	— 0.05	+ 0.04	— 0.13
			— 2	— 0.01	+ 0.02	— 0.02	— 0.01
0	0	5	—13		+ 0.01	+ 0.01	— 0.01
			—12	— 0.03	— 0.01	+ 0.19	— 0.10
			—11	+ 0.08	+ 0.14	— 0.17	+ 0.13
			—10	0.11	+ 0.06	— 0.08	0.13
			— 9	0.16	— 0.08	— 0.01	0.15
			— 8	+ 0.14	— 0.06	+ 0.04	+ 0.09
			— 7	— 0.10	+ 0.22	— 0.22	— 0.09
			— 6	+ 0.01	0.55	— 0.52	0.00
			— 5	+ 0.35	0.66	— 0.61	+ 0.32
			— 4	— 0.06	+ 0.05	— 0.04	— 0.05

Table 2 (continued)

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i_4	i_3	i_1	i_2	(42) $= \delta\varphi_2$ sin	(42) $= \delta\varphi_2$ cos	(43) $= \delta w_2$ sin	(43) $= \delta w_2$ cos
0	0	6	−12				+0.02
			−11	+0.02	−0.03	+0.02	0.02
			−10	0.01	−0.05	0.05	+0.01
			− 9		−0.04	+0.05	
			− 8	0.08	+0.06	− 0.06	+0.08
			− 7	0.24	+0.04	− 0.04	0.22
			− 6	0.28	−0.07	+ 0.07	0.25
			− 5	+0.01	+0.03	− 0.04	+0.01
0	0	7	−11	−0.02	−0.01	+ 0.01	− 0.01
			−10	−0.02	0.00	0.00	− 0.01
			− 9	+0.04	−0.04	0.04	+ 0.04
			− 8	0.04	−0.10	0.09	+0.04
			− 7	0.00	−0.11	+0.10	− 0.01
			− 6	+0.02			+ 0.02
0	0	8	−11			− 0.01	
			−10	−0.01	−0.02	+ 0.02	− 0.01
			− 9	−0.04	−0.03	0.02	− 0.03
			− 8	−0.04	−0.01	+0.01	− 0.04
0	0	9	−10	−0.02	+0.02	− 0.01	− 0.01
			− 9	−0.01	0.02	− 0.02	− 0.01
0	1	0	− 3	−0.02			+ 0.04
			− 2	−0.34	0.03	+ 0.05	0.59
			− 1	−2.76	0.51	+ 1.49	7.93
			0	+0.26	0.10	− 0.67	0.95
			1	0.03	0.02	− 0.06	+ 0.06
0	2	0	− 4	0.01	0.03	+ 0.06	− 0.03
			− 3	0.26	0.56	0.87	− 0.39
			− 2	2.91	6.93	11.81	− 4.94
			− 1	+0.15	+1.33	8.21	− 1.08
			0		−0.20	0.39	+ 0.01
			1		−0.02	0.03	
0	3	0	− 5	−0.01			0.01
			− 4	−0.12	+0.02	+ 0.02	0.17
			− 3	−1.50	+0.09	− 0.02	2.32
			− 2	−9.29	−4.86	−10.10	+19.35
			− 1	−0.27	−0.60	+21.46	−16.51
			0	−0.47	−0.60	1.22	− 0.97
			1	−0.04	−0.05	0.09	− 0.07
			2			+ 0.01	
0	4	0	− 5	−0.01		− 0.01	+ 0.01
			− 4	−0.17	−0.11	− 0.12	0.22
			− 3	−0.18	+0.44	+ 0.67	0.28
			− 2	−0.30	+0.03	0.14	+ 0.91
			− 1	+0.01		+ 0.03	
0	5	0	− 5	+0.03	−0.06	− 0.06	− 0.02
			− 4	−0.11	0.00	+ 0.01	+ 0.13
			− 3	+0.07	+0.24	0.41	− 0.12
			− 2	−0.03	+0.03	0.22	+ 0.21
			− 1	+0.01	−0.01	0.01	+ 0.01
0	6	0	− 6	+0.02		+ 0.01	− 0.02
			− 5	0.00	−0.04	− 0.04	+ 0.01
			− 4	−0.05	+0.02	+ 0.02	0.06
			− 3	−0.09	−0.06	− 0.13	0.20
			− 2	+0.01		+ 0.11	0.02
			− 1			0.01	

Table 2 (continued)

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i_4	i_3	i_1	i_2	(42) = $\delta\varrho_2$		(43) = δw_2	
				sin	cos	sin	cos
0	7	0	−4			0″.02	0″.01
			−3				+0.02
1	0	0	−3			0.01	−0.01
			−2	0″.06	+0″.05	0.09	−0.09
			−1	+0.54	+0.50	1.17	−1.27
			0	−0.04	−0.03	0.10	−0.17
			1	−0.01		0.01	−0.01
2	0	0	−4				+0.01
			−3	−0.06			0.09
			−2	−0.72	+0.05	0.07	1.10
			−1	−0.33	0.03	0.08	0.96
			0	+0.03		0.01	0.05
3	0	0	−3	−0.05	+0.06	+0.06	+0.06
			−2	+0.02	−0.06	−0.10	−0.03
			−1			−0.04	+0.01

Table 3

i_1	i_2	i_4	i_3	(44) = $\delta\varrho_3$		(45) = δw_3	
				sin	cos	sin	cos
0	0	0	0	—	″ − 4.81T	—	″ +20.32T
			1	−37″.18T	−55.59T	+111″.36T	−74.32T
			2	− 2.62T	− 3.94T	6.54T	− 4.36T
			3	− 0.14T	− 0.28T	0.40T	− 0.27T
			4			+ 0.03T	− 0.02T
0	1	0	−4		− 0.04T	− 0.09T	+ 0.01T
			−3	− 0.01T	− 0.19T	− 1.83T	0.15T
			−2	+ 0.43T	+ 0.63T	− 1.39T	0.98T
			−1	0.03T	0.04T	− 0.08T	0.05T
0	2	0	−5	+ 0.02T	+ 0.02T	− 0.05T	+ 0.05T
			−4	− 0.02T		0.00	− 0.03T
			−3	− 0.01T		− 0.01T	− 0.02T
			−2				
			−1		− 0.01T	− 0.01T	
2	−6	0	1			− 0.01T	+ 0.01T
			2	− 0.07T	− 0.11T	− 0.24T	0.14T
			3	+ 0.03T	+ 0.01T	− 0.40T	0.80T
			4	0.02T	0.01T	− 0.02T	+ 0.04T
2	−5	0	−1		0.01T	+ 0.03T	− 0.01T
0	0	0	0	—	+92.12	—	+ 3.12
			1	−13.72	−11.50	+ 29.78	−33.42
			2	− 0.92	− 0.82	1.61	− 1.84
			3	− 0.02	− 0.08	0.10.	− 0.10
			4			+ 0.02	+ 0.01
0	0	1	−4				− 0.01
			−3	+ 0.07	− 0.05	− 0.08	− 0.11
			−2	0.99	− 0.70	− 1.28	− 1.76
			−1	+ 7.58	− 5.41	− 19.87	−27.87

Table 3 (continued)

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i_1	i_2	i_4	i_3	(44) $= \delta\rho_3$ sin	(44) $= \delta\rho_3$ cos	(45) $= \delta w_3$ sin	(45) $= \delta w_3$ cos
				"	"	"	"
0	0	1	0	— 0.92	+ 0.78	— 2.47	— 2.78
			1	— 0.10	+ 0.09	— 0.16	— 0.18
			2			— 0.01	— 0.01
0	0	2	—4	— 0.05	— 0.02	— 0.03	+ 0.09
			—3	— 0.74	— 0.25	— 0.50	1.47
			—2	— 8.74	— 3.04	—10.04	+28.79
			—1	+ 1.34	+ 0.37	+ 0.12	— 0.60
			0				— 0.05
0	0	3	—5		— 0.04	— 0.06	+ 0.02
			—4	— 0.18	— 0.64	— 0.97	0.28
			—3	— 2.72	— 9.00	—14.28	4.33
			—2	— 2.32	— 4.62	—17.41	8.63
			—1	+ 0.12	+ 0.25	— 0.54	0.23
			0		0.02	— 0.03	0.01
0	0	4	—6			+ 0.01	0.01
			—5	— 0.12	0.16	0.21	0.17
			—4	— 1.50	1.91	2.67	2.06
			—3	— 0.69	+ 1.31	+ 4.45	+ 2.37
			—2	+ 0.08	— 0.18	— 0.13	— 0.02
0	0	5	—7				— 0.01
			—6	0.07		+ 0.01	— 0.09
			—5	+ 0.71	+ 0.07	0.08	— 0.84
			—4	— 1.86	0.25	0.42	+ 2.95
			—3	— 0.70	+ 0.23	0.84	2.82
			—2	+ 0.04		+ 0.01	0.07
0	0	6	—7	— 0.02	— 0.02	— 0.04	0.02
			—6	— 0.16	— 0.28	— 0.31	+ 0.18
			—5	+ 0.38	+ 0.40	+ 0.56	— 0.52
			—4	+ 0.18	+ 0.14	0.48	— 0.62
			—3	— 0.02	— 0.02		+ 0.02
0	0	7	—8		+ 0.02	0.02	+ 0.01
			—7	— 0.06	+ 0.14	+ 0.15	+ 0.07
			—6	+ 0.04	— 0.19	— 0.23	— 0.05
			—5		+ 0.32	+ 0.52	+ 0.01
			—4		+ 0.11	0.45	— 0.06
			—3			+ 0.01	
0	0	8	—8	+ 0.07	— 0.02	— 0.02	— 0.08
			—7	— 0.09	+ 0.05	+ 0.04	+ 0.10
			—6	+ 0.09	— 0.05	— 0.08	— 0.12
			—5	+ 0.02	— 0.02	— 0.09	— 0.09
0	0	9	—9	— 0.02	— 0.02	— 0.03	+ 0.03
			—8	+ 0.02	+ 0.02	+ 0.03	— 0.05
			—7	— 0.04	— 0.02	— 0.02	+ 0.05
			—6	+ 0.04			— 0.07
0	1	0	—6			+ 0.02	+ 0.01
			—5	— 0.12	+ 0.18	0.30	0.20
			—4	— 1.74	2.56	4.99	3.43
			—3	— 8.10	+12.52	101.72	75.75
			—2	+12.06	—63.06	142.51	+19.81
			—1	+36.44	+ 2.52	12.57	—18.58
			—0	— 0.09	— 0.66	0.46	+ 1.41
			1	+ 0.01	— 0.03	0.03	+ 0.04

Table 3 (continued)

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i_1	i_2	i_4	i_3	(44) $= \delta\gamma_3$		(45) $= \delta w_3$	
				sin	cos	sin	cos
					"	"	"
0	2	0	—7		+ 0.01	0.03	— 0.01
			—6	+ 0.05	+ 0.13	0.58	— 0.14
			—5	— 0.87	— 0.82	+ 2.12	— 2.20
			—4	+ 1.32	+ 0.08	— 0.09	+ 1.92
			—3	2.00	— 1.10	+ 1.26	2.31
			—2	1.80	— 3.84	3.75	+ 1.66
			—1	+ 0.84	— 0.04	0.34	— 0.74
			0	— 0.01	— 0.01	0.01	+ 0.02
0	3	0	—6	+ 0.05	— 0.03	0.04	0.06
			—5	+ 0.04	— 0.16	0.16	+ 0.06
			—4	— 0.14	— 0.44	0.42	— 0.13
			—3	— 0.73	— 0.55	0.50	— 0.66
			—2	0.00	— 0.06	0.06	— 0.01
			—1	+ 0.03		0.01	— 0.04
0	4	0	—6	— 0.04	— 0.02	+ 0.02	— 0.04
			—5	— 0.14	+ 0.01	— 0.01	— 0.13
			—4	— 0.19	0.17	— 0.15	— 0.18
0	5	0	—7	— 0.01	0.01	— 0.01	— 0.01
			—6	0.00	0.05	— 0.04	0.00
			—5	+ 0.04	0.07	— 0.06	+ 0.03
			—4	— 0.01	+ 0.01	— 0.01	— 0.01
0	6	0	—7	+ 0.02		— 0.01	+ 0.02
			—6	+ 0.03	— 0.01	+ 0.01	+ 0.02
1	—5	0	1	— 0.16			— 0.15
			2	— 0.01			
1	—4	0	2	+ 0.01	+ 0.03	+ 0.04	— 0.02
			3	0.03	— 0.03	— 0.05	— 0.05
			4	+ 0.03	+ 0.03	+ 0.17	— 0.13
1	—3	0	1	— 0.02	+ 0.01	0.04	+ 0.21
			2	— 0.06			— 0.15
1	—2	0	—1	— 0.02		0.01	— 0.09
			0		— 0.02	0.02	— 0.01
			1		— 0.03	+ 0.02	
1	—1	0	0	+ 0.01			— 0.01
			1	+ 0.01	— 0.05	— 0.04	— 0.01
			2	— 0.01	— 0.04	— 0.03	+ 0.01
1	0	0	—4	+ 0.01	— 0.02	+ 0.01	+ 0.01
			—3	0.19	— 0.30	— 0.16	— 0.02
			—2	1.71	— 3.65	— 3.24	— 1.32
			—1	+21.46	—48.93	—48.62	—21.32
			0	— 0.47	+ 0.87	+ 1.23	+ 0.52
			1	— 0.01	— 0.01	0.04	
1	1	0	—4	— 0.04	+ 0.02	0.02	0.03
			—3	— 0.02	0.03	0.03	+ 0.02
2	—7	0	2	+ 0.06			— 0.06
			3	0.05	0.01	+ 0.01	— 0.06
			4	+ 0.04	0.00	— 0.01	— 0.06
			5	— 0.09	0.10	+ 0.23	+ 0.21
2	—6	0	0	— 0.01	+ 0 02	0.03	0.02
			1	— 0.15	— 0.01	0.53	0.36
			2	— 2.72	+ 3.06	6.39	+ 5.68

Table 3 (continued)

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i_1	i_2	i_4	i_5	(44) = $\delta\rho_3$		(45) = δw_3	
				sin	cos	sin	cos
				"	"	"	"
2	—6	0	3	+0.04	—1.18	29.28	—1.66
			4		—0.66	1.32	—0.01
			5		—0.05	0.08	
2	—5	0	—1	+0.02			—0.04
			0	—0.02		+ 0.02	+0.34
			1	+0.02	+0.16	— 0.41	+0.06
			2		0.01	— 0.01	
2	—4	0	—3	+0.03	+0.05	+ 0.29	—0.18
			—2	—0.27	—0.17	+ 0.38	—0.69
			—1	+0.13	—0.24	— 0.08	—0.12
2	—3	0	—5	—0.01		— 0.02	—0.04
			—4	+0.01	—0.02	+ 0.03	+ 0.02
			—3	0.00	—0.04	0.04	0.00
			—2	—0.04	—0.05	+ 0.04	—0.04
			—1		—0.03	— 0.03	—0.01
2	—2	0	—1		+0.01		
2	0	0	—4	—0.02	0.01	— 0.01	—0.01
			—3	—0.07	0.05	— 0.04	—0.06
			—2	—0.15	+ 0.10	— 0.22	—0.18
			—1	+0.51	—1.17	— 1.18	—0.52
			0	—0.01	+0.03	+ 0.03	+0.01
3	—5	0	—2	+0.01	0.01		
			—1	+0.11	0.12	0.12	—0.11
3	—2	0	—1	—0.01	+0.01		
3	0	0	—4	0.01			+0.01
			—3	0.03	—0.01	+ 0.01	+0.02
			—2	0.00	0.00	0.00	0.00
			—1	+0.02	—0.04	— 0.04	—0.02

CHAPTER III

RIGHT-HAND MEMBERS OF THE EQUATIONS OF MOTION

9. Derivatives of the perturbing Function

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The expressions $a'\delta R$, $a'\delta\frac{\partial R}{\partial \rho}$, and $a'\delta\frac{\partial R}{\partial W}$, include first and second derivatives of the perturbing function, with respect to the logarithm of the radius vector and longitude in the orbit of the perturbed and perturbing planets. In Part One we obtained expansions

$$(a'R)_0,\ \left(a'\frac{\partial R}{\partial \rho'}\right)_0,\ \left(a'\frac{\partial R}{\partial W'}\right)_0,\ \left(a'\frac{\partial R}{\partial W}\right)_0$$

for the combinations Pluto-Jupiter, Pluto-Saturn, and Pluto-Uranus (see Tables 5, 12, and 17 of Part One). The expansion of $(a'\frac{\sigma R}{\sigma \rho})$ can be obtained from the relation:

$$a'\frac{\partial R}{\partial \rho} = -a'R - a'\frac{\partial R}{\partial \rho'}. \qquad (*)$$

Tables 4, 5, and 6 give the coefficients of expansions $10^7 ma'\frac{\partial R}{\partial \rho'}$, $10^7 ma'\frac{\partial R}{\partial W'}$, $10^7 ma'\frac{\partial R}{\partial \rho}$, $10^7 ma'\frac{\partial R}{\partial W}$ for the combinations Pluto-Jupiter, Pluto-Saturn, and Pluto-Uranus; m is the mass of the perturbing planets. The arguments are: in Table 4, the mean anomalies of Jupiter and Pluto; in Table 5, the mean anomalies of Saturn and Pluto; and in Table 6, the mean anomalies of Uranus and Pluto.

To calculate the second derivatives of the perturbing function, we used the expansion of the perturbing function and its derivatives, with respect to the argument N = (i + k)L' - (i - k)L + m'M' + mM.

This expansion has the form:

$$\begin{aligned} a'R &= \sum P_N \cos N, \\ a'DR &= \sum DP_N \cos N, \\ a'\frac{\partial R}{\partial L'} &= \sum -(i+k)\,P_N \sin N, \\ a'\frac{\partial R}{\partial L} &= \sum -(-i+k)\,P_N \cos N. \end{aligned} \qquad (52)$$

Here,

$$P_N = \sigma^{2k} \sum P^{(k)nn'}_{mm'} e^n e'^{n'},$$

$$DP_N = \sigma^{2k} \sum DP^{(k)nn'}_{mm'} e^n e'^{n'}.$$

Furthermore, we have:

$$a'D'R = \sum -(P_N + DP_N) \cos N. \tag{53}$$

As is known, /37

$$\frac{\partial R}{\partial L'} = \frac{\partial R}{\partial W'} = \frac{\partial R}{\partial \omega'}, \quad \frac{\partial R}{\partial L} = \frac{\partial R}{\partial W} = \frac{\partial R}{\partial \omega}.$$

Differentiating the first formula from Equation (52), with respect to L' and L, we obtain expressions for determining second-order derivatives of a'R:

$$\begin{aligned} a' \frac{\partial^2 R}{(\partial W')^2} &= \sum -(i+k)^2 P_N \cos N, \\ a' \frac{\partial^2 R}{\partial W' \partial W} &= \sum -(i+k)(-i+k) P_N \cos N. \end{aligned} \tag{54}$$

For the purpose of verification, the same quantities can be obtained by differentiating the third formula from Equation (52) with respect to L' and L, and then the fourth formula with respect to L'.

In the same way, using Formula (53), we obtain

$$\begin{aligned} a' \frac{\partial^2 R}{\partial \zeta' \partial W'} &= \sum -(i+k)(P_N + DP_N) \sin N, \\ a' \frac{\partial^2 R}{\partial \zeta' \partial W} &= \sum -(-i+k)(P_N + DR_N) \sin N. \end{aligned} \tag{55}$$

The second derivative of the perturbing function with respect to ρ is calculated by the formula

$$a'D^2R = \sum D^2 P_N \cos N$$

on punched-card machinery, using tables of the coefficients of operators and derivatives of the coefficients of expansions of circular motion, for the cases of Pluto-Jupiter, Pluto-Saturn, and Pluto-Uranus obtained in Part One.

Table 4

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i_1	i'	$(01) = 10^7 m_1 a' \frac{\partial R}{\partial \rho'}$ sin	cos	$(02) = 10^7 m_1 a' \frac{\partial R}{\partial W'}$ sin	cos	$(03) = 10^7 m_1 a' \frac{\partial R}{\partial \rho_1}$ sin	cos	$(04) = 10^7 m_1 a' \frac{\partial R}{\partial W_1}$ sin	cos
0	0	—	— 9672	—	0	—	+ 83	—	0
	1	+ 89	— 2595	— 74	— 44	— 46	135	+ 76	+ 44
	2	35	— 678	— 31	— 16	— 16	55	38	23
	3	11	— 195	— 11	+ 4	— 4	18	16	10
	4	+ 3	— 58	— 3	— 1	— 1	6	7	4
	5		— 18	— 1		+ 1	+ 2	3	2
	6		— 6					+ 1	
	7	— 1	— 1			+ 1	— 1		
1	—8	+ 8	+ 13	+ 10	— 6	— 14	— 23	— 10	6
	—7	31	52	35	— 19	— 51	— 87	— 35	20
	—6	90	152	120	— 67	— 159	— 274	— 120	68
	—5	348	597	495	— 280	— 631	— 1093	— 495	281
	—4	1386	2396	2129	— 1214	— 2605	— 4528	— 2129	1215
	—3	6134	10654	9971	— 5704	— 11849	— 20630	— 9970	5706
	—2	32482	56576	55009	— 31530	— 64028	—111578	— 54998	31534
	—1	+261434	+455684	+452704	—259601	—521078	—908228	—452539	+259547
	0	—101173	—170885	—178322	+ 99134	+202349	+341791	+170798	—101233
	1	+ 4782	— 5991	+ 13300	533	— 9544	11920	6192	+ 4912
	2	510	— 892	1469	148	— 1005	1759	898	514
	3	88	— 182	253	34	— 170	352	176	85
	4	19	— 43	52	9	— 36	81	40	17
	5	5	— 12	13	3	— 9	23	11	4
	6	+ 2	— 3	3	+ 1	— 3	6	+ 3	1
2	—9	— 3	— 2	1	— 2	+ 2	1	— 1	2
	—8	— 10	— 5	3	— 7	7	3	— 3	7
	—7	— 19	— 9	6	— 12	+ 13	+ 6	— 6	12
	—6	— 31	— 5	28	— 34	+ 10	— 16	— 28	34
	—5	— 48	+ 15	84	— 87	— 10	— 85	— 84	87
	—4	— 29	134	267	— 223	— 142	— 374	— 267	223
	—3	+ 328	856	1070	— 724	— 970	— 1880	— 1070	723
	—2	2826	5226	5446	— 3238	— 5977	— 10620	— 5444	3237
	—1	+ 25170	+ 43823	+ 43723	— 25055	— 50241	— 87537	— 43705	+ 25055
	0	— 9746	— 16472	— 17201	+ 9560	+ 19510	+ 32956	+ 16481	— 9772
	1	+ 469	— 576	+ 1291	53	— 930	1156	599	+ 471
	2	51	— 84	143	15	— 99	169	88	48
	3	10	— 18	27	4	— 19	37	18	9
	4	2	— 13	6	1	— 4	+ 17	+ 4	+ 2
3	—4	52	+ 14	22	+ 21	— 53	— 34	— 22	— 21
	—3	79	78	90	— 32	— 123	— 166	— 90	+ 32
	—2	225	429	442	— 267	— 483	— 868	— 442	267
	—1	+ 2048	+ 3570	+ 3568	— 2046	— 4094	— 7138	— 3568	+ 2046
	0	— 796	— 1344	— 1404	+ 779	+ 1592	+ 2688	+ 1344	— 795
	1	+ 38	— 51	+ 108	6	— 76	102	51	+ 38
	2	4	— 8	12	+ 2	— 8	+ 16	+ 8	4
4	—2	19	+ 34	34	— 21	— 39	— 69	— 34	21
	—1	+ 161	+ 281	281	— 161	— 322	— 562	— 281	+ 161
	0	— 60	— 105	— 105	+ 60	+ 120	+ 210	+ 105	— 60

Table 5

i_2	i'	(05) $= 10^7 m_2 a' \frac{\partial R}{\partial \gamma'}$ sin	(05) cos	(06) $= 10^7 m_2 a' \frac{\partial R}{\partial W'}$ sin	(06) cos	(07) $= 10^7 m_2 a' \frac{\partial R}{\partial \gamma_2}$ sin	(07) cos	(08) $= 10^7 m_2 a' \frac{\partial R}{\partial W_2}$ sin	(08) cos
0	0	—	— 2989	—	+ 1	—	+ 90	—	0
	1	— 89	— 885	— 39	44	+ 46	110	+ 38	— 42
	2	— 57	— 241	— 13	29	32	40	19	— 23
	3	— 31	— 70	— 3	16	18	13	7	— 10
	4	— 15	— 19	0	8	9	2	3	— 5
	5	— 7	— 6	0	4	4	+ 1	+ 1	— 2
	6	— 3	— 1	+ 1	1	2			— 1
	7	— 1				1			
1	— 8	— 3	+ 1	1		3	— 1		
	— 7	— 11	4	3	— 1	11	— 5	— 2	+ 1
	— 6	— 31	15	8	— 1	33	— 20	— 6	+ 1
	— 5	— 89	56	29	+ 15	109	— 81	— 27	— 15
	— 4	— 286	216	123	110	406	— 355	— 121	— 110
	— 3	— 1047	879	612	647	1710	— 1487	— 611	— 647
	— 2	— 4829	4251	3580	3981	8827	— 7829	— 3581	— 3981
	— 1	—35844	+31950	+30581	+34344	+70174	—62531	—30581	—34332
	0	+13099	—11953	—11906	—13711	—26208	+23943	+11977	+13119
	1	515	+ 376	+ 147	+ 1046	— 1004	— 723	— 325	462
	2	85	41	+ 7	110	— 152	— 72	— 28	64
	3	21	8		17	— 33	— 12	— 4	11
	4	6	2		3	— 8	— 3	— 1	2
	5	2				— 2			+ 1
	6	1				— 1			
2	— 9			— 1	2	+ 1		+ 1	— 2
	— 8			— 4	7	3	+ 1	4	— 7
	— 7			— 9	15	+ 7	+ 3	8	— 15
	— 6	45	19	— 12	32	— 29	— 15	11	— 32
	— 5	92	33	— 16	69	— 56	— 30	15	— 69
	— 4	163	58	— 11	140	— 86	— 63	+ 11	— 140
	— 3	+ 200	127	+ 45	278	— 20	— 189	— 46	— 277
	— 2	— 178	466	400	676	+ 751	— 876	— 401	— 675
	— 1	— 3973	+ 3488	+ 3468	+ 3861	+ 7846	— 6956	— 3472	— 3858
	0	+ 1458	— 1356	— 1334	— 1535	— 2930	+ 2694	+ 1333	+ 1475
	1	52	+ 34	+ 17	+ 119	— 108	— 75	— 41	55
	2	7	+ 2	1	13	— 15	— 6	— 5	8
	3	+ 1			2	— 3	— 1	— 1	2
	4				+ 1	— 1			1
3	—10			3	— 1	— 1	— 1	— 3	1
	— 9			8	— 2	— 1	— 3	— 8	3
	— 8			14	— 3	— 1	— 4	— 14	3
	— 7			21	— 8	— 3	— 7	— 22	8
	— 6			35	— 16	— 5	— 11	— 35	17
	— 5			51	— 27	— 9	— 16	— 50	27
	— 4	— 45	— 73	58	— 31	+ 37	+ 53	— 58	31
	— 3	— 33	— 37	43	— 9	37	+ 18	— 41	+ 10
	— 2	— 18	+ 65	25	+ 65	71	— 99	— 22	— 64
	— 1	— 375	+ 337	+ 328	+ 367	+ 743	— 665	— 325	— 367
	0	+ 139	— 126	— 127	— 146	— 279	+ 254	+ 128	+ 140
	1	+ 5	+ 4	+ 1	+ 5	— 10	— 8	— 3	5
	2				1	— 1			+ 1
4	— 2	— 1	5	3	6	+ 6	— 8	— 3	— 7
	— 1	— 34	+ 31	+ 30	+ 34	+ 68	— 61	— 30	— 34
	0	+ 12	— 11	— 11	— 12	— 24	+ 22	+ 11	+ 12
	1	1			+ 1	— 2			1

To determine the second derivatives with respect to ρ and ρ' we used the formulas:

$$\begin{aligned} a'\frac{\partial^2 R}{\partial \rho'^2} &= -2a'\frac{\partial R}{\partial \rho'} + a'\frac{\partial^2 R}{\partial \rho^2} - a'R, \\ a'\frac{\partial^2 R}{\partial \rho' \partial \rho} &= a'\frac{\partial R}{\partial \rho'} - a'\frac{\partial^2 R}{\partial \rho^2} + a'R, \\ a'\frac{\partial^2 R}{\partial \rho \partial W'} &= -a'\frac{\partial R}{\partial W'} - a'\frac{\partial^2 R}{\partial \rho' \partial W'}, \end{aligned} \tag{56}$$

which are easily derived by means of relation (*).

Tables 7, 8, 9, 10, 11, and 12 give $10^7 ma'\frac{\partial^2 R}{\partial \rho'^2}$, $10^7 ma'\frac{\partial^2 R}{\partial \rho' \partial W}$, $10^7 ma'\frac{\partial^2 R}{\partial \rho' \partial \rho}$, $10^7 ma'\frac{\partial^2 R}{\partial \rho^2}$, $10^7 ma'\frac{\partial^2 R}{\partial W'^2}$, $10^7 ma'\frac{\partial^2 R}{\partial W' \partial \rho'}$, $10^7 ma'\frac{\partial^2 R}{\partial W' \partial W}$, $10^7 ma'\frac{\partial^2 R}{\partial W' \partial \rho}$.

To check the calculation of the derivatives of a'R we used the following relations:

$$\begin{aligned} a'D_t'\frac{\partial R}{\partial W'} &= a'\frac{\partial^2 R}{\partial W' \partial \rho'}\frac{d\rho'}{dt} + a'\frac{\partial^2 R}{(\partial W')^2}\frac{dw'}{dt}, \\ a'D_t'\frac{\partial R}{\partial W} &= a'\frac{\partial^2 R}{\partial W \partial \rho'}\frac{d\rho'}{dt} + a'\frac{\partial^2 R}{\partial W \partial W'}\frac{dw'}{dt}, \\ a'D_t'\frac{\partial R}{\partial \rho'} &= a'\frac{\partial^2 R}{\partial \rho'^2}\frac{d\rho'}{dt} + a'\frac{\partial^2 R}{\partial \rho' \partial W'}\frac{dw'}{dt}, \\ a'D_t'\frac{\partial R}{\partial \rho} &= a'\frac{\partial^2 R}{\partial \rho \partial \rho'}\frac{d\rho'}{dt} + a'\frac{\partial^2 R}{\partial \rho \partial W'}\frac{dw'}{dt}, \\ a'D_t\frac{\partial R}{\partial \rho} &= a'\frac{\partial^2 R}{\partial \rho^2}\frac{d\rho}{dt} + a'\frac{\partial^2 R}{\partial \rho \partial W}\frac{dw}{dt}. \end{aligned}$$

We find the derivative $a'\frac{\partial^2 R}{\partial \rho \partial W}$, which we need for verification, from the equality: /53

$$a'\frac{\partial^2 R}{\partial \rho \partial W} = -a'\frac{\partial R}{\partial W} - a'\frac{\partial^2 R}{\partial W \partial \rho'}.$$

10. Perturbations of Plutonian Coordinates and Certain Auxiliary Expressions

The expansions of $\delta\rho'$, $\delta w'$ and $m\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt + C_2\right]$, which we need for determining the second members, are given in Table 13. The mean anomalies of Jupiter, Saturn, Uranus, and Pluto serve as the arguments.

The coefficients of expansions $\frac{a'}{r'}$ and $\left(\frac{r'}{a'}\right)^2$ with respect to cos jM' (for e' = 0.24707348) are given in Table 14.

Table 6

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i_3	i'	(09) $= 10^7 m_3 a' \frac{\partial R}{\partial \rho'}$ sin	(09) cos	(10) $= 10^7 m_3 a' \frac{\partial R}{\partial W'}$ sin	(10) cos	(11) $= 10^7 m_3 a' \frac{\partial R}{\partial \rho_3}$ sin	(11) cos	(12) $= 10^7 m_3 a' \frac{\partial R}{\partial W_3}$ sin	(12) cos
0	0	—	— 527	—	+ 1	—	+ 62	—	— 1
	1	— 32	— 159	+ 9	14	+ 19	39	— 10	— 14
	2	— 30	— 30	12	13	20	+ 3	— 6	— 6
	3	— 20	+ 3	9	8	14	— 8	— 3	— 2
	4	— 10	7	6	4	7	— 7	— 1	
	5	— 4	6	3	1	3	— 5		
	6	— 1	3	2		1	— 2		
	7	— 1	+ 2	+ 1		+ 1	— 2		
1	— 8	+ 3			2	— 3			
	— 7	5		— 1	3	— 4			— 1
	— 6	7	— 3	0	5	— 6	+ 2	— 1	— 2
	— 5	+ 5	— 14	+ 4	8	— 4	10	— 3	— 3
	— 4	— 11	— 38	9	11	+ 11	30	— 8	— 5
	— 3	— 83	— 100	+ 11	17	87	92	— 10	— 12
	— 2	— 347	— 288	— 55	101	439	347	+ 55	— 102
	— 1	—1801	—1329	—918	+1232	+3033	+2248	+918	—1231
	0	+ 520	+ 392	+422	— 550	—1065	— 781	—392	+ 546
	1	— 11	52	— 35	+ 14	+ 14	— 73	— 23	— 3
	2	+ 1	22	0	— 1	— 1	— 21	+ 1	
	3	2	10	+ 1	— 1	— 2	— 9	2	
	4	2	4		— 1	— 2	— 3	+ 1	
	5	2	+ 1		— 1	— 2	— 1		
	6	1				— 1			
2	— 9		— 4	3			+ 3	— 1	+ 1
	— 8	+ 1	— 8	5	+ 1	— 2	7	— 4	1
	— 7	— 2	— 9	6	0	0	7	— 4	2
	— 6	— 18	— 11	8	— 10	+ 11	10	— 5	11
	— 5	— 49	— 10	+ 8	— 29	33	+ 10	— 5	30
	— 4	— 109	+ 6	— 2	— 65	75	— 1	+ 4	65
	— 3	— 190	40	— 26	— 114	134	— 24	26	113
	— 2	— 228	+ 59	— 55	— 119	177	— 26	56	+ 118
	— 1	— 108	— 101	— 92	+ 144	+ 247	+ 196	+ 95	— 148
	0	+ 71	+ 50	+ 39	— 52	— 117	— 83	— 30	+ 41
	1	11	9	— 5	+ 1	— 8	— 10		— 5
	2	6	+ 2	— 1		— 5	— 2		— 2
	3	2		— 1		— 2			— 1
	4	+ 1	— 1			— 1	+ 1		
3	—10	— 5	— 1	+ 1	— 4	+ 4	1	+ 1	+ 3
	— 9	— 10	— 4	3	— 7	8	4	— 1	7
	— 8	— 11	— 1	+ 1	— 8	8	+ 2	+ 1	8
	— 7	— 21	+ 17	— 11	— 15	16	— 12	13	14
	— 6	— 36	42	— 29	— 26	28	— 31	30	25
	— 5	— 47	81	— 56	— 35	36	— 61	57	34
	— 4	— 48	113	— 81	— 37	36	— 86	80	35
	— 3	— 27	+ 88	— 65	— 23	+ 19	— 67	+ 64	+ 20
	— 2	+ 12	— 27	+ 17	+ 4	— 9	+ 22	— 18	— 9
	— 1	7	— 15	— 7	+ 14	+ 7	+ 21	+ 4	— 21
	0	11	— 1	+ 3	— 3	— 14	— 3	— 5	0
	1	+ 2	— 2		+ 1	— 2	+ 2	— 1	— 1
	2		— 1				+ 1		
4	—11	— 2	+ 6	— 5	— 2	+ 2	— 5	+ 5	
	—10	— 13	12	— 9	— 10	12	— 9	10	+ 8
	— 9	— 9	12	— 9	— 7	+ 8	— 9	10	+ 6
	— 8	+ 6	27	— 21	+ 4	— 4	— 21	21	— 5
	— 7	14	46	— 36	10	— 11	— 37	35	— 11
	— 6	28	60	— 48	20	— 23	— 48	+ 47	— 21
	— 5	35	57	— 46	26	— 28	— 45	— 40	+ 30
	— 4	+ 20	+ 24	— 22	+ 14	— 17	— 19	+ 19	— 16
	— 3	— 9	— 21	+ 12	— 9	+ 7	+ 18	— 15	+ 8
	— 2	+ 4	— 11	3	+ 2	— 3	9	— 7	— 3

Table 6 (continued)

i_3	i'	(09) $= 10^7 m_3 a' \frac{\partial R}{\partial \rho'}$		(10) $= 10^7 m_3 a' \frac{\partial R}{\partial W'}$		(11) $= 10^7 m_3 a' \frac{\partial R}{\partial \rho_3}$		(12) $= 10^7 m_3 a' \frac{\partial R}{\partial W_3}$	
		sin	cos	sin	cos	sin	cos	sin	cos
4	— 1	+ 1	— 4	1		— 1	3	— 2	
	0		— 1	+ 1			+ 1	— 1	
	1	— 1				+ 1			
5	—12	+13	+ 1	— 1	10	—11	— 1	+ 1	—10
	—11	12	13	—11	10	—10	—11	11	—10
	—10	5	12	—10	4	— 4	—10	10	— 4
	— 9	24	6	— 5	19	—20	— 5	5	—19
	— 8	36	9	— 8	30	—30	— 7	8	—30
	— 7	40	8	— 8	33	—33	— 7	7	—32
	— 6	32	+ 2	— 3	26	—27	— 1	+ 2	—26
	— 5	+ 7	— 3	0	+ 7	— 6	+ 3	— 2	— 6
	— 4	—15	— 5	+ 1	—10	+13	5	— 3	+12
	— 3	— 3	— 6	3	— 1	3	5	— 4	+ 2
	— 2	— 1	— 1			+ 1	1	— 1	
6	—13	+ 9	—11	9	+ 8	— 8	9	— 9	— 8
	—12	19	— 3	+ 2	16	—16	+ 2	— 2	—16
	—11	13	+ 7	— 7	11	—11	— 6	+ 7	—11
	—10	16	—11	+ 9	14	—14	+ 9	— 9	—14
	— 9	18	—18	15	17	—15	15	—15	—16
	— 8	13	—13	11	13	—11	11	—11	—11
	— 7	+ 6	— 8	+ 7	+12	— 5	+ 7	— 7	—14
	— 6	— 4	+ 1	0	— 1	+ 4	— 1	0	+ 3
	— 5	— 8	+ 5	— 4	— 5	7	— 4	+ 4	6
	— 4	— 4	— 1	+ 1	— 2	4	+ 1	— 1	3
	— 3	— 1				1			
7	—14	— 4	—15	13	— 3	4	13	—13	3
	—13	— 5	—22	19	— 5	4	19	—19	5
	—12	— 2	— 8	7	— 2	+ 2	7	— 7	+ 2
	—11	+ 1	—12	10	+ 1	— 1	11	—10	— 1
	—10	+ 1	—18	16	+ 1	— 1	16	—16	— 1
	— 9	— 1	—11	10	— 1	+ 1	10	—10	+ 1
	— 8	+ 1	— 5	+ 4	+ 1	— 1	+ 4	— 4	— 1
	— 7	+ 1	+ 1	0	+ 1	— 1	— 1	0	— 1
	— 6	0	+ 2	— 2	0	1	— 2	+ 2	0
	— 5	— 1			— 1	0			+ 1
8	—12	— 1	— 5	+ 4	— 1	+ 1	+ 4	— 4	1
	—11	— 1	— 6	5	— 1	1	5	— 5	1
	—10		— 1	1		1	1	— 1	
	— 9		— 1	0			1	0	
	— 8		— 1	+ 1			1	— 1	
	— 7			0				0	
	— 6			— 1				+ 1	

Table 7

i_1	i'	(13) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho'^2}$		(14) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho' \partial W_1}$		(15) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho' \partial \rho_1}$		(16) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho_1^2}$	
		sin	cos	sin	cos	sin	cos	sin	cos
0	0	—	+9925	—	0	—	—253	—	+170
	1	—181	2936	—154	—89	+93	—341	—47	206
	2	— 64	808	— 78	—47	28	—130	—12	75
	3	— 14	238	— 34	—21	+ 4	— 42	0	24
	4	— 1	71	— 13	— 9	— 2	— 12	+ 3	6
	5	+ 2	22	— 6	— 4	— 3	— 4	2	2
	6	4	6	— 2	— 1	— 3		3	
	7	2				— 1	+ 1		

Table 7 (continued)

i_1	i'	(13) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho'^2}$ sin	(13) cos	(14) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho' \partial W}$ sin	(14) cos	(15) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho' \partial \rho_1}$ sin	(15) cos	(16) $= 10^7 m_1 a' \frac{\partial \cdot R}{\partial \rho_1^2}$ sin	(16) cos
1	— 9	8	13	— 3	+ 2	— 8	— 13	8	13
	— 8	+ 5	+ 7	— 14	8	— 12	— 20	26	43
	— 7	— 3	— 3	— 48	29	— 27	— 49	78	136
	— 6	+ 48	+ 95	— 162	102	— 137	— 246	296	520
	— 5	208	392	— 604	362	— 557	— 989	1188	2082
	— 4	1023	1843	— 2407	1407	— 2410	— 4238	5015	8766
	— 3	5235	9234	— 10661	6162	— 11369	— 19888	23218	40518
	— 1	30529	53349	— 56597	32521	— 63011	—109926	127039	221504
	— 1	+257819	+449340	—455676	+261430	—519252	—905023	+1040330	+1815251
	0	—101190	—170959	+170803	—101231	+202363	+341843	— 404712	— 685654
	1	+ 4750	— 5845	6240	+ 4949	— 9531	11837	+ 19075	— 23757
	2	475	— 834	926	529	— 985	1726	1990	— 3485
	3	74	— 156	159	91	— 162	338	332	— 690
	4	11	— 31	45	18	— 31	74	67	— 155
	5	+ 3	— 6	11	+ 5	— 7	18	16	— 41
	6	— 1	— 2	4		— 1	5	4	— 11
	7	+ 1	— 4				+ 4		— 4
	8		— 1						
2	— 9	8	+ 6	4	— 6	— 5	— 4	3	+ 3
	— 8	30	15	9	— 21	— 20	— 11	13	8
	— 7	72	42	13	— 40	— 52	— 34	39	28
	— 6	132	75	22	— 76	— 102	— 71	92	87
	— 5	270	162	+ 32	— 139	— 223	— 178	233	263
	— 4	593	458	— 53	— 191	— 565	— 592	707	966
	— 3	1293	1377	— 720	+ 77	— 1621	— 2234	2591	4114
	— 2	3828	5766	— 5075	2567	— 6654	— 10993	12631	21613
	— 1	+ 24924	+ 43575	— 43856	+ 25208	— 50094	— 87395	+ 100335	+ 174932
	0	— 9788	— 16521	+ 16443	— 9755	+ 19534	+ 32993	— 39044	— 65949
	1	+ 451	— 596	583	+ 475	— 919	1171	+ 1849	— 2327
	2	45	— 90	81	51	— 95	174	194	— 343
	3	6	— 19	15	8	— 14	37	33	— 74
	4	1	+ 12	3		— 3	1	+ 7	— 18
	5	+ 1	— 6				+ 6		— 6
	6		— 1						
3	—10	— 5			7	+ 5		— 5	
	— 9	— 12		2	17	12		— 12	
	— 8	— 20	+ 2	4	28	20	— 2	— 20	+ 2
	— 7	— 36	5	5	47	36	— 5	— 36	5
	— 6	— 60	4	6	78	60	— 4	— 60	4
	— 5	— 77	25	+ 3	118	77	— 25	— 77	25
	— 4	— 163	33	— 10	141	+ 112	— 48	— 59	82
	— 3	— 74	85	— 65	118	— 5	— 163	+ 128	329
	— 2	+ 339	459	— 415	182	— 564	— 888	1047	1756
	— 1	+ 2030	+ 3535	— 3556	+ 2039	— 4078	— 7105	+ 8172	+ 14243
	0	— 790	— 1337	+ 1342	— 796	+ 1586	+ 2682	— 3178	— 5370
	1	+ 41	— 45	50	+ 36	— 80	95	+ 156	— 197
	2	4	— 6	8	+ 4	— 7	13	15	— 29
	3	3	— 6	+ 2	—	— 3	6	3	— 6
	4	1	— 2						
4	—11			— 2	— 3	— 3		3	
	—10			— 6	— 8	— 6	5	6	— 5
	— 9			— 7	— 10	— 8	6	8	— 6
	— 8			— 10	— 12	— 9	7	9	— 7
	— 7			— 13	— 17	— 14	11	14	— 11
	— 6			— 16	— 20	— 16	12	16	— 12
	— 5			— 13	— 14	— 11	11	+ 11	— 11
	— 4			— 5	+ 3	+ 2	+ 3	— 2	— 3
	— 3	2	+ 29	— 3	21	— 2	— 29	+ 2	+ 29
	— 2	24	35	— 33	17	— 43	— 70	82	139
	— 1	+ 141	+ 248	— 271	+ 157	— 303	— 528	+ 625	+ 1090
	0	— 62	— 94	+ 103	— 61	+ 123	+ 199	— 243	— 409

Table 7 (continued)

i_1	i'	(13) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho'^2}$ sin	(13) cos	(14) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho' \partial W_1}$ sin	(14) cos	(15) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho' \partial \rho_1}$ sin	(15) cos	(16) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial \rho_1^2}$ sin	(16) cos
4	1	+11	−15	4	+ 3	−11	15	+11	−15
	2	1	− 3				+ 3		− 3
5	−12			2			− 2		+ 2
	−11			4	2	+ 2	− 4	− 2	4
	−10			4	+ 2	+ 2	− 4	− 2	4
	− 9			3	0	0	− 3	0	3
	− 8			4	0	0	− 3	0	3
	− 7			4	0	0	− 3	0	3
	− 6			+ 3	0	0	− 1	0	+ 1
	− 5			0	− 3	− 2	0	+ 2	0
	− 4			− 2	− 2	− 1	+ 1	+ 1	− 1
	− 3			0	0	+ 2	0	− 2	0
	− 2			0	0	0	0	0	0
	− 1			−21	+11	−47	−82	+47	+82

Table 8

i_2	i'	(17) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial \rho'^2}$ sin	(17) cos	(18) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial \rho' \partial W_2}$ sin	(18) cos	(19) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial \rho' \partial \rho_2}$ sin	(19) cos	(20) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial \rho_2^2}$ sin	(20) cos
0	0	—	+ 3271	—	+ 1	—	− 282	—	+ 192
	1	+ 194	1194	− 78	87	− 104	− 310	+ 58	200
	2	143	351	− 39	51	− 86	− 109	54	69
	3	85	100	− 15	24	− 54	− 31	36	18
	4	45	23	− 5	11	− 29	− 3	20	+ 1
	5	21	+ 4	− 2	5	− 13	+ 2	9	− 3
	6	8	− 2	− 1	+ 1	− 6	3	4	− 3
	7	2	− 1			− 2	+ 1	+ 1	− 1
1	− 8	3	+ 1	− 1	− 3	− 1	− 4	− 2	+ 5
	− 7	18	6	− 3	− 15	− 6	− 11	− 5	16
	− 6	45	8	− 11	− 41	− 14	− 23	− 19	43
	− 5	84	8	− 51	− 107	+ 6	− 64	− 115	145
	− 4	+ 115	29	− 209	− 318	171	− 245	− 577	580
	− 3	− 158	318	− 870	− 1097	1206	− 1197	− 2916	2684
	− 2	− 3005	2833	− 4244	− 4881	7834	− 7084	− 16661	14913
	− 1	−32710	+29094	−31946	−35836	+68555	−61043	−138729	+123574
	0	+13142	−12086	+11957	+13111	−26240	+24040	+ 52448	− 47983
	1	451	+ 294	− 344	485	− 965	− 670	1969	+ 1393
	2	+ 34	+ 10	− 37	81	− 120	− 51	272	123
	3	− 7	− 2	− 7	20	− 14	− 6	47	18
	4	− 7	− 1	− 2	6	+ 1	0	7	3
	5	− 4		− 1	0	1	− 1	1	+ 1
	6	− 1			1			+ 1	
2	− 9	− 2	− 3	− 2	7	2	+ 3	− 3	− 3
	− 8	− 11	− 13	− 16	23	12	13	− 15	− 14
	− 7	− 25	− 25	− 33	47	25	25	− 32	− 28
	− 6	− 149	− 73	− 51	97	104	52	− 75	− 37
	− 5	− 316	− 106	− 79	199	224	73	− 168	− 43
	− 4	− 602	− 150	− 117	361	439	+ 72	− 353	− 9
	− 3	− 998	− 78	− 183	503	797	− 50	− 777	+ 239
	− 2	− 1413	+ 324	− 488	+ 130	1592	− 788	− 2343	1664
	− 1	− 3719	+ 3473	− 3474	− 4018	+ 7692	− 6960	− 15538	+ 13916
	0	+ 1498	− 1294	+ 1367	+ 1450	− 2957	+ 2650	+ 5887	− 5344
	1	69	+ 60	− 26	47	− 120	− 94	228	+ 169
	2	14	11	+ 1	5	− 21	− 13	36	19
	3	5	3	1	+ 1	− 5	− 3	8	4
	4	1	1			− 2		3	
3	−10	2	7	10	− 5	− 2	− 7	3	8

Table 8 (continued)

i_2	i'	(17) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial \rho'^2}$		(18) $= 10^7 m_2 a' \frac{\partial_2 R}{\partial \rho' \partial W_2}$		(19) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial \rho' \partial \rho_2}$		(20) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial \rho_2^2}$	
		sin	cos	sin	cos	sin	cos	sin	cos
3	— 9	3	26	37	— 9	— 3	— 26	4	29
	— 8	5	45	62	— 11	— 5	— 45	6	49
	— 7	20	69	94	— 31	— 20	— 68	23	75
	— 6	46	107	149	— 67	— 46	—108	51	119
	— 5	79	149	210	—112	— 79	—149	88	165
	— 4	186	312	228	—142	—141	—238	104	185
	— 3	+105	+190	+127	— 96	— 73	—153	+ 36	135
	— 2	—135	— 57	—111	+ 19	+154	— 8	— 225	107
	— 1	—353	+303	—348	—380	+727	—639	—1470	+1304
	0	+143	—134	+123	+137	—280	+260	+ 559	— 514
	1	7	+ 1	— 5	5	— 12	— 5	22	+ 13
	2	+ 2	+ 1	— 1	+ 1	— 2	— 1	3	+ 1
	3	— 1							
4	—11	+ 4	— 9	— 12	— 4	— 4	+ 9	4	— 9
	—10	17	— 25	— 31	— 20	— 17	25	17	— 25
	— 9	25	— 32	— 39	— 29	— 25	32	25	— 32
	— 8	26	— 43	— 52	— 30	— 26	43	26	— 43
	— 7	29	— 64	— 77	— 35	— 29	64	29	— 64
	— 6	30	— 79	— 95	— 36	— 30	79	30	— 79
	— 5	22	— 66	— 80	— 29	— 22	66	22	— 66
	— 4	+ 5	— 15	— 18	— 11	— 6	+ 15	+ 6	— 15
	— 3	— 6	+ 35	+ 44	+ 7	+ 6	— 35	— 6	+ 35
	— 2	— 19	— 1	— 6	+ 7	20	— 4	— 26	12
	— 1	— 35	+ 29	— 31	— 34	+ 68	— 60	— 136	+ 121
	0	+ 13	— 11	+ 11	+ 13	— 26	+ 23	+ 50	— 45
	1	— 3				+ 2			
5	—12	— 9	+ 5	6	10	9	— 5	— 9	+ 5
	—11	— 24	5	6	27	24	— 5	— 24	5
	—10	— 25	2	2	29	25	— 2	— 25	2
	— 9	— 23	3	4	28	23	— 3	— 23	3
	— 8	— 27	7	8	32	27	— 7	— 27	7
	— 7	— 27	9	10	31	27	— 9	— 27	9
	— 6	— 15	+ 5	+ 5	+ 18	+ 15	— 5	— 15	+ 5
	— 5	+ 3	— 3	— 4	— 3	— 3	+ 3	+ 3	— 3
	— 4	+ 11	— 5	— 7	— 14	— 11	+ 5	+ 11	— 5
	— 3	0	+ 5	+ 5	— 1	0	— 5	0	+ 5
	— 2	— 1			+ 1	+ 1		— 1	
6	—13			3	— 8	— 7	— 3	+ 7	3
	—12			10	— 15	— 13	— 9	13	9
	—11			10	— 10	— 9	— 9	9	9
	—10			7	— 6	— 5	— 6	5	6
	— 9			7	— 6	— 5	— 6	5	6
	— 8			6	— 7	— 6	— 4	6	4
	— 7			+ 2	— 2	— 2	— 2	+ 2	+ 2
	— 6			— 2	+ 4	+ 4	+ 2	— 3	— 2
	— 5			— 2	+ 3	+ 3	2	— 3	— 2
	— 4			— 1	— 2	— 2	1	+ 2	— 1
7	—14			— 5	+ 2	+ 2	4	— 2	— 4
	—13			— 7	3	3	6	— 3	— 6
	—12			— 3	+ 1	+ 1	2	— 1	— 2
	—11			— 1	— 2	— 2	1	+ 2	— 1
	—10			— 3	— 3	— 2	2	2	— 2
	— 9			— 2	— 1	0	+ 2	0	— 2
	— 8			— 1	0	0	0	0	0
	— 7			+ 1	0	0	— 1	0	+ 1
	— 6			+ 1	0	0	— 1	0	+ 1
	— 5			— 1	+ 1	0	+ 1	0	— 1
	— 4					— 1		+ 1	
8	—12			— 2	1	+ 1	2	— 1	— 2
	—11			— 2	+ 1	+ 1	2	— 1	— 2
	—10				0	0		0	
	— 9				— 1	— 1		+ 1	
	— 8				— 1	— 1		1	

Table 9

i_3	i'	(21) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial \varphi'^2}$		(22) $= -10^7 m_3 a' \frac{\partial^2 R}{\partial \varphi' \partial W_3}$		(23) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial \varphi' \partial \varphi_3}$		(24) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial \varphi_3^2}$	
		sin	cos	sin	cos	sin	cos	sin	cos
0	0	—	+739	—	+ 2	—	— 211	—	+ 149
	1	+ 95	300	+ 25	32	— 64	— 141	+ 45	+ 102
	2	119	+ 30	16	14	— 88	0	68	— 3
	3	94	— 54	8	+ 3	— 73	+ 52	59	— 44
	4	47	— 51	3	— 1	— 36	44	29	— 37
	5	20	— 42	1	— 2	— 16	36	+ 13	— 31
	6	1	— 21	+ 1		0	17	— 1	— 15
	7	+ 3	— 12			— 3	10	+ 2	— 8
1	— 8	— 20	— 3		+ 3	+ 18	3	— 15	— 3
	— 7	— 32	— 14	— 1	7	27	13	— 23	— 13
	— 6	— 55	— 2	— 1	9	48	+ 5	— 42	— 7
	— 5	— 68	+ 32	+ 10	12	63	— 19	— 59	+ 9
	— 4	— 64	93	38	+ 6	75	— 55	— 86	25
	— 3	+ 47	195	106	— 54	36	— 94	— 123	+ 2
	— 2	+ 235	+ 280	300	— 315	112	+ 8	— 551	— 355
	— 1	—478	—396	+1334	—1806	+2280	+1726	—5313	—3974
	0	+ 627	+ 366	— 380	+ 517	—1147	— 759	+2213	+1540
	1	+ 16	— 88	— 48	— 15	— 5	+ 35	— 9	+ 38
	2	— 11	— 96	— 22	— 2	+ 11	74	— 10	— 53
	3	— 18	— 49	— 11	0	15	40	— 13	— 31
	4	— 15	— 25	— 5	+ 2	14	20	— 12	— 17
	5	— 11	— 5	— 2	1	10	4	— 8	— 3
	6	— 6	— 3	— 1	+ 1	5	+ 2	— 4	— 2
	7			— 1	— 1				
2	— 9	— 11	+ 35	+ 9		10	— 29	— 10	+ 26
	— 8	— 28	46	18	+ 6	27	— 39	— 25	32
	— 7	— 21	55	20	+ 3	+ 24	— 46	— 24	39
	— 6	+ 36	74	28	— 27	— 18	— 63	+ 7	53
	— 5	141	83	38	— 88	— 91	— 73	58	63
	— 4	346	+ 25	+ 1	— 213	— 237	— 32	162	+ 33
	— 3	613	—133	— 64	— 372	— 422	+ 94	288	— 70
	— 2	+674	—269	— 139	— 443	— 445	209	+ 268	— 183
	— 1	—234	—125	+ 98	— 82	+ 342	+ 227	— 589	— 423
	0	— 53	— 31	— 65	+ 90	— 19	— 19	+ 136	+ 102
	1	— 73	— 32	— 16	23	+ 61	+ 23	— 53	— 13
	2	— 34	— 6	— 5	11	29	+ 5	— 24	— 3
	3	— 18	+ 1	0	5	15	— 1	— 13	+ 1
	4	— 5	5	+ 11	7	4	— 5	— 3	4
	5	— 2	2	1	+ 1	+ 2	— 2	— 2	2
3	—10	+ 32	27	3	— 15	— 27	— 26	+ 23	25
	— 9	44	55	20	— 31	— 35	— 48	27	44
	— 8	46	+ 38	+ 14	— 31	— 34	— 38	26	+ 36
	— 7	99	— 40	— 38	— 62	— 78	+ 24	62	— 12
	— 6	169	—142	— 108	— 115	— 133	101	105	— 70
	— 5	210	—328	— 227	— 150	— 163	247	127	— 186
	— 4	192	—474	— 333	— 145	— 145	362	109	— 276
	— 3	+ 87	—378	— 271	— 71	— 59	+ 290	+ 40	— 223
	— 2	— 94	+116	+ 79	+ 47	+ 82	— 89	— 73	+ 67
	— 1	—109	33	27	41	103	— 17	— 110	— 4
	0	— 43	29	7	25	32	— 29	— 18	+ 32
	1	— 12	16	6	7	10	— 14	— 8	12
	2	— 3	9	3	+ 1	+ 3	— 8	— 3	7
	3	+ 2	3	1	— 1	— 2	— 2	+ 2	2
	4	1	+ 2	+ 1		— 1	— 1	1	+ 1
4	—11	38	— 27	— 25	— 10	— 35	+ 21	33	— 16
	—10	109	— 56	— 48	— 63	— 97	43	85	— 34
	— 9	+ 79	— 54	— 42	— 50	— 70	40	+ 62	— 31
	— 8	— 11	—133	— 101	+ 17	+ 5	105	— 1	— 84
	— 7	— 52	—229	— 179	45	38	183	— 27	— 146
	— 6	—139	—307	— 239	102	112	247	— 89	— 199
	— 5	—188	—282	— 225	141	152	225	— 124	— 180

Table 9 (continued)

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i_3	i'	(21) $= 10^7 m_3 a' \frac{\partial R}{\partial \varepsilon'_{23}}$ sin	(21) cos	(22) $= 10^7 m_3 a' \frac{\partial R}{\partial \varepsilon' \partial W_3}$ sin	(22) cos	(23) $= 10^7 m_3 a' \frac{\partial R}{\partial \varepsilon' \partial \varepsilon_{33}}$ sin	(23) cos	(24) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial \varepsilon_{23}^2}$ sin	(24) cos
4	— 4	—120	—106	— 91	+ 90	+100	+ 82	— 83	— 63
	— 3	+ 39	+136	+ 89	— 34	— 29	—114	+ 22	+ 96
	— 2	— 23	75	44	+ 16	+ 19	— 63	— 16	54
	— 1	— 6	28	15	+ 1	+ 6	— 24	— 5	21
	0	+ 5	8	4	— 2	— 5	— 8	+ 5	7
	1	4	+ 3	+ 1	— 2	— 4	— 3	3	+ 3
	2	2			— 1	— 2		2	
	3	+ 1			— 1	— 1		+ 1	
5	—12	— 77	— 4	— 3	+ 63	+ 65	+ 4	— 54	— 3
	—11	— 66	— 88	— 74	52	54	75	— 44	— 64
	—10	— 19	— 81	— 67	12	13	68	— 9	— 58
	— 9	—137	— 34	— 27	110	114	28	— 94	— 23
	— 8	—212	— 54	— 47	175	176	44	—146	— 37
	— 7	—188	— 41	— 41	197	148	+ 33	—115	— 26
	— 6	—190	+ 5	— 2	159	159	— 7	—132	+ 8
	— 5	— 37	37	+ 23	+ 35	+ 31	— 34	— 25	31
	— 4	+100	42	25	— 74	— 85	— 38	+ 72	33
	— 3	25	41	28	— 15	— 22	— 35	+ 19	30
	— 2	10	+ 9	+ 3	— 4	— 9	— 8	8	+ 7
	— 1	4	— 2	— 1	— 2	— 3	+ 2	3	— 2
	0	+ 3	— 1	— 2	— 1	— 2	0	+ 2	0
	1		— 2	— 1			+ 1		— 1
6	—13	— 66	+ 79	+ 67	+ 56	+ 57	— 68	— 49	+ 59
	—12	—134	+ 7	+ 2	114	116	— 3	—100	+ 1
	—11	— 99	— 71	— 63	84	85	+ 62	— 74	— 56
	—10	—111	+ 74	+ 62	93	95	— 63	— 81	+ 54
	— 9	—118	126	107	104	99	—108	— 84	93
	— 8	— 81	90	76	74	68	— 76	— 57	65
	— 7	— 27	+ 57	+ 49	+ 31	+ 21	— 49	— 16	+ 42
	— 6	+ 40	— 4	— 2	— 27	— 37	+ 4	+ 33	— 3
	— 5	70	— 36	— 29	— 50	— 62	+ 31	55	— 27
	— 4	29	+ 9	+ 8	— 20	— 27	— 7	23	+ 6
	— 3	9	1	1	— 4	— 8	— 1	7	1
	— 2	2			— 1	— 2		2	
	— 1	2			— 1	— 2		2	
7	—14	29	120	105	— 25	— 26	—105	22	92
	—13	43	177	154	— 37	— 37	—155	33	136
	—12	+ 15	62	54	— 13	— 13	— 54	+ 11	47
	—11	— 7	90	78	+ 5	+ 6	— 79	— 5	68
	—10	— 4	142	123	+ 3	+ 3	—124	— 2	108
	— 9	+ 6	88	77	— 6	— 6	— 77	+ 5	67
	— 8	— 6	+ 37	+ 32	+ 4	+ 5	— 32	— 4	+ 28
	— 7	— 10	— 6	— 5	+ 8	+ 8	+ 6	— 7	— 5
	— 6	+ 1	— 18	— 16	— 1	— 1	+ 16	+ 1	— 14
	— 5	9	+ 3	+ 3	— 7	— 8	— 2	7	+ 2
	— 4	2			— 1	— 1		1	
8	—12	10	40	35	— 8	— 8	— 35	7	31
	—11	12	49	42	— 10	— 10	— 42	9	37
	—10	+ 2	7	6	— 1	— 1	— 7	+ 1	6
	— 9	— 3	4	4	+ 3	+ 2	— 4	— 2	3
	— 8	— 2	+ 9	+ 7	+ 2	+ 1	— 8	— 1	+ 7
	— 7	0	— 1	— 1	0	0	+ 1	0	— 1
	— 6	0	— 3	— 3	0	0	+ 3	0	— 3
	— 5	+ 1	+ 1	+ 1	— 1	— 1	— 1	+ 1	+ 1
	—10	1	5	5	— 1	— 1	— 4	1	4
	— 9	1	+ 5	+ 4	— 1	— 1	+ 1		+ 4
	— 8		— 1	— 1			— 1		— 1
	— 7		— 1	— 1					— 1

Table 10

i_1	i'	(25) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W'^2}$ sin	(25) cos	(26) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W' \partial \varphi'}$ sin	(26) cos	(27) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W' \partial W_1}$ sin	(27) cos	(28) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W' \partial \varphi_1}$ sin	(28) cos
0	0	—	0	—	0	—	0	—	0
	1	+ 44	— 76	+ 148	+ 89	— 45	+ 79	— 74	— 45
	2	+ 12	— 27	57	27	— 24	40	— 26	— 11
	3	— 1	— 6	16	+ 4	— 11	16	— 5	— 8
	4	— 2	— 1	4	— 2	— 5	7	— 1	+ 3
	5	— 3		+ 1	— 2	— 2	4	0	2
	6	— 3		— 1	— 3			+ 1	3
1	— 9	— 2	— 3	+ 3	— 2	+ 2	3	— 3	2
	— 8	— 7	— 11	13	— 7	6	11	— 23	13
	— 7	— 17	— 30	47	— 25	15	29	— 82	44
	— 6	— 67	— 118	162	— 98	63	118	— 282	165
	— 5	— 276	— 493	603	— 357	273	493	— 1098	637
	— 4	— 1204	— 2122	2405	— 1401	1198	2121	— 4534	2615
	— 3	— 5687	— 9957	10661	— 6156	5683	9957	— 20632	11860
	— 2	— 31507	— 54979	56611	— 32517	31503	54993	—111620	64047
	— 1	—259646	—452549	+455834	—261387	+259504	+452713	—908538	+520988
	0	+101177	+170905	—178313	+ 99130	— 99076	—178425	+356635	—198264
	1	— 4761	5928	+ 13399	535	— 684	+ 13565	— 26699	— 1069
	2	— 494	864	1516	154	— 168	1501	— 2985	— 302
	3	— 81	170	302	38	— 38	259	— 555	— 72
	4	— 15	36	61	11	— 9	54	— 113	— 20
	5	— 4	9	15	3	— 2	12	— 28	— 6
	6	— 1	+ 3	+ 4		— 1	3	— 7	— 1
2	— 9	— 3	— 3	— 4	6	+ 4	3	+ 3	— 4
	— 8	— 12	— 7	— 10	20	13	6	7	— 13
	— 7	— 30	— 16	— 14	39	31	15	+ 8	— 27
	— 6	— 65	— 42	— 23	75	66	41	— 5	— 41
	— 5	— 144	— 108	— 32	138	145	107	— 52	— 51
	— 4	— 332	— 322	+ 51	+ 190	333	322	— 318	+ 33
	— 3	— 890	— 1161	717	— 75	890	1161	— 1787	799
	— 2	— 3410	— 5548	5072	— 2566	3410	5550	— 10518	5804
	— 1	— 25035	— 43695	+ 43859	— 25202	+ 25031	+ 43709	— 87582	+ 50257
	0	+ 9760	+ 16482	— 17201	+ 9561	— 9557	— 17207	+ 34402	— 19121
	1	— 463	578	+ 1291	55	— 64	+ 1306	— 2582	— 108
	2	— 50	84	145	14	— 15	146	— 288	— 29
	3	— 8	17	25	+ 4	— 4	25	— 52	— 8
	4	— 1	4	+ 6		— 1	6	— 12	— 1
	5		+ 1				1		
3	—10	+ 5			— 6	— 5			+ 6
	— 9	12		— 1	— 16	— 13		+ 1	16
	— 8	21	— 3	— 3	— 27	— 21	3	3	27
	— 7	33	— 3	— 4	— 46	— 33	3	4	46
	— 6	58	— 5	— 6	— 78	— 58	5	6	78
	— 5	85	— 10	— 2	— 119	— 85	10	+ 2	119
	— 4	91	— 25	+ 10	— 142	— 91	25	— 32	121
	— 3	+ 11	— 88	67	— 121	— 11	89	— 157	153
	— 2	— 293	— 445	419	— 187	+ 293	445	— 861	454
	— 1	— 2039	— 3558	+ 3562	— 2044	+ 2040	+ 3560	— 7130	+ 4090
	0	+ 794	+ 1342	— 1401	+ 779	— 778	— 1400	+ 2805	— 1558
	1	— 38	48	+ 105	6	— 5	+ 107	— 213	— 12
	2	— 4	8	11	2	— 2	11	— 23	— 4
	3	— 1	2	2		— 1	+ 2	— 2	
4	—11	— 3		2	3	+ 3		— 2	— 3
	—10	— 6	5	6	8	6	— 5	— 6	— 8
	— 9	— 7	5	7	10	7	— 5	— 7	— 10
	— 8	— 9	8	10	12	9	— 8	— 10	— 12
	— 7	— 14	11	13	17	14	— 11	— 13	— 17
	— 6	— 17	13	16	21	17	— 13	— 16	— 21
	— 5	— 13	11	13	+ 14	+ 13	— 11	— 13	— 14
	— 4	— 1	+ 3	4	— 2	0	— 3	— 4	+ 2
	— 3	+ 8	— 11	2	— 19	— 8	+ 11	— 2	19

Table 10 (continued)

i_1	i'	(25) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W'^2}$ sin	(25) cos	(26) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W' \partial \zeta'}$ sin	(26) cos	(27) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W' \partial W_1}$ sin	(27) cos	(28) $= 10^7 m_1 a' \frac{\partial^2 R}{\partial W' \partial \zeta_1}$ sin	(28) cos
4	− 2	− 23	− 35	33	− 16	+ 23	35	− 67	37
	− 1	−157	−272	+272	−157	+157	+272	−553	+318
	0	+ 61	+103	−107	+ 59	− 59	−107	+212	−119
	1	− 2	4	+ 9	+ 1	− 2	+ 9	− 9	− 1
	2		+ 1				1		
5	−12		− 2	− 2			2	+ 2	
	−11	+ 2	− 4	− 4	− 2	− 2	4	4	+ 2
	−10	+ 1	− 3	− 4	− 2	− 1	3	4	+ 2
	− 9	0	− 3	− 3	0	0	3	3	0
	− 8	0	− 3	− 4	0	0	3	4	0
	− 7	0	− 3	− 4	0	0	3	4	0
	− 6	0	− 2	− 3	0	0	+ 2	+ 3	0
	− 5	− 1	0	0	+ 2	+ 1	0	0	− 2
	− 4	− 2	+ 2	+ 2	+ 1	+ 2	− 2	− 2	− 1
	− 3	+ 1	0	0	0	− 1	0	0	0
	− 2	0	0	0	0	0	0	0	0
	− 1	− 11	− 21	21	− 11	+ 11	+ 21	− 21	+ 11

Table 11

i_2	i'	(29) $= 10^7 m_2 a' \frac{\partial^2 K}{\partial W'^2}$ sin	(29) cos	(30) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial W' \partial \zeta'}$ sin	(30) cos	(31) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial W' \partial W_2}$ sin	(31) cos	(32) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial W' \partial \zeta_2}$ sin	(32) cos
0	0	—	0	—	− 2	—	0	—	+ 1
	1	− 42	− 40	+ 80	− 91	+ 41	+ 39	− 41	48
	2	− 36	− 8	+ 21	− 70	25	19	− 8	41
	3	− 23	+ 3	− 1	− 43	12	7	+ 4	27
	4	− 12	5	− 6	− 23	6	2	6	15
	5	− 6	4	− 5	− 10	3	1	5	6
	6	− 2	2	− 3	− 4	1		2	3
	7	− 1	+ 1	− 1	− 1			+ 1	+ 1
1	−8		− 2	− 1	+ 2		1		− 2
	−7	− 3	− 7	− 1	14	3	3	− 2	− 13
	−6	− 6	− 15	+ 3	39	+ 7	11	− 11	− 38
	−5	+ 4	− 40	41	106	− 3	33	− 70	− 121
	−4	89	− 136	197	318	− 90	128	− 320	− 428
	−3	614	− 623	862	1097	− 614	618	− 1474	− 1744
	−2	3945	− 3583	4244	4884	− 3946	3584	− 7824	− 8865
	−1	34337	−30582	+31941	+35852	−34348	+30580	−62522	−70196
	0	−13109	+11985	−11893	−13693	+13701	−11915	+23799	+27404
	1	− 488	− 350	+ 158	+ 1107	− 1020	+ 172	− 305	− 2153
	2	− 67	− 31	11	141	− 108	10	− 18	− 251
	3	− 12	− 5	2	31	− 17	+ 1	− 2	− 48
	4	− 2	− 1	1	8	− 3		− 1	− 11
	5		− 1	1	3	− 1		− 1	− 3
	6				+ 1				− 1
2	−9	+ 3	+ 3	3	− 5	− 4	− 1	− 2	+ 3
	−8	13	13	18	− 20	− 16	− 11	− 14	13
	−7	29	26	35	− 44	− 31	− 25	− 25	29
	−6	63	37	54	− 94	− 66	− 35	− 42	62
	−5	138	51	82	− 197	− 140	− 48	− 66	128
	−4	270	+ 55	119	− 360	− 271	− 54	− 108	220
	−3	478	− 5	186	− 504	− 478	+ 6	− 231	+ 226
	−2	878	− 382	495	− 133	− 879	383	− 895	− 543
	−1	+ 3831	− 3470	+ 3488	+ 4012	− 3833	+ 3471	− 6956	− 7873
	0	− 1469	+ 1342	− 1333	− 1539	+ 1534	− 1335	+ 2667	+ 3074
	1	− 55	− 39	+ 17	+ 119	− 117	+ 19	− 34	− 238
	2	− 8	− 3	+ 1	13	− 13	2	− 2	− 26
	3	− 2			2	− 2			− 4
	4	− 1			1	− 1			− 2

Table 11 (continued)

i_2	i'	(29) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial W'^2}$		(30) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial W' \partial p'}$		(31) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial W' \partial W_2}$		(32) $= 10^7 m_2 a' \frac{\partial^2 R}{\partial W' \partial p_2}$	
		sin	cos	sin	cos	sin	cos	sin	cos
3	−10	− 3	− 7	− 10	4	+ 4	7	+ 7	− 3
	− 9	− 4	− 28	− 36	6	6	28	28	− 4
	− 8	− 6	− 47	− 61	9	7	47	47	− 6
	− 7	− 21	− 71	− 94	29	22	71	73	− 21
	− 6	− 48	−113	−150	66	49	112	115	− 50
	− 5	− 82	−158	−211	111	82	158	160	− 84
	− 4	−100	−175	−231	141	101	174	173	−110
	− 3	− 55	−113	−133	+ 93	56	111	+ 90	− 84
	− 2	+ 95	+ 3	+101	− 23	− 93	− 5	−126	− 42
	− 1	+365	−329	+334	+379	−364	+326	−662	−746
	0	−139	+127	−125	−145	+146	−128	+252	+291
	1	− 6	− 3	+ 2	+ 11	− 11	+ 1	− 3	− 16
	2	− 1		1	1	− 1		− 1	− 2
4	−11	− 3	+ 9	11	5	+ 3	− 10	− 11	− 5
	−10	− 17	24	30	20	16	− 25	− 30	− 20
	− 9	− 25	31	39	29	24	− 31	− 39	− 29
	− 8	− 25	42	52	31	25	− 42	− 52	− 31
	− 7	− 29	62	77	36	29	− 62	− 77	− 36
	− 6	− 31	76	95	38	30	− 76	− 95	− 38
	− 5	− 24	65	81	31	24	− 64	− 81	− 31
	− 4	− 11	+ 16	+ 19	+ 14	+ 10	− 15	− 19	− 14
	− 3	+ 1	− 33	− 42	− 3	− 3	+ 34	+ 42	+ 3
	− 2	8	+ 1	+ 9	− 3	− 10		− 12	− 3
	− 1	+ 34	− 31	+ 30	+ 34	− 34	+ 31	− 60	− 68
	0	− 13	+ 11	− 11	− 13	+ 12	− 11	+ 22	+ 25
	1								− 1
5	−12	+ 9	− 5	− 6	− 10	− 9	+ 5	6	+ 10
	−11	23	− 5	− 6	− 27	− 23	5	6	27
	−10	25	− 1	− 2	− 29	− 25	1	2	29
	− 9	23	− 3	− 4	− 28	− 23	3	4	28
	− 8	27	− 7	− 8	− 32	− 27	7	8	32
	− 7	27	− 7	− 9	− 32	− 27	8	9	32
	− 6	+ 16	− 3	− 5	− 19	− 15	+ 4	+ 5	+ 19
	− 5	− 1	+ 4	+ 5	+ 2	+ 1	− 3	− 5	− 2
	− 4	− 10	+ 6	+ 7	+ 13	+ 10	− 6	− 7	− 13
	− 3	+ 1	− 4	− 5	0	0	+ 5	+ 5	0
	− 2	+ 1	+ 1	+ 1	− 1	− 1	− 1	− 1	+ 1
6	−13	− 7	− 2	− 3	+ 8	+ 7	+ 2	+ 3	− 8
	−12	− 13	− 9	− 10	15	13	9	10	− 15
	−11	− 9	− 9	− 10	10	9	9	10	− 10
	−10	− 5	− 6	− 7	6	5	6	7	− 6
	− 9	− 6	− 7	− 7	6	5	6	7	− 6
	− 8	− 6	− 6	− 6	7	6	5	6	− 7
	− 7	− 2	− 3	− 2	+ 2	+ 2	+ 2	+ 2	− 2
	− 6	+ 4	+ 1	+ 1	− 4	− 3	− 1	− 1	+ 4
	− 5	+ 3	2	2	− 3	− 3	− 2	− 2	+ 3
	− 4	− 2	1	1	+ 2	+ 2	− 1	− 1	− 2
7	−14	+ 2	4	5	− 2	− 2	− 4	− 5	+ 2
	−13	3	6	7	− 3	− 3	− 6	− 7	3
	−12	+ 1	2	3	− 1	− 1	− 2	− 3	+ 1
	−11	− 2	1	1	+ 2	+ 2	− 1	− 1	− 2
	−10	− 2	2	3	2	2	− 2	− 3	− 3
	− 9	− 1	2	2	+ 1	+ 1	− 2	− 2	− 1
	− 8	0	+ 1	+ 1	0	0	− 1	− 1	0
	− 7	0	− 1	− 1	0	0	+ 1	+ 1	0
	− 6	0	− 1	− 1	0	0	0	+ 1	0
	− 5	+ 1	+ 1	+ 1	− 1	− 1	− 1	− 1	+ 1
8	−12	1	2	2	− 1	− 1	− 2	− 2	1
	−11	+ 1	2	2	− 1	− 1	− 2	− 2	+ 1
	−10	0			0	0			0
	− 9	− 1			+ 1	+ 1			− 1
	− 8	− 1			1	1			− 1

Table 12

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i_3	i'	(33) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial W'^2}$ sin	(33) cos	(34) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial W' \partial \rho'}$ sin	(34) cos	(35) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial W' \partial W_3}$ sin	(35) cos	(36) $= 10\, m_3 a' \frac{\partial^2 R}{\partial W' \partial p_3}$ sin	(36) cos
0	0	—	+ 1	—	— 5	—	— 1	—	+ 4
	1	— 14	10	— 22	— 38	+ 13	— 11	+ 13	24
	2	— 19	19	— 41	— 45	+ 5	— 6	29	32
	3	— 12	17	— 38	— 31	0	— 3	29	23
	4	— 4	10	— 25	— 13	— 1	— 1	19	9
	5	0	6	— 14	— 2	— 1		11	+ 1
	6	+ 2	3	— 8	+ 3	— 1		6	— 3
	7	1	1	— 4	+ 1			+ 3	— 1
1	— 8	4	1	+ 2	— 8	— 2		— 2	+ 6
	— 7	8	6	4	— 19	— 4	— 2	— 3	16
	— 6	14	+ 5	+ 6	— 25	— 6	— 3	— 6	20
	— 5	26	— 2	— 11	— 38	— 12	+ 2	+ 7	30
	— 4	38	— 11	— 43	— 38	— 20	9	31	+ 27
	— 3	47	— 17	— 113	+ 32	— 34	+ 13	102	— 49
	— 2	118	+ 48	— 305	317	— 121	— 49	360	— 418
	— 1	+ 1230	+ 919	— 1340	+ 1802	— 1230	— 918	+ 2258	— 3034
	0	— 544	— 392	+ 395	— 526	+ 549	+ 423	— 817	+ 1076
	1	+ 4	— 22	— 80	+ 33	— 13	— 35	+ 115	— 47
	2	1	0	— 25	12	0	+ 1	25	— 11
	3	2	0	— 9	8	+ 1	2	8	— 7
	4	1	— 1	— 1	5	+ 1	1	+ 1	— 4
	5	1	— 1	+ 1	3			— 1	— 2
	6	1	— 1	1	2			— 1	— 2
	7			+ 5	+ 4			— 5	— 4
2	— 9	4	— 12	— 19	— 6		5	+ 16	+ 6
	— 8	13	— 18	— 28	— 18	— 5	12	23	17
	— 7	+ 13	— 18	— 31	— 16	— 5	13	25	+ 16
	— 6	— 6	— 31	— 47	+ 15	+ 13	20	39	— 5
	— 5	— 45	— 41	— 58	80	51	29	50	— 51
	— 4	— 121	— 21	— 4	215	123	+ 13	+ 6	— 150
	— 3	— 227	+ 38	+ 62	377	225	— 36	— 36	— 263
	— 2	— 254	101	+ 129	449	+ 255	— 102	— 74	— 330
	— 1	+ 159	+ 87	— 114	+ 100	— 158	— 88	+ 206	— 244
	0	— 52	— 36	+ 43	— 46	+ 54	+ 38	— 82	+ 98
	1		— 2	— 2	+ 3	— 1	— 5	+ 7	— 4
	2		— 1	+ 3	0	0	— 1	— 2	0
	3		— 1	+ 3	— 1	0	— 1	— 2	+ 1
	4		— 1	— 18	— 12	+ 1	— 1	+ 18	+ 12
	5			+ 1				— 1	
3	— 10	— 15	— 11	— 16	+ 21	11	+ 2	+ 15	— 17
	— 9	— 22	— 28	— 37	31	22	16	34	— 24
	— 8	— 19	— 20	— 27	30	21	+ 12	+ 26	— 22
	— 7	— 48	+ 15	+ 25	68	44	— 24	— 14	— 53
	— 6	— 90	66	98	123	85	— 74	— 69	— 97
	— 5	— 115	153	222	156	110	— 157	— 166	— 121
	— 4	— 112	238	336	152	112	— 237	— 255	— 115
	— 3	— 65	+ 195	+ 271	+ 86	+ 62	— 197	— 206	— 63
	— 2	+ 11	— 52	— 72	— 21	— 16	+ 53	+ 55	+ 17
	— 1	+ 14	+ 9	— 11	— 8	— 21	— 6	+ 18	— 6
	0	— 3	— 3	+ 1	— 12	— 0	+ 4	— 4	+ 15
	1			— 1	— 4	— 2	+ 1	+ 1	3
	2				— 3	— 1			3
	3				— 1	—			1
	4				— 1	—			+ 1
4	— 11	— 18	+ 17	+ 21	+ 25	+ 6	— 19	— 16	— 23
	— 10	— 65	34	43	83	50	— 37	— 34	— 73
	— 9	— 46	29	40	+ 60	+ 39	— 31	— 31	— 53

Table 12 (continued)

i_3	i'	(33) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial W'^2}$		(34) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial W' \partial \rho'}$		(35) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial W' \partial W_3}$		(36) $= 10^7 m_3 a' \frac{\partial^2 R}{\partial W' \partial \rho_2}$	
		sin	cos	sin	cos	sin	cos	sin	cos
4	— 8	+ 7	75	99	— 9	— 12	— 76	— 78	+ 5
	— 7	28	140	182	— 39	— 32	—140	—146	29
	— 6	70	189	242	— 97	— 72	—189	—194	77
	— 5	103	185	234	—137	—103	—183	—188	111
	— 4	+ 61	+ 86	+105	— 82	— 64	— 82	— 83	+ 68
	— 3	— 37	— 43	— 67	+ 41	+ 34	+ 50	+ 55	— 32
	— 2	+ 6	— 5	— 21	— 11	— 8	13	18	+ 9
	— 1	1	— 2	— 8	— 1	— 1	4	7	1
	0		— 1	— 3	— 1		2	2	1
	1			— 1			+ 1	+ 1	
5	—12	51	+ 3	+ 3	— 63	— 51	— 3	— 2	53
	—11	40	63	74	— 52	— 40	— 63	— 63	42
	—10	7	57	67	— 12	— 7	— 57	— 57	8
	— 9	87	21	27	—110	— 88	— 21	— 22	91
	— 8	147	45	51	—178	—146	— 43	— 43	148
	— 7	159	44	48	—198	—160	— 41	— 40	165
	— 6	132	15	+ 12	—161	—131	— 11	— 9	135
	— 5	+ 36	+ 1	— 9	— 41	— 34	+ 5	+ 9	+ 34
	— 4	— 47	— 1	— 12	+ 64	+ 51	7	11	— 54
	— 3	— 1	— 9	— 19	7	4	13	16	— 6
	— 2	0	— 1	— 3	2	1	2	3	— 2
	— 1	— 1		— 1	+ 1	+ 1	1	1	— 1
6	—13	+ 47	— 57	— 67	— 56	— 47	+ 57	+ 58	+ 48
	—12	98	+ 1	— 2	—114	— 98	— 1	0	98
	—11	72	+ 56	+ 63	— 84	— 72	— 56	— 56	73
	—10	79	— 53	— 62	— 93	— 79	+ 53	+ 53	79
	— 9	98	— 92	—108	—110	— 94	92	93	93
	— 8	77	— 67	— 77	— 84	— 72	66	66	71
	— 7	+ 45	— 43	— 51	— 42	— 40	42	+ 44	+ 30
	— 6	— 3	— 3	+ 1	+ 13	+ 9	+ 2	— 1	— 12
	— 5	— 23	+ 23	+ 29	38	29	— 23	— 25	— 33
	— 4	— 6	— 6	— 8	13	10	+ 6	+ 7	— 11
	— 3	— 1	— 1	— 1	2	1	1	1	— 2
	— 2				1				— 1
7	—14	— 22	— 92	—105	25	22	92	92	— 22
	—13	— 32	—133	—154	37	32	133	135	— 32
	—12	— 11	— 46	— 54	+ 13	+ 11	46	47	— 11
	—11	+ 5	— 67	— 78	— 5	— 5	67	68	+ 4
	—10	+ 2	—107	—123	— 3	— 2	107	107	+ 2
	— 9	— 5	— 66	— 77	+ 6	+ 5	66	67	— 5
	— 8	+ 4	— 26	— 32	— 4	— 4	+ 26	+ 28	+ 3
	— 7	9	+ 3	+ 5	— 10	— 8	— 3	— 5	9
	— 6	+ 2	+ 15	+ 16	— 1	— 1	— 15	— 14	+ 1
	— 5	— 5	— 4	— 3	+ 6	+ 6	+ 4	+ 3	— 5
	— 4	— 1	— 1			1	1		
8	—12	— 7	— 29	— 35	8	7	29	31	— 7
	—11	— 9	— 38	— 42	10	9	38	37	— 9
	—10	— 1	— 6	— 6	+ 1	+ 1	6	5	— 1
	— 9	+ 2	— 2	— 4	— 3	— 2	2	4	+ 3
	— 8	2	— 5	— 7	— 2	— 2	+ 5	+ 5	+ 2
	— 7	0	0	+ 1	0	0	0	— 1	0
	— 6	+ 1	+ 4	+ 3	0	— 1	— 4	— 2	0
	— 5		— 2	— 1	+ 1		+ 2	+ 1	— 1
9	—10	— 1	— 4	— 5	1	+ 1	4	5	— 1
	— 9	— 1	— 4	— 4	1	1	+ 4	+ 4	— 1
	— 8		0	+ 1			0	— 1	
	— 7		+ 2	1			— 2	— 1	

Table 13

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i_1	i_2	i_3	i'	(37) = $\delta\rho'$ sin	(37) = $\delta\rho'$ cos	(38) = $\delta w'$ sin	(38) = $\delta w'$ cos	(59) = $10^7\, m \left[n' \int \left(a' \frac{\partial R}{d W'} \right)_0 dt + C_2 \right]$ sin	(59) cos
					"		"		
0	0	0	0	—	—846.82	—	— 495.89	—	—19565
			0	—	— 0.01$n't$	—	+1480.77$n't$	—	
			1	—1086.80	—365.06	+773.36	—2208.90	+ 14	+ 103
			1	+ 357.69$n't$	+ 0.06$n't$	— 0.14$n't$	+ 726.78$n't$		
			2	— 399.26	—135.45	+234.16	— 676.19	14	16
			2	+ 131.44$n't$	+ 0.02$n't$	— 0.04$n't$	+ 222.50$n't$		
			3	— 138.52	— 47.16	+ 74.19	— 215.15	7	1
			3	+ 45.64$n't$	+ 0.01$n't$	— 0.01$n't$	+ 70.81$n't$		
			4	— 47.25	— 16.14	+ 23.92	— 69.52	3	
			4	+ 15.56$n't$			+ 22.88$n't$		
			5	— 15.99	— 5.47	7.77	— 22.65	+ 1	
			5	+ 5.26$n't$			+ 7.46$n't$		
			6	— 5.30	— 1.83	2.54	— 7.40		
			6	+ 1.69$n't$			+ 2.44$n't$		
			7	— 1.80	— 0.63	0.84	— 2.43		
			7	+ 0.56$n't$			+ 0.79$n't$		
			8	— 0.57	— 0.21	0.27	— 0.80		
			8	+ 0.24$n't$			+ 0.28$n't$		
			9	— 0.18	— 0.04	0.10	— 0.27		
			9	+ 0.07$n't$			+ 0.09$n't$		
			10	— 0.07		0.01	— 0.09		
			10	+ 0.02$n't$			+ 0.02$n't$		
			11	— 0.02			— 0.01		
0	0	1	—13			0.01			
			—12	— 0.01	+ 0.02	0.02	+ 0.01		
			—11	— 0.03	0.05	0.06	0.04		
			—10	— 0.09	0.14	0.19	0.12		
			— 9	— 0.26	0.41	0.58	0.36		
			— 8	— 0.79	1.23	1.74	1.11		
			— 7	— 2.33	3.64	5.35	3.41	— 1	
			— 6	— 6.81	10.68	16.55	10.55	— 2	
			— 5	— 19.60	30.79	51.98	33.18	— 4	2
			— 4	— 53.26	83.89	169.53	108.48	— 10	9
			— 3	— 11.80	+ 22.55	679.44	459.39	—387	254
			— 2	+ 47.01	—101.39	210.72	90.93	+105	57
			— 1	20.61	— 35.02	66.13	25.80	+629	+ 469
			0	5.62	— 13.34	19.92	9.73	—186	— 143
			1	2.02	— 4.57	6.61	3.02	+ 3	+ 9
			2	0.68	— 1.56	2.16	0.98		
			3	0.23	— 0.52	0.71	0.32		
			4	0.07	— 0.18	0.23	0.11		
			5	0.03	— 0.06	0.08	0.04		
			6	0.01	— 0.02	0.03	+ 0.01		
			7		— 0.01	0.01			
0	0	2	—15	0.01			— 0.01		
			—14	0.02		0.01	— 0.03		
			—13	0.07	+ 0.01	0.02	— 0.09		
			—12	0.20	0.04	0.05	— 0.28		
			—11	0.60	0.11	0.15	— 0.85		
			—10	1.77	0.33	0.48	— 2.60		
			— 9	5.19	0.97	1.50	— 8.03		1
			— 8	14.98	2.79	4.68	— 25.21		3
			— 7	40.96	7.61	15.34	— 82.17		5
			— 6	+ 22.71	+ 4.05	64.44	— 326.94	+112	+ 90
			— 5	— 68.77	— 8.65	17.81	— 144.26	— 32	— 9
			— 4	— 23.52	— 3.49	5.85	— 40.95	— 34	+ 1
			— 3	— 7.37	— 1.42	2.12	— 12.01	— 39	9

Table 13 (continued)

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i_1	i_2	i_3	i'	(37) $= \delta p'$ sin	(37) $= \delta p'$ cos	(38) $= \delta w'$ sin	(38) $= \delta w'$ cos	(39) $= 10^7 m \left[n' \int \left(a' \frac{\partial R}{d W'} \right)_0 dt + C_2 \right]$ sin	(39) cos
				"	"	"	"		
0	0	2	− 2	− 2.19	−0.59	0.83	− 3.56	− 31	14
			− 1	− 0.74	−0.14	0.32	− 1.26	+ 29	+ 19
			0	− 0.30	−0.08	0.08	− 0.38	− 9	− 7
			1	− 0.10	−0.03	0.04	− 0.13		
			2	− 0.03	−0.01	+0.01	− 0.05		
			3	− 0.01			− 0.02		
			4	− 0.01			− 0.01		
0	0	3	−16	+ 0.01			− 0.02		
			−15	0.04			− 0.06		
			−14	0.13			− 0.18		
			−13	0.38		−0.01	− 0.56		
			−12	1.11	−0.01	−0.02	− 1.72		
			−11	3.20	−0.04	−0.06	− 5.41		
			−10	8.73	−0.08	−0.24	−17.66	+ 3	
			− 9	+ 6.60	−0.76	−5.73	−75.65	+ 55	+ 27
			− 8	−13.05	+2.74	−5.88	−27.29	− 9	− 1
			− 7	− 4.38	+0.48	−0.98	− 7.62	− 8	+ 6
			− 6	− 1.32	−0.12	+0.05	− 2.16	− 9	10
			− 5	− 0.34	−0.25	0.25	− 0.57	− 9	14
			− 4	− 0.06	−0.22	0.22	− 0.13	− 7	17
			− 3	− 0.01	−0.12	0.12	− 0.03	− 4	+ 11
			− 2	− 0.01	−0.01	0.03	− 0.02	0	− 2
			− 1	− 0.01		0.01	− 0.01	+ 2	+ 1
			0	− 0.01					
0	0	4	−15		−0.01				
			−14		−0.03				
			−13	+ 0.01	−0.09	0.06	− 0.01		
			−12	+ 0.03	−0.41	+3.93	+ 0.05		
			−11	− 0.26	+2.94	−7.18	− 0.41	− 2	6
			−10	+ 0.15	0.79	−1.58	+ 0.19	− 6	5
			− 9	+ 0.08	+0.18	−0.35	+ 0.09	− 3	3
			− 8	− 0.01	−0.03	−0.01	0.00	+ 1	5
			− 7	− 0.03	−0.08	+0.08	− 0.03	2	7
			− 6	− 0.04	−0.08	0.08	− 0.04	3	8
			− 5	− 0.03	−0.06	0.06	− 0.04	4	7
			− 4	− 0.02	−0.02	0.03	− 0.02	+ 2	+ 3
			− 3					− 1	− 1
0	0	5	−15	− 0.01					
			−14	+ 0.04			+ 0.05		
			−13	− 0.03	−0.01	0.02	− 0.06		
			−12	− 0.11	−0.02	0.04	− 0.13	+ 3	
			−11	− 0.08	−0.05	0.05	− 0.08	3	+ 3
			−10	− 0.04	−0.03	0.04	− 0.05	1	2
			− 9	− 0.04	−0.02	0.02	− 0.05	3	1
			− 8	− 0.04	−0.01	0.01	− 0.05	4	1
			− 7	− 0.03	−0.01	+0.01	− 0.03	4	+ 1
			− 6	− 0.02			− 0.02	3	
			− 5	− 0.01			− 0.01	+ 1	
			− 4					− 1	
0	1	0	−10	− 0.01	+0.01	−0.02	− 0.01		
			− 9	+ 0.02	+0.08	+0.09	− 0.08		
			− 8	− 0.15	−0.26	+0.92	− 0.14		− 1
			− 7	+ 0.37	+0.03	−0.06	+ 0.58	− 1	− 2
			− 6	0.30	+0.02	−0.12	0.31	− 1	− 3
			− 5	0.32	−0.09	−0.18	+ 0.08	+ 4	− 8
			− 4	0.58	−0.35	−0.41	− 0.27	25	− 28
			− 3	1.38	−1.12	−1.13	− 1.14	119	−113

Table 13 (continued)

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i_1	i_2	i_3	i'	(37) $= \delta\rho'$ sin	(37) $= \delta\rho'$ cos	(38) $= \delta w'$ sin	(38) $= \delta w'$ cos	(39) $= 10^7 m \left[n' \int \left(a' \frac{\partial R}{\partial W'}\right)_0 dt + C_2\right]$ sin	(39) $= 10^7 m \left[n' \int \left(a' \frac{\partial R}{\partial W'}\right)_0 dt + C_2\right]$ cos
				"	"	"	"		
0	1	0	−2	3.69	− 3.23	− 3.21	− 3.53	619	− 557
			−1	+ 9.81	− 8.73	− 8.70	− 9.74	+ 4620	− 4114
			0	− 1.27	+ 1.17	+ 1.17	+ 1.38	− 1626	+ 1412
			1	− 0.31	0.05	0.10	− 0.07	+ 111	− 16
			2	− 0.07	0.02	+ 0.02	− 0.05	11	− 1
			3	− 0.02	0.03		− 0.02	1	
			4	− 0.01	+ 0.01		− 0.01		
0	2	0	−8				− 0.01	1	+ 1
			−7				− 0.01	1	1
			−6	− 0.02			− 0.02	3	1
			−5	− 0.03	− 0.01		− 0.03	6	1
			−4	− 0.03	− 0.02	− 0.01	− 0.05	11	+ 1
			−3	− 0.01	− 0.03	− 0.02	− 0.08	20	− 3
			−2	+ 0.06	− 0.08	− 0.09	− 0.14	45	− 27
			−1	+ 0.28	− 0.22	− 0.25	− 0.28	+ 243	− 218
			0	− 0.03	+ 0.06	− 0.03	− 0.04	− 91	+ 79
			1		+ 0.01		− 0.01	+ 7	− 1
			2					+ 1	
0	3	0	−8						− 1
			−7					− 1	− 1
			−6					− 1	− 2
			−5					− 1	− 3
			−4					− 1	− 3
			−3					0	− 2
			−2					+ 3	− 1
			−1					+ 15	− 13
			0					− 6	+ 5
1	0	0	−8			− 0.02		0	− 1
			−7	− 0.02	− 0.02	− 0.04	+ 0.02	− 1	− 3
			−6	− 0.06	− 0.10	− 0.10	0.06	− 4	− 8
			−5	− 0.16	− 0.30	− 0.32	0.18	− 18	− 31
			−4	− 0.49	− 0.91	− 0.91	0.53	− 72	− 126
			−3	− 1.50	− 2.64	− 2.66	1.52	− 318	− 556
			−2	− 4.33	− 7.58	− 7.62	4.37	− 1664	− 2904
			−1	−11.82	−20.62	−20.66	11.86	−13017	−22699
			0	+ 1.63	+ 2.76	+ 2.84	− 1.58	+ 4733	+ 8514
			1	− 0.02	0.63	− 0.26	− 0.22	24	− 606
			2	− 0.08	0.20	− 0.14	− 0.06	6	− 64
			3	− 0.02	+ 0.06	− 0.06	− 0.02	+ 1	− 11
			4			− 0.02			− 2
2	0	0	−6					− 1	− 1
			−5					− 2	− 2
			−4		− 0.02	− 0.02	+ 0.02	− 6	− 7
			−3	− 0.02	− 0.06	− 0.06	0.04	− 19	− 27
			−2	− 0.10	− 0.24	− 0.18	0.12	− 81	− 137
			−1	− 0.26	− 0.47	− 0.49	+ 0.30	− 613	− 1059
			0	+ 0.04	+ 0.12	+ 0.06	− 0.04	+ 228	+ 411
			1	0.02	0.06			+ 1	− 30
			2		0.02				− 3
			3						− 1
3	0	0	−3					− 1	− 2
			−2					− 4	− 7
			−1	0.02	0.02	− 0.02	+ 0.02	− 33	− 58
			0	+ 0.08	0.12			+ 12	+ 22
			1	− 0.02	0.06				− 2
			2	+ 0.06					
			3	0.02					

Table 14

	(46) $= \frac{a'}{r'}$	(47) $= \left(\frac{r'}{a'}\right)^2$
	cos	cos
0	1.000000	1.091568
1	0.245193	—0.490386
2	0.059813	—0.029906
3	0.016393	—0.003643
4	0.004726	—0.000591
5	0.001403	—0.000112
6	0.000461	—0.000026
7	—	—0.000006

11. Multiplication of Series

As was shown in Section 1, the right-hand members of equations which serve to determine second-order perturbations of the logarithm of the radius vector and longitude in orbit, have this form:

$$a'\delta Q = 2\int a'\delta D_t' R dt + a'\delta\frac{\partial R}{\partial \rho'} + \frac{1+m'}{m}\left[-\frac{d^2}{(dn't)^2}\left(\frac{r_0'}{a'}\delta\rho'\right)^2 + \right.$$
$$\left. + \frac{1}{2}\left(\frac{r_0'}{a'}\right)^{-1}(\delta_2\rho')^2\right],$$

$$\frac{d^2\delta_2 w'}{dt} = \left(\frac{a'}{r_0'}\right)^2\left\{\frac{m}{1+m'}\left[n'\int a'\delta\frac{\partial R}{\partial W'}dt + \delta C_2\right] - 2\delta_1\rho'\frac{m}{1+m'}\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt + (C_2)\right] - \right.$$
$$\left. - 2\cos\varphi'\left[\delta_2\rho' - (\delta_1\rho')^2\right]\right\}. \qquad (57)$$

We will examine the series entering these equations. As is known from Section 4, in $a'\delta R$, $a'\delta\frac{\partial R}{\partial \rho'}$, and $a'\delta\frac{\partial R}{\partial W'}$, we can restrict ourselves to the following forms if we determine the perturbations of Pluto from three planets -- Jupiter, Saturn, and Uranus:

$$\left.\begin{aligned}
a'\delta R &= a'\frac{\partial R}{\partial \rho'}\delta\rho' + a'\frac{\partial R}{\partial W'}\delta w' + a'\frac{\partial R}{\partial \rho_1}\delta\rho_1 + a'\frac{\partial R}{\partial W_1}\delta w_1 + a'\frac{\partial R}{\partial \rho_2}\delta\rho_2 + a'\frac{\partial R}{\partial W_2}\delta w_2 + \\
&+ a'\frac{\partial R}{\partial \rho_3}\delta\rho_3 + a'\frac{\partial R}{\partial W_3}\delta w_3, \\
a'\delta\frac{\partial R}{\partial \rho'} &= a'\frac{\partial^2 R}{\partial \rho'^2}\delta\rho' + a'\frac{\partial^2 R}{\partial \rho'\partial W'}\delta w' + a'\frac{\partial^2 R}{\partial \rho'\partial \rho_1}\delta\rho_1 + a'\frac{\partial^2 R}{\partial \rho'\partial W_1}\delta w_1 + a'\frac{\partial^2 R}{\partial \rho'\partial \rho_2}\delta\rho_2 + \\
&+ a'\frac{\partial^2 R}{\partial \rho'\partial W_2}\delta w_2 + a'\frac{\partial^2 R}{\partial \rho'\partial \rho_3}\delta\rho_3 + a'\frac{\partial^2 R}{\partial \rho'\partial W_3}\delta w_3, \\
a'\delta\frac{\partial R}{\partial W'} &= a'\frac{\partial^2 R}{\partial W'\partial \rho'}\delta\rho' + a'\frac{\partial^2 R}{\partial W'^2}\delta w' + a'\frac{\partial^2 R}{\partial W'\delta\rho_1}\delta\rho_1 + a'\frac{\partial^2 R}{\partial W'\partial W_1}\delta w_1 + a'\frac{\partial^2 R}{\partial W'\partial \rho_2}\delta\rho_2 + \\
&+ a'\frac{\partial^2 R}{\partial W'\partial W_2}\delta w_2 + a'\frac{\partial^2 R}{\partial W'\partial \rho_3}\delta\rho_3 + a'\frac{\partial^2 R}{\partial W'\partial W_3}\delta w_3.
\end{aligned}\right\} \qquad (58)$$

The first and second derivatives of the perturbing function, with respect to the coordinates of the planets entering Equation (58) and perturbations of the coordinates, were derived in preceding sections in the form of series with respect to mean anomalies. Consequently, the determination of $a'\delta R$, $a'\delta\frac{\partial R}{\partial \rho'}$, and $a'\delta\frac{\partial R}{\partial W'}$ reduces to multiplication of the trigonometric series:

$$a'\delta R = [(01) + (05) + (09)] \times (37) + [(02) + (06) + (10)] \times (38) + (03) \times (40) +$$
$$+ (04) \times (41) + (07) \times (42) + (08) \times (43) + (11) \times (44) + (12) \times (45),$$

$$a'\delta\frac{\partial R}{\partial r'}=[(13)+(17)+(21)]\times(37)+[(26)+(30)+(34)]\times(38)+(15)\times(40)+ \qquad /54$$
$$+(14)\times(41)+(19)\times(42)+(18)\times(43)+(23)\times(44)+(22)\times(45),$$
$$a'\delta\frac{\partial R}{\partial W'}=[(26)+(30)+(34)]\times(37)+[(25)+(29)+(33)]\times(38)+(28)\times(40)+$$
$$+(27)\times(41)+(32)\times(42)+(31)\times(43)+(36)\times(44)+(35)\times(45).$$

The determination of the quantities $(\delta_1\rho')^2$, $\left(\frac{r_0'}{a'}\delta_1\rho'\right)^2$, $\frac{1}{2}\left(\frac{r_0'}{a'}\right)^{-1}(\delta_1\rho')^2$, $2\delta\rho'\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt+(C_2)_0\right]$ also reduces to multiplication of the series:

$$(\delta_1\rho')^2=(37)\times(37),$$
$$\left(\frac{r_0'}{a'}\right)^2\times(\delta_1\rho')^2=(47)\times[(37)\times(37)],$$
$$\frac{1}{2}\left(\frac{r_0'}{a'}\right)^{-1}(\delta_1\rho')^2=\frac{1}{2}(46)\times[(37)\times(37)],$$
$$2\delta_1\rho'\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt+(C_2)_0\right]=2(39)\times(37).$$

The method of multiplication of trigonometric series on punched-card machinery has been thoroughly presented by a number of authors (Kulikov, 1949; Gontkovskaya, 1958). We emphasize that the so-called method of characteristics, usually employed when multiplying series on punched-card machinery, has an appreciable effect since the number of cards is reduced by at least 75%.

Table 15 characterizes the initial series.

At the first stage of the work we obtained 28 products of the series. The derivatives of a'R and the series $m\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt+(C_2)_0\right]$ were expressed in a radian measure to the seventh decimal place, and the functions of r' to the sixth decimal place.

Perturbations from the coordinates of the planets were taken to the second decimal place and expressed in seconds of arc. The products of the coefficients of these series were obtained to the fourth decimal place in seconds of arc.

We will give the main formula which we used in multiplying the series. It includes all our forms of the arguments.

Let two series be multiplied:

$$f(M_\mu, M') = \sum_{i_\mu, i'} a_{(i_\mu, i')} \sin\left(\sum_{\mu=1}^{4} i_\mu M_\mu + i'M'\right) + \sum_{i_\mu, i'} b_{(i_\mu, i')} \cos\left(\sum_{\mu=1}^{4} i_\mu M_\mu + i'M'\right),$$
$$\varphi(M_\mu, M') = \sum_{k_\mu, k'} A_{(k_\mu, k')} \sin\left(\sum_{\mu=1}^{4} k_\mu M_\mu + k'M'\right) + \sum_{k_\mu, k'} B_{(k_\mu, k')} \cos\left(\sum_{\mu=1}^{4} k_\mu M_\mu + k'M'\right),$$

here μ is the number of the planet. The resulting series has the form

$$g(M_\mu, M') = f(M_\mu, M') \times \varphi(M_\mu, M') = \sum_{p_\mu, p'} \alpha_{(p_\mu, p')} \sin\left(\sum_{\mu=1}^{4} p_\mu M_\mu + p'M'\right) + \sum_{p_\mu, p'} \beta_{(p_\mu, p')} \cos\left(\sum_{\mu=1}^{4} p_\mu M_\mu + b'M'\right), \quad (59)$$

where $\alpha_{(p_\mu, p')}$ and $\beta_{(p_\mu, p')}$ are functions of the coefficients of the series being multiplied, and

$$p_\mu = i_\mu \pm k_\mu, \; p' = i' \pm k'.$$

Table 16 characterizes the products of the series. /57

All stages of multiplying the series were checked by comparing two, simultaneously calculated specimens of the work (2 hands).

The resulting series were checked by two methods.

The first method, based on comparing the first and second members of relation (59) with certain particular values in the argument, is described in the work of M. G. Polozova (1958); in our case we used the following variants of the values of M:

$$M_1 = M_2 = M_3 = M_4 = M' = 0;$$
$$M_1 = 90°, \; M_2 = M_3 = M_4 = M' = 0;$$
$$M_2 = 90°, \; M_1 = M_3 = M_4 = M' = 0;$$
$$M_3 = 90°, \; M_1 = M_2 = M_4 = M' = 0;$$
$$M_4 = 90°, \; M_1 = M_2 = M_3 = M' = 0;$$
$$M' = 90°, \; M_1 = M_2 = M_3 = M_4 = 0.$$

Furthermore, we calculated and compared the values of the series multiplied, and the resulting series, in each case for the instant of osculation 1930 Sept. 20.0; for this verification we used the values of $\begin{matrix}\sin\\ \cos\end{matrix}(i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M')$, obtained for determining the constants of integration.

This method of verification is not completely reliable since it either does

Table 15

Series	Periodic part: No. of terms	Periodic part: Term with maximal coefficient	Secular and mixed part: No. of terms	Secular and mixed part: Terms with maximal coefficient	General form of argument
$(01)+(05)+(09)=\Sigma m_k\left(a'\frac{\partial R}{\partial \rho'}\right)_0$	190	+0.0455684 cos (M_1-M')	—	—	$i_1M_1+i_2M_2+i_3M_3+i'M'$
$(03)=m_1\left(a'\frac{\partial R}{\partial \rho_1}\right)_0$	47	−0.0908228 cos (M_1-M')	—	—	$i_1M_1+i'M'$
$(07)=m_2\left(a'\frac{\partial R}{\partial \rho_2}\right)_0$	54	+0.0070174 sin (M_2-M')	—	—	$i_2M_2+i'M'$
$(11)=m_3\left(a'\frac{\partial R}{\partial \rho_3}\right)_0$	100	0.0003033 sin (M_3-M')	—	—	$i_3M_3+i'M'$
$(02)+(06)+(10)=\Sigma m_k\left(a'\frac{\partial R}{\partial W'}\right)_0$	190	+0.0452704 sin (M_1-M')	—	—	$i_1M_1+i_2M_2+i_3M_3+i'M'$
$(04)=m_1\left(a'\frac{\partial R}{\partial W_1}\right)_0$	45	−0.0452539 sin (M_1-M')	—	—	$i_1M_1+i'M'$
$(08)=m_2\left(a'\frac{\partial R}{\partial W_2}\right)_0$	51	−0.0034332 cos (M_2-M')	—	—	$i_2M_2+i'M'$
$(12)=m_3\left(a'\frac{\partial R}{\partial W_3}\right)_0$	90	−0.0001231 cos (M_3-M')	—	—	$i_3M_3+i'M'$
$(13)+(17)+(21)=\Sigma m_k\left(a'\frac{\partial^2 R}{\partial \rho'^2}\right)_0$	256	+0.0449340 cos (M_1-M')	—	—	$i_1M_1+i_2M_2+i_3M_3+i'M'$
$(15)=m_1\left(a'\frac{\partial^2 R}{\partial \rho' \partial \rho_1}\right)_0$	79	−0.0905023 cos (M_1-M')	—	—	$i_1M_1+i'M'$
$(19)=m_2\left(a'\frac{\partial^2 R}{\partial \rho' \partial \rho_2}\right)_0$	98	+0.0068555 sin (M_2-M')	—	—	$i_2M_2+i'M'$
$(23)=m_3\left(a'\frac{\partial^2 R}{\partial \rho' \partial \rho_3}\right)_0$	118	0.0002280 sin (M_3-M')	—	—	$i_3M_3+i'M'$
$(26)+(30)+(34)=\Sigma m_k\left(a'\frac{\partial^2 R}{\partial \rho' \partial W'}\right)_0$	285	+0.0455834 sin (M_1-M')	—	—	$i_1M_1+i_2M_2+i_3M_3+i'M'$
$(14)=m_1\left(a'\frac{\partial^2 R}{\partial \rho' \partial W_1}\right)_0$	73	−0.0455676 sin (M_1-M')	—	—	$i_1M_1+i'M'$
$(18)=m_2\left(a'\frac{\partial^2 R}{\partial \rho' \partial W_2}\right)_0$	95	−0.0035836 cos (M_2-M')	—	—	$i_2M_2+i'M'$
$(22)=m_3\left(a'\frac{\partial^2 R}{\partial \rho' \partial W_3}\right)_0$	118	−0.0001806 cos (M_3-M')	—	—	$i_3M_3+i'M'$

Table 15 (continued)

Series	Periodic part: No. of terms	Periodic part: Term with maximal coefficient	Secular and mixed part: No. of terms	Secular and mixed part: Term with maximal coefficient	General form of argument
$(25)+(29)+(33)=\Sigma m_k\left(a'\frac{\partial^2 R}{\partial W'^2}\right)_0$	281	$-0.0452549\cos(M_1-M')$	—	—	$i_1M_1+i_2M_2+i_3M_3+i'M'$
$(27)=m_1\left(a'\frac{\partial^2 R}{\partial W'\partial W_1}\right)_0$	76	$+0.0452713\cos(M_1-M')$	—	—	$i_1M_1+i'M'$
$(31)=m_2\left(a'\frac{\partial^2 R}{\partial W'\partial W_2}\right)_0$	94	$-0.0034348\sin(M_2-M')$	—	—	$i_2M_2+i'M'$
$(35)=m_3\left(a'\frac{\partial^2 R}{\partial W'\partial W_3}\right)_0$	104	$-0.0001230\sin(M_3-M')$	—	—	$i_3M_3+i'M'$
$(28)=m_1\left(a'\frac{\partial^2 R}{\partial W'\partial \rho_1}\right)_0$	74	$-0.0908538\sin(M_1-M')$	—	—	$i_1M_1+i'M'$
$(32)=m_2\left(a'\frac{\partial^2 R}{\partial W'\partial \rho_2}\right)_0$	98	$-0.0070196\cos(M_2-M')$	—	—	$i_2M_2+i'M'$
$(36)=m_3\left(a'\frac{\partial^2 R}{\partial W'\partial \rho_3}\right)_0$	113	$-0.0003034\cos(M_3-M')$	—	—	$i_3M_3+i'M'$
$(37)=\delta\rho'$	138	$-1086''.80\sin M'$	11	$+357''.69\ n't\sin M'$	$i_1M_1+i_2M_2+i_3M_3+i'M'$
$(38)=\delta w'$	133	$-2208''.90\cos M'$	11	$+1480''.77\ n't$	$i_1M_1+i_2M_2+i_3M_3+i'M'$
$(40)=\delta\rho_1$	100	$+103''.53\cos(2M_1-2M_2)$	32	$-0''.6988\ n_1t\sin M_1$	$i_1M_1+i_2M_2+i_3M_3+i_4M_4$
$(41)=\delta w_1$	108	$+1091''.35\cos(2M_1-5M_2)$	39	$-1''.3986\ n_1t\cos M_1$	$i_1M_1+i_2M_2+i_3M_3+i_4M_4$
$(42)=\delta\rho_2$	118	$-404''.25\sin(2M_1-4M_2)$	42	$-5''.3662\ n_2t\sin M_2$	$i_1M_1+i_2M_2+i_3M_3+i_4M_4$
$(43)=\delta w_2$	128	$-2681''.44\cos(2M_1-5M_2)$	52	$-10''.7458\ n_2t\cos M_2$	$i_1M_1+i_2M_2+i_3M_3+i_4M_4$
$(44)=\delta\rho_3$	131	$+92''.12$	16	$-55''.59\ T\cos M_3$	$i_1M_1+i_2M_2+i_3M_3+i_4M_4$
$(45)=\delta w_3$	135	$142''.51\sin(M_2-2M_3)$	18	$+111''.36\ T\sin M_3$	$i_1M_1+i_2M_2+i_3M_3+i_4M_4$
$\frac{1}{2}(46)=\frac{1}{2}\frac{a'}{r_0'}$	8	0.500000	—	—	$i'M'$
$(47)=\left(\frac{r_0'}{a'}\right)^2$	8	1.091568	—	—	$i'M'$
$(48)=\left(\frac{a'}{r_0'}\right)^2$	10	+1.0319950	—	—	$i'M'$
$(39)=\Sigma m_k\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt+C_2\right]$	113	$-0.0022699\cos(M_1-M')$	—	—	$i_1M_1+i_2M_2+i_3M_3+i'M'$

not include all coefficients or it does not verify all significant digits of the coefficients.

However, an appreciable part of the calculations can be thus verified.

We will investigate in greater detail the calculation of $a'\delta D'_t R$. It is known that $D'_t R$ is determined as the derivative of the perturbing function, with respect to time entering in terms of the coordinates of the perturbed planet:

$$a' D'_t R = a' \frac{\partial R}{\partial \rho'} \frac{d\rho'}{dt} + a' \frac{\partial R}{\partial W'} \frac{dw'}{dt}. \tag{60}$$

Since

$$\left.\begin{aligned} a' \frac{\partial R}{\partial \rho'} &= \left(a' \frac{\partial R}{\partial \rho'}\right)_0 + a'\delta \frac{\partial R}{\partial \rho'}, \\ a' \frac{\partial R}{\partial W'} &= \left(a' \frac{\partial R}{\partial W'}\right)_0 + a'\delta \frac{\partial R}{\partial W'}, \\ \frac{d\rho'}{dt} &= \frac{\partial \rho'_0}{dt} + \frac{d\delta\rho'}{dt}, \\ \frac{dw'}{dt} &= \frac{dw'_0}{dt} + \frac{d\delta w'}{dt}, \end{aligned}\right\} \tag{61}$$

then, substituting into Equation (60) the values of Equation (61) and restricting ourselves to terms of the first order, we obtain:

$$a'\delta D'_t R = a'\delta \frac{\partial R}{\partial \rho'} \frac{d\rho'_0}{dt} + a'\delta \frac{\partial R}{\partial W'} \frac{dw'_0}{dt} + \left(a' \frac{\partial R}{\partial \rho'}\right)_0 \frac{d\delta\rho'}{dt} + \left(a' \frac{\partial R}{\partial W'}\right)_0 \frac{d\delta w'}{dt}. \tag{62}$$

On the other hand, we resort to the expression (for simplicity, we take two planets):

$$a'\delta R = \left(a' \frac{\partial R}{\partial \rho'}\right)_0 \delta\rho' + \left(a' \frac{\partial R}{\partial W'}\right)_0 \delta w' + \left(a' \frac{\partial R}{\partial \rho}\right)_0 \delta\rho + \left(a' \frac{\partial R}{\partial W}\right)_0 \delta w. \tag{63}$$

We take the derivative of $a'\delta R$ with respect to time entering this expression through the coordinates of the perturbed planet. This time will enter the expressions $\left(a' \frac{\partial R}{\partial \rho'}\right)_0$, $\left(a' \frac{\partial R}{\partial W'}\right)_0$, $\left(a' \frac{\partial R}{\partial \rho}\right)_0$ and $\left(a' \frac{\partial R}{\partial W}\right)_0$ through M'; it will not at all enter $\delta\rho$ and δw; and it will enter $\delta\rho'$ and $\delta w'$ through M and M'.

/60

Table 16

Series	Periodic Part: No. of Terms	Periodic Part: Term with maximal coefficient
$[(01)+(05)+(09)]\times(37)=\left[\Sigma m_k\left(a'\frac{\partial R}{\partial \rho'}\right)\right]_0\times\delta\rho'$	1200	$-33.6645''\sin(M_1-M')$
$[(26)+(30)+(34)]\times(37)=\left[\Sigma m_k\left(a'\frac{\partial^2 R}{\partial\rho'\partial W'}\right)_0\right]\times\delta\rho'$	1610	$+33.6789\cos(M_1-M')$
$[(35)+(17)+(21)]\times(37)=\left[\Sigma m_k\left(a'\frac{\partial^2 R}{\partial\rho'^2}\right)_0\right]\times\delta\rho'$	1500	$-33.1418\sin(M_1-M')$
$(37)\times(37)=(\delta\rho')^2$	360	$+8.9142\sin M'$
$[(02)+(06)+(10)]\times(38)=\left[\Sigma m_k\left(a'\frac{\partial R}{\partial W'}\right)_0\right]\times\delta w'$	1320	$-55.1058\sin M_1$
$[(25)+(29)+(33)]\times(38)=\left[\Sigma m_k\left(a'\frac{\partial^2 R}{\partial W'^2}\right)_0\right]\times\delta w'$	1830	$+53.0356\cos M_1$
$[(26)+(30)+(34)]\times(38)=\left[\Sigma m_k\left(a'\frac{\partial^2 R}{\partial\rho'\partial W'}\right)_0\right]\times\delta w'$	1880	$-55.6100\sin M_1$
$(15)\times(40)=\left[m_1\left(a'\frac{\partial^2 R}{\partial\rho'\partial\rho_1}\right)_0\right]\times\delta\rho_1$	1140	$-7.6100\cos(M_1-M')$
$(28)\times(40)=\left[m_1\left(a'\frac{\partial^2 R}{\partial W'\partial\rho_1}\right)_0\right]\times\delta\rho_1$	1170	$-7.6424\sin(M_1-M')$
$(03)\times(40)=\left[m_1\left(a'\frac{\partial R}{\partial\rho_1}\right)_0\right]\times\delta\rho_1$	1050	$-7.6376\cos(M_1-M')$
$(14)\times(41)=\left[m_1\left(a'\frac{\partial^2 R}{\partial\rho'\partial W_1}\right)_0\right]\times\delta w_1$	1280	$-31.5438\sin(3M_1-5M_2-M')$
$(27)\times(41)=\left[m_1\left(a'\frac{\partial^2 R}{\partial W'\partial W_1}\right)_0\right]\times\delta w_1$	1270	$+31.3400\cos(3M_1-5M_2-M')$
$(04)\times(41)=\left[m_1\left(a'\frac{\partial R}{\partial W_1}\right)_0\right]\times\delta w_1$	1100	$-31.3274\sin(3M_1-5M_2-M')$
$(19)\times(42)=\left[m_2\left(a'\frac{\partial^2 R}{\partial\rho'\partial\rho_2}\right)_0\right]\times\delta\rho_2$	1060	$-1.3860\cos(2M_1-5M_2+M')$
$(32)\times(42)=\left[m_2\left(a'\frac{\partial^2 R}{\partial W'\partial\rho_2}\right)_0\right]\times\delta\rho_2$	1100	$+1.4132\sin(2M_1-5M_2+M')$
$(07)\times(42)=\left[m_2\left(a'\frac{\partial R}{\partial\rho_2}\right)_0\right]\times\delta\rho_2$	820	$-1.4188\cos(2M_1-5M_2+M')$
$(18)\times(43)=\left[m_2\left(a'\frac{\partial^2 R}{\partial\rho'\partial W_2}\right)_0\right]\times\delta w_2$	1230	$+6.7905\cos(2M_1-4M_2-M')$
$(31)\times(43)=\left[m_2\left(a'\frac{\partial^2 R}{\partial W'\partial W_2}\right)_0\right]\times\delta w_2$	1180	$6.5114\sin(2M_1-4M_2-M')$
$(08)\times(43)=\left[m_2\left(a'\frac{\partial R}{\partial W_2}\right)_0\right]\times\delta w_2$	860	$6.5098\cos(2M_1-4M_2-M')$
$(23)\times(44)=\left[m_3\left(a'\frac{\partial^2 R}{\partial\rho'\partial\rho_3}\right)_0\right]\times\delta\rho_3$	950	$+0.0210\sin(M_3-M')$
$(36)\times(44)=\left[m_3\left(a'\frac{\partial^2 R}{\partial W'\partial\rho_3}\right)_0\right]\times\delta\rho_3$	810	$-0.0280\cos(M_3-M')$
$(11)\times(44)=\left[m_3\left(a'\frac{\partial R}{\partial\rho_3}\right)_0\right]\times\delta\rho_3$	470	$+0.0278\sin(M_3-M')$
$(22)\times(45)=\left[m_3\left(a'\frac{\partial^2 R}{\partial\rho'\partial W_3}\right)_0\right]\times\delta w_3$	1210	$-0.0148\sin(M_2-4M_3+M')$

Table 16 (continued) /59

Secular and mixed part				
No. of terms	Term with maximal coefficient	No. of terms	Term with maximal coefficient	General form of argument
260	+ 8.6624 $n't \sin M_1$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i'M'$
350	— 8.3014 $n't \cos M_1$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i'M'$
330	+ 8.5208 $n't \sin M_1$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i'M'$
140	— 2.9345 $n't \sin M_1$	10	+0".3576 $(n't)^2$	$i_1M_1 + i_2M_2 + i_3M_3 + i'M'$
270	+62.8274 $n't \sin (M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i'M'$
380	—62.8502 $n't \cos (M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i'M'$
400	+63.3596 $n't \sin (M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i'M'$
180	+ 0.0479 $n't \sin (2M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
190	— 0.0481 $n't \cos (2M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
180	+ 0.0481 $n't \sin (2M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
230	+ 0.0489 $n't \sin (2M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
230	— 0.0486 $n't \cos (2M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
220	+ 0.0486 $n't \sin (2M_1 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
200	— 0.0287 $n't \cos M'$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
200	+ 0.0289 $n't \sin M'$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
150	— 0.0293 $n't \cos M'$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
270	+ 0.0299 $n't \cos M'$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
250	— 0.0282 $n't \sin M'$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
180	+ 0.0287 $n't \cos M'$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
120	— 0.0100 $n't \sin (2M_3 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
110	+ 0.0127 $n't \cos (2M_3 - M')$	—	—	$i_1M^1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
90	— 0.0127 $n't \sin (2M_3 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$
120	— 0.0149 $n't \sin (2M_3 - M')$	—	—	$i_1M_1 + i_2M_2 + i_3M_3 + i_4M_4 + i'M'$

Table 16 (continued) /60

Series	Periodic part: No. of terms	Periodic part: Term with maximal coefficient
$(35)\times(45)=\left[m_3\left(a'\frac{\partial^2 R}{\partial W'\partial W_3}\right)_0\right]\times\delta w_3$	1020	$-0''.0110\cos(M_2-4M_3+M')$
$(12)\times(45)=\left[m_3\left(a'\frac{\partial R}{\partial W_3}\right)_0\right]\times\delta w_3$	570	$-0''.0109\sin(M_2-4M_3+M')$
$(47)\times[(37)\times 37)]=\left(\frac{r_0'}{a'}\right)^2\times(\delta\rho')^2$	390	$+8''.5608\sin M'$
$(46)\times[(37)\times(37)]=\frac{1}{2}\frac{a'}{r_0'}\times(\delta\rho')^2$	390	$4''.6608\sin M'$
$(39)\times(37)=\left\{\Sigma m_k\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt+C_2\right]\right\}\times\delta\rho'$	600	$2''.1182\sin M'$

Table 16 (continued) /61

Secular and mixed part: No. of terms	Term of maximal coefficient	No. of terms	Term with maximal coefficient	General form of argument
100	$+0''.0100\ n't\cos(2M_3-M')$	—	—	$i_1M_1+i_2M_2+i_3M_3+i_4M_4+i'M'$
80	$-0''.0100\ n't\sin(2M_3-M')$	—	—	$i_1M_1+i_2M_2+i_3M_3+i_4M_4+i'M'$
110	$-2''.8174\ n't\sin M'$	10	$0''.3301\ (n't)^2$	$i_1M_1+i_2M_2+i_3M_3+i'M'$
110	$-1''.5344\ n't\sin M'$	10	$0''.1906\ (n't)^3$	$i_1M_1+i_2M_2+i_3M_3+i'M'$
150	$-0''.6993\ n't\sin M'$	—	—	$i_1M_1+i_2M_2+i_3M_3+i'M'$

Then:

$$a'\delta D_t' R=a'D_t'\left(\frac{\partial R}{\partial\rho'}\right)_0\cdot\delta\rho'+a'D_t'\left(\frac{\partial R}{\partial W'}\right)_0\cdot\delta w'+a'D_t'\left(\frac{\partial R}{\partial\rho}\right)_0\cdot\delta\rho+a'D_t'\left(\frac{\partial R}{\partial W}\right)_0\cdot\delta w+ \quad /60$$
$$+\left(a'\frac{\partial R}{\partial\rho'}\right)_0\left[\frac{\partial\delta\rho'}{\partial M'}\frac{dM'}{dt}+\frac{\partial\delta\rho'}{\partial M}\frac{dM}{dt}\right]+\left(a'\frac{\partial R}{\partial W'}\right)_0\left[\frac{\partial\delta w'}{\partial M'}\frac{dM'}{dt}+\frac{\partial\delta w'}{\partial M}\frac{dM}{dt}\right] \qquad (64)$$

Since perturbations of Jupiter, Saturn, and Uranus do not depend on the coordinates of Pluto, Equation (64) can be rewritten in the following manner:

$$a'\delta D_t' R=a'\frac{\partial\delta R}{\partial M'}\frac{dM'}{dt}+\left(a'\frac{\partial R}{\partial\rho'}\right)_0\frac{\partial\delta\rho'}{dM}\frac{dM}{dt}+\left(a'\frac{\partial R}{\partial W'}\right)_0\frac{\partial\delta w'}{dM}\frac{dM}{dt}. \qquad (65)$$

The derivative of a' δR, with respect to time, is determined by Formula (65). Furthermore, the quantity of a' $\delta D_t'$R can be calculated also by Formula (62), using the previously obtained series:

$$a'\delta\frac{\partial R}{\partial \rho'},\ a'\delta\frac{\partial R}{\partial W'},\ \left(a'\frac{\partial R}{\partial \rho'}\right)_0,\ \left(a'\frac{\partial R}{\partial W'}\right)_0.$$

Formula (62) was used for verifying the calculations of the quantities $a'\delta D_t' R$, $a'\delta\frac{\partial R}{\partial \rho'}$, $a'\delta\frac{\partial R}{\partial W'}$; the products of the series were verified in the same manner. This verification was accomplished for the periodic part of the coefficients for:

$$\begin{matrix}\sin\\ \cos\end{matrix}(M_1 + i'M'),$$
$$\begin{matrix}\sin\\ \cos\end{matrix}(M_2 + i'M'),$$
$$\begin{matrix}\sin\\ \cos\end{matrix}(M_3 + i'M').$$

Let us examine in more detail the course of the calculations of the derivative of a' δR, with respect to time, for the periodic part of the series and for the secular and mixed terms.

Any of the terms of the series a' δR (periodic part) has the form:

$$a^s \sin\left(\sum_{k=1}^{4} i_k M_k + i'M'\right) + a^c \cos\left(\sum_{k=1}^{4} i_k M_k + i'M'\right).$$

The derivative, with respect to time, entering in terms of M' will be: /61

$$-n'i'a^c \sin\left(\sum_{k=1}^{4} i_k M_k + i'M'\right) + n'i'a^s \cos\left(\sum_{k=1}^{4} i_k M_k + i'M'\right).$$

Now we must take the derivative of perturbations $\delta\rho$' and δw', with respect to time, entering in terms of M, and then multiply it by $(a'\frac{\partial R}{\partial \rho'})_0$ and $(a'\frac{\partial R}{\partial W'})_0$. This derivative for any of the terms of the series (periodic part) has the form:

$$-n_k i_k B^c \sin\left(\sum_{k=1}^{3} i_k M_k + i'M'\right) + n_k i_k B^s \cos\left(\sum_{k=1}^{3} i_k M_k + i'M'\right),$$

where B^s and B^c are coefficients of the series of perturbations of Pluto. We

represent this in the form:

$$-n'\frac{n_k}{n'}\,i_kB^c\sin\left(\sum_{k=1}^{3}i_kM_k+i'M'\right)+n'\frac{n_k}{n'}\,i_kB^s\cos\left(\sum_{k=1}^{3}i_kM_k+i'M'\right).$$

We designate by b^s and b^c the coefficients of the sum of the multiplied series $\left(a'\frac{\partial R}{\partial W'}\right)_0\frac{\partial\delta w'}{\partial M}\frac{dM}{dt}$ and $\left(a'\frac{\partial R}{\partial\rho'}\right)_0\frac{\partial\delta\rho'}{\partial M}\frac{dM}{dt}$.

We will write out all parts representing any term of the quantity $\frac{1}{n'}a'\delta D_t'R$:

$$-i'a^c\sin\left(\sum_{k=1}^{4}i_kM_k+i'M'\right)+i'a^s\cos\left(\sum_{k=1}^{4}i_kM_k+i'M'\right)-\frac{n_k}{n'}\,i_kb^c\sin\left(\sum_{k=1}^{3}i_kM_k+i'M'\right)+$$
$$+\frac{n_k}{n'}\,i_kb^s\cos\left(\sum_{k=1}^{3}i_kM_k+i'M'\right).$$

Thus, the obtainment of the derivative $\frac{1}{n'}a'\delta D_t'R$ reduces to multiplying the series $a'\delta R$ by i' and to multiplying the series $\left(a'\frac{\partial R}{\partial\rho'}\right)_0\delta\rho'$ and $\left(a'\frac{\partial R}{\partial W'}\right)_0\delta w'$ by i_k, which is taken from the series of perturbations ϕ' and $\delta w'$, and by $\frac{n_k}{n'}$ (in this case the coefficients a^s and a^c, as well as b^s and b^c, change places, and the sign of the coefficients a^c and b^c change to the opposite).

The derivative $\frac{1}{n'}a'\delta D_t'R$, for the secular and mixed parts of the series $a'\delta R$, is calculated by the same formula, but does not have the products $\left(a'\frac{\partial R}{\partial\rho'}\right)_0\frac{\partial\delta\rho'}{\partial M}\frac{dM}{dt}$ and $\left(a'\frac{\partial R}{\partial W'}\right)_0\frac{\partial\delta w'}{\partial M}\frac{dM}{dt}$. The general term of the secular and mixed parts of the series $a'\delta R$: /62

$$n'tA\sin\left(\sum_{k=1}^{4}i_kM_k+i'M'\right)+n'tB\cos\left(\sum_{k=1}^{4}i_kM_k+i'M'\right)$$

gives for the quantities $\frac{1}{n'}a'\delta D_t'R$.

$$A\sin\left(\sum_{k=1}^{4}i_kM_k+i'M'\right)+B\cos\left(\sum_{k=1}^{4}i_kM_k+i'M'\right)-$$
$$-n'ti'B\sin\left(\sum_{k=1}^{4}i_kM_k+i'M'\right)+n'ti'A\cos\left(\sum i_kM_k+i'M'\right).$$

The first two terms of this expression give the correction to the periodic

part of $\frac{1}{n'} a'\delta D'_t R$.

The second derivative with respect to n't of $\left(\frac{r'_0}{a'}\right)^2 (\delta\rho')^2$ is derived from the formula:

$$\frac{d^2}{(dn't)^2}\left[\left(\frac{r'_0}{a'}\right)^2 (\delta\rho')^2\right] = -\sum (a^s_\mu \sin C_\mu + a^c_\mu \cos C_\mu) \sum_{k=1} \left(\frac{i_k n_k + i'n'}{n'}\right)^2,$$

where a^s_μ and a^c_μ are the coefficients of the expansion of $\left(\frac{r'_0}{a'}\right)^2 (\delta\rho')^2$ for the corresponding trigonometric functions; $C_\mu = \sum_{k=1}^{5} i_k M_k + i'M'$. On calculating this quantity from secular and mixed terms of the first order of the series $\left(\frac{r'_0}{a'}\right)^2 (\delta\rho')^2$ we obtain terms with n't and the correction to the periodic part of $a'\delta Q$. Upon calculating this derivative from mixed terms of the second order, we obtain terms with $(n't)^2$ and corrections to mixed terms of the first order and to the periodic part of $a'\delta Q$.

CHAPTER IV

SECOND-ORDER PERTURBATIONS OF PLUTONIAN COORDINATES WITH RESPECT TO THE PERTURBING MASSES OF JUPITER, SATURN, AND URANUS

12. Integration of the Equations of Motion

To determine second-order perturbations of the logarithm of the radius vector and longitude in orbit, we used Formulas (18) derived in Section 1: /63

$$\delta_2\rho' = \frac{m}{1+m'}\left(\frac{a'}{r_0'}\right)^2\left\{qn'\int p\,(a'\delta Q + 2a'\delta C_1)\,dt - \right.$$
$$\left. - pn'\int q\,(a'\delta Q + 2a'\delta C_1)\,dt + a'\delta K_1 p + a'\delta K_2 q,\right.$$

$$\delta_2 w' = n'\int\left(\frac{a'}{r_0'}\right)^2\left\{\frac{m}{1+m'}\left(n'\int a'\delta\,\frac{\partial R}{\partial W'}\,dt + \delta C_2\right) - \right.$$
$$- 2\delta_1\rho'\frac{m}{1+m'}\left[n'\int\left(a'\,\frac{\partial R}{\partial W'}\right)_0 dt + (C_2)_0\right] -$$
$$\left. - 2\cos\varphi'\left[\delta_2\rho' - (\delta_1\rho')^2\right]\right\}dt + \delta C_3.$$

The calculations were performed on punched-card machinery, according to the schemes in Section 11 of Part One.

The integrating factors, given in Table 20, and the coefficients of the expansions of p, q, $(\frac{a'}{r_0'})^2$, $p(\frac{a'}{r_0'})^2$, and $q(\frac{a'}{r_0'})^2$, given in Table 17, were calculated on the basis of elements derived in Section 22 or Part One.

From the general scheme of integration, from the very beginning we selected groups containing long-period terms. These groups were calculated separately and the following section will be devoted to them.

13. Long-period Perturbations

As was indicated in Section 6, in order to avoid a loss of accuracy in the presence of pronounced covariances of the mean motions, we can represent the long-period perturbations as secular. In determining second-order perturbations of Pluto from major planets, 13 covariances were isolated. Table 18 gives the basic characteristics of these covariances.

The formulas derived in Section 6 are the working formulas. As an example, we will present the basic stages of calculations for the group of terms containing $\nu = 5n_3 - 15n'$. The calculations were carried out with an accuracy to the fourth decimal place of a second of arc.

Scheme I and Scheme II give part of the perturbations $\delta_2 \log r'$ and $\delta_2 w'$ of group $5n_3$, independent of the terms of pronounced covariances. For determining the part dependent on pronounced covariances, we use Formulas (42) and (48).

Table 17

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j	p cos	q sin	$(48)=\left(\frac{a'}{r'}\right)^2$ cos	$p\left(\frac{a'}{r'}\right)^2$ cos	$q\left(\frac{a'}{r'}\right)^2$ sin
0	—0.3706102	—	+1.0319950	—0.1254819	—
1	+0.9772049	+0.9923887	0.5061168	+0.9310096	+0.9770466
2	0.1185664	0.1210421	0.1540122	0.3432111	0.3568691
3	0.0216068	0.0221166	0.0487039	0.1187397	0.1230938
4	0.0046691	0.0047870	0.0156423	0.0402989	0.0417174
5	0.0011088	0.0011380	0.0050651	0.0135574	0.0140263
6	0.0002796	0.0002872	0.0016448	0.0045419	0.0046937
7	0.0000730	0.0000752	0.0005365	0.0015225	0.0015566
8	0.0000197	0.0000204	0.0001917	0.0005016	0.0005177
9	0.0000064	0.0000064	0.0000638	0.0001768	0.0001794
10	0.0000018	0.0000018		0.0000590	0.0000612

Table 18

i_1	i_2	i_3	i_4	i'	$\nu=\sum_{k=1}^{4}(i_k n_k+i'n')$	Period (in centuries)	$\frac{n'}{\nu}$
0	0	0	2	— 3	+0".4453	80	+ 31.9036560
		1	—4	3	—1.2764	28	— 11.1302883
		1	—2	0	—0.8311	43	— 17.0938491
		1	0	— 3	—0.3858	92	— 36.8239969
		1	2	— 6	+0.0595	600	+238.7680336
		2	—2	— 3	—1.2169	29	— 11.6745008
		2	0	— 6	—0.7716	46	— 18.4119984
		2	2	— 9	—0.3263	109	— 43.5387619
		3	0	— 9	—1.1574	31	— 12.2746674
		3	2	—12	—0.7121	50	— 19.9504255
		4	0	—12	—1.5432	23	— 9.2060005
		5	0	—15	—1.9290	18	— 7.3648004
1	—2	0	0	— 4	+1.3916	26	+ 10.2088962

Scheme I

i_3	i'	$10^4\frac{m}{n'}a'\delta D_l' R$ sin	cos	$n't$ sin	$n't$ cos	$10^4 m\int 2a'\delta D_l' R dt$ sin	cos	$n't$ sin	$n't$ cos
5	—16	+ 2					+ 4		
	—15	— 9	+ 20	+ 15		—	—	—	—
	—14	— 53	31	49	+ 14	+203	160	+ 32	—113
	—13	—298	+ 27	102	46	96	346	49	—126
	—12	—346	— 31	294	120	+ 50	271	84	—205
	—11	—363	— 98	302	264	— 10	223	136	—156
	—10	—490	+ 32	290	215	+ 59	220	88	—119
	— 9	—602	348	495	144	148	213	49	—169
	— 8	—710	496	576	140	169	213	41	—167
	— 7	—456	434	480	105	126	119	27	—122
	— 6	—112	252	267	+ 16	64	+ 26	+ 7	— 60
	— 5	—130	63	+ 50	— 12	+ 14	— 26	— 3	— 10
	— 4	+ 92	0	— 56	— 14	— 1	— 17	— 3	+ 10
	— 3	+ 54	2	— 9	— 12	0	— 6	— 2	2
	— 2	— 11	+ 5		+ 1	+ 1	+ 2		
	— 1	— 5	0				1		
	0		— 3						

Scheme I (continued)

i_s	i'	$ma' \frac{\partial R}{\partial \rho'} + \left[- \frac{d^2}{(dn't)^2} \left(\frac{r_0'}{a'} \delta\rho' \right)^2 + \frac{1}{2} \left(\frac{r_0'}{a'} \right)^{-1} \delta\rho'^2 \right.$				$ma'\delta Q$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−21		+ 38				+ 38		
	−20		− 69				− 69		
	−19	− 9	− 65			− 9	− 65		
	−18	+ 17	− 77			+ 17	− 77		
	−17	18	− 93			18	− 93		
	−16	14	− 14	+ 1	− 2	14	− 10	+ 1	− 2
	−15	11	+ 44	3	− 4	—	—	—	—
	−14	26	+120	7	− 18	229	+279	39	−131
	−13	59	− 22	18	− 52	154	325	67	−178
	−12	6	− 53	60	−142	+ 57	217	144	−347
	−11	8	+ 42	157	−134	− 2	265	293	−290
	−10	133	153	132	−139	+172	374	220	−258
	− 9	429	454	82	−310	578	668	131	−479
	− 8	568	441	103	−411	735	655	144	−578
	− 7	469	+268	80	−416	594	387	107	−538
	− 6	339	− 19	+ 16	−288	404	+ 8	+ 23	−348
	− 5	+ 166	−270	− 16	− 49	+180	−295	− 19	− 59
	− 4	− 52	− 41	− 24	+ 88	− 53	− 57	− 27	+ 98
	− 3	− 13	− 63	− 27	+ 13	− 13	− 69	− 29	+ 15
	− 2	− 20	+ 6	− 2	0	− 19	+ 9	− 2	0
	− 1	− 20	8		0	− 20	10		0
	0	− 8	6		− 1	− 8	7		− 1
	1	0	3		− 2	0	3		− 2
	2	+ 1	1			+ 1	1		
	3	1				1			

Scheme I (continued)

i_s	i'	$mpa'\delta Q$				$mqa'\delta Q$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−23		+ 1			− 1			
	−22		+ 14			− 14			
	−21	− 1	− 53			+ 39	− 1		
	−20	− 3	+ 7			57	− 3		
	−19	+ 12	− 52			12	+ 9		
	−18	1	− 52			+ 10	15		
	−17	9	− 10		− 2	− 42	3	+ 2	
	−16	21	− 25	+ 3	− 10	− 74	6	11	+ 3
	−15	+130	+146	26	− 31	−173	116	80	26
	−14	− 6	71	32	− 63	−180	+ 78	113	46
	−13	+ 86	144	83	−189	+ 8	− 83	129	72
	−12	85	256	139	−129	+ 15	− 73	+ 70	124
	−11	167	261	82	−236	−103	+ 89	− 20	+ 44
	−10	274	377	145	−344	−226	334	+113	− 80
	− 9	269	296	153	−287	−142	319	174	− 52
	− 8	336	295	80	−324	+169	+ 24	+ 31	− 27
	− 7	382	208	51	−287	385	−192	−144	− 72
	− 6	275	85	43	−197	390	−259	−285	− 75
	− 5	147	+113	+ 11	−138	+ 68	−272	−263	− 34
	− 4	+133	−150	− 11	− 86	−107	−129	− 64	− 7
	− 3	− 15	− 13	− 4	+ 34	− 50	0	+ 40	+ 13
	− 2	− 9	− 38	− 15	11	− 45	− 4	11	16
	− 1	− 8	− 3	− 3	+ 2	− 5	+ 7	2	3
	0	− 8	+ 3		− 1	+ 4	11	+ 1	
	1	− 5	3		+ 1	4	5	0	
	2		1		− 1	1		− 1	

Scheme I (continued)

i_3	i'	$mn' \int pa'\delta Qdt$				$mn' \int qa'\delta Qdt$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−22	− 2					− 2		
	−21	+ 9					+ 6		
	−20	− 1	− 1			+ 1	11		
	−19	+ 13	+ 3			− 2	3		
	−18	17	0			− 5	+ 3		
	−17	5	4	+ 1		− 1	− 20		+ 1
	−16	24	+ 10	+ 9	+ 3	+ 4	− 63	− 3	+ 10
	−15	—	—	—	—	—	—	—	—
	−14	125	− 77	− 73	− 37	+241	+270	+ 53	−131
	−13	101	−100	−101	− 45	− 8	17	39	− 69
	−12	106	− 46	− 45	− 49	− 16	10	43	− 24
	−11	73	− 59	− 61	− 21	+ 22	30	+ 11	+ 5
	−10	84	− 71	− 71	− 30	74	43	− 16	− 23
	− 9	54	− 54	− 49	− 26	59	+ 22	− 9	− 30
	− 8	45	− 56	− 47	− 12	+ 4	− 26	− 4	− 5
	− 7	27	− 54	− 36	− 6	− 26	− 50	− 9	+ 18
	− 6	11	− 34	− 22	− 5	− 33	− 45	− 8	32
	− 5	+ 5	− 16	− 14	− 1	− 31	− 7	− 3	27
	− 4	− 14	− 13	− 8	+ 1	− 13	+ 10	− 1	+ 6
	− 3	− 1	+ 1	+ 3	0	0	4	+ 1	− 3
	− 2	− 3	1	1	1	0	3	1	− 1
	− 1		1			+ 1			
	0		1			1			

Scheme I (continued)

i_3	i'	$\left(\frac{a'}{r_0'}\right)^2 qmn' \int pa'\delta Qdt$				$\left(\frac{a'}{r_0'}\right)^2 pmn' \int qa'\delta Qdt$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−24		+ 1						
	−23		1				+ 1		
	−22		5				5		
	−21	− 1	4				4		
	−20	− 1	5			− 1	3		
	−19	− 1	9		+ 1		+ 2		
	−18	0	3		+ 1	+ 6	− 12		− 1
	−17	+ 3	13	+ 2	− 2	14	− 7	+ 3	− 4
	−16	23	25	11	−20	39	+ 47	12	−27
	−15	**66**	**74**	**30**	**−62**	**+112**	**+ 99**	**34**	**−69**
	−14	64	70	34	−64	− 34	− 33	18	−18
	−13	+ 1	+ 9	+13	− 3	+115	+133	41	−63
	−12	−17	−16	−12	+15	66	73	23	−55
	−11	+ 4	−23	−13	− 4	46	41	20	−48
	−10	− 5	−28	− 8	+11	28	19	+11	−19
	− 9	−13	−36	−16	21	27	+ 1	− 4	− 5
	− 8	−12	−32	−16	19	+ 21	− 8	− 9	− 2
	− 7	−21	−33	−11	24	− 2	− 19	− 9	+10
	− 6	−30	−24	− 8	22	− 18	− 20	− 6	15
	− 5	−25	−22	− 5	18	− 20	− 25	− 6	16
	− 4	−18	−11	− 1	15	− 20	− 13	− 3	17
	− 3	−12	+ 2		8	− 14	0	− 1	10
	− 2	− 4	1		+ 1	− 5	+ 3		3
	− 1	− 1	2		− 1	− 2	3		
	0		1				1		

Scheme I (continued)

i_3	i'	$10^4\,(\delta_2\rho')''$				$10^9 i_2 \log r'$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−21	−0.0001					+ 1		
	−20	0.0000	+0.0002						
	−19	−0.0001	0.0007		+0.0001		+ 1		
	−18	−0.0006	0.0015		0.0002	− 1	3		
	−17	−0.0011	+0.0020	−0.0001	0.0002	− 2	+ 4		
	−16	−0.0016	−0.0022	−0.0001	0.0007	− 3	− 5		+ 1
	−15	−0.0046	−0.0025	−0.0004	+0.0037	−10	− 5	−1	+ 1
	−14	+0.0098	+0.0103	+0.0016	−0.0046	+21	+22	+3	−10
	−13	−0.0114	−0.0124	−0.0028	+0.0060	−24	−26	−6	+13
	−12	−0.0083	−0.0089	−0.0035	0.0070	−70	−17	−7	15
	−11	−0.0042	−0.0064	−0.0033	0.0044	− 9	−13	−7	9
	−10	−0.0033	−0.0047	−0.0019	0.0030	− 7	−10	−4	6
	− 9	−0.0040	−0.0037	−0.0012	0.0026	− 8	− 8	−3	5
	− 8	−0.0033	−0.0024	−0.0007	0.0021	− 7	− 5	−1	4
	− 7	−0.0019	−0.0014	−0.0002	0.0014	− 4	− 3		3
	− 6	−0.0012	−0.0004	−0.0002	0.0007	− 3	− 1		1
	− 5	−0.0005	+0.0003	+0.0001	+0.0002	− 1	+ 1		
	− 4	+0.0002	0.0002	0.0002	−0.0002				
	− 3	0.0002	+0.0002	0.0001	−0.0002				
	− 2	0.0001	−0.0002		−0.0002				
	− 1	0.0001	−0.0001		−0.0001				

Scheme II

i_3	i'	$ma'\delta \frac{\partial R}{\partial W'}$				$mn' \int a'\delta \frac{\partial R}{\partial W'} dt$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−17		+ 2			− 1			
	−16	− 2	8	+ 1		− 6	− 2		+ 1
	−15	− 6	16	4	+ 2	—	—	—	—
	−14	− 28	26	14	6	+49	+40	+ 6	−16
	−13	−105	+ 22	41	16	+24	62	8	−22
	−12	−120	− 40	113	50	0	48	18	−40
	−11	−124	− 92	104	134	−16	42	34	−27
	−10	−204	− 26	110	116	− 1	46	24	−22
	− 9	−292	+134	255	73	+31	52	12	−44
	− 8	−395	248	348	94	44	60	14	−50
	− 7	−320	231	336	79	35	42	10	−43
	− 6	−114	144	230	32	20	+13	4	−26
	− 5	+ 75	+ 21	+ 48	+ 8	+ 2	− 8	1	− 5
	− 4	49	− 30	− 62	− 4	− 4	− 4		+ 6
	− 3	+ 35	+ 9	− 6	− 14		− 3		
	− 2	− 8	+ 2	+ 2	− 1				
	− 1	− 2	− 6	− 2	+ 1				
	0	0	− 3						
	1	+ 2							

Scheme II (continued)

i_2	i'	$I = 2\cos\varphi'\,(\delta_1\rho'^2 - \delta_2\rho') - 2\delta_1\rho' m\left[n'\int\left(a'\frac{\partial R}{\partial W'}\right)_0 dt + (C_2)_0\right]$				$I + mn'\int a'\delta\frac{\partial R}{\partial W'}\,dt$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−21	+ 2				+ 2			
	−20	0	− 12			0	− 12		
	−19	2	− 28		− 2	2	− 28		− 2
	−18	16	− 53		− 4	16	− 53		− 4
	−17	27	− 73	+ 2	− 4	26	− 73	+ 2	− 4
	−16	27	+ 46	2	− 14	+ 21	+ 45	+ 2	− 13
	−15	+ 56	+179	+ 8	− 12	—	—	—	—
	−14	−200	− 72	−31	+ 86	−150	− 32	− 25	+ 70
	−13	+231	+294	+54	−125	+255	+256	+ 62	−147
	−12	175	126	71	−140	176	174	88	−180
	−11	103	103	67	− 82	87	144	102	−109
	−10	92	85	34	− 55	92	131	57	− 78
	− 9	110	80	20	− 51	140	132	33	− 94
	− 8	88	52	14	− 42	132	111	28	− 93
	− 7	55	+ 26	4	− 24	91	68	14	− 67
	− 6	35	− 2	+ 4	− 9	55	+ 11	+ 8	− 35
	− 5	+ 16	− 16	− 2	− 2	+ 18	− 24	− 1	− 7
	− 4	− 6	− 6	− 4	+ 2	− 10	− 10	− 4	+ 8
	− 3	− 4	− 3	− 2	4	− 4	− 6	− 2	4
	− 2	− 2	+ 4		4	− 2	4		4
	− 1	− 2	2		2	− 2	2		2

Scheme II (continued)

i_2	i'	$\frac{d\delta_2 w'}{dt}$				$\delta_2 w'$			
		sin	cos	$n't$ sin	$n't$ cos	sin	cos	$n't$ sin	$n't$ cos
5	−22	+ 1	− 2						
	−21	2	− 7			+0.0001			
	−20	4	− 25		− 1	0.0005	+0.0001		
	−19	9	− 50		− 3	0.0012	0.0002	+0.0001	
	−18	27	− 77	+ 1	− 6	0.0025	0.0008	0.0002	
	−17	34	− 78	2	− 7	+0.0037	0.0015	0.0003	+0.0001
	−16	+ 25	+ 28	4	− 14	−0.0022	0.0011	0.0012	0.0004
	−15	− 6	21	+ 3	− 1	—	—	—	—
	−14	− 70	51	− 1	+ 16	+0.0058	+0.0102	+0.0019	+0.0001
	−13	+281	315	+ 89	−191	0.0195	−0.0206	−0.0102	−0.0048
	−12	268	293	136	−255	0.0119	−0.0125	−0.0089	−0.0047
	−11	189	259	149	−197	0.0077	−0.0062	−0.0051	−0.0039
	−10	183	234	104	−158	0.0052	−0.0045	−0.0032	−0.0021
	− 9	221	219	66	−160	0.0039	−0.0043	−0.0027	−0.0011
	− 8	209	181	49	−150	0.0028	−0.0033	−0.0022	−0.0007
	− 7	156	113	28	−113	0.0014	−0.0022	−0.0014	−0.0004
	− 6	98	+ 34	15	− 64	+0.0004	−0.0012	−0.0007	−0.0002
	− 5	41	− 16	+ 2	− 22	−0.0002	−0.0004	−0.0002	
	− 4	+ 1	− 14	− 4	+ 1	−0.0001			
	− 3	− 5	− 9	− 3	4	−0.0001			
	− 2	− 5	+ 1	− 1	7				
	− 1	− 3	3		3				
	0	− 1	1		1				

Since

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$$\begin{array}{llll} A=-0''.0009, & A'=0''.0011, & A_1=0''.0130, & A_2=-0''.0173, \\ B=0.0020, & B'=0.0044, & B_1=0.0146, & B_2=0.0116, \\ C=0.0015, & C'=0.0003, & C_1=0.0026, & C_2=0.0080, \\ D=0.0000, & D'=-0.0004, & D_1=-0.0081, & D_2=0.0026, \end{array}$$

then, according to Formulas (38) and (40),

$$\begin{array}{lll} a_0=-0''.0028, & b_0=-0''.0034, & c_0=-0''.0198, \\ a_1=-0.0034, & b_1=0.0106, & c_1=0.0021, \\ a_2=0.0011, & b_2=-0.0004, & c_2=0.0013, \\ a_3=0.0001, & b_3=-0.0001, & c_3=-0.0001. \end{array}$$

Hence from Formula (42) we find $\Delta_{1.p.}\delta\rho'$ (Scheme III).

Scheme III

i	i'	$\Delta_{1.p.}\delta\rho'$							
		sin	cos	$n't$ sin	$n't$ cos	$(n't)^2$ sin	$(n't)^2$ cos	$(n't)^3$ sin	$(n't)^3$ cos
0	0	—	−0″.0048	—	−0″.0064	—	+0″.0012	—	+0″.0002
	1	−0″.0018	−0.0013	−0″.0013	+0.0173	+0″.0059	−0.0006	−0″.0002	−0.0004
	2	−0.0006	−0.0005	−0.0005	0.0063	0.0021	−0.0002		−0.0001
	3	−0.0002	−0.0002	−0.0002	0.0023	0.0008	−0.0001		
	4	−0.0001		−0.0001	0.0008	0.0002			
					0.0003	0.0001			
					0.0001				

Since

$$\begin{array}{lll} A_w=-0''.0006, & A'_w=0''.0056, & A''_w=-0''.0006, \\ B_w=0.0016, & B'_w=0.0179, & B''_w=0.0021, \\ C_w=0.0004, & C'_w=0.0008, & C''_w=0.0003, \\ D_w=0.0002, & D'_w=-0.0012, & D''_w=-0.0001, \end{array}$$

then from Equations (45) and Equations (47):

$$\begin{array}{ll} a'_0=-0''.0106, & a''_0=-0''.0020, \\ a'_1=0.0019, & a''_1=0.0004, \\ a'_2=0.0001, & a''_2=0.0001, \\ a'_3=0.0000, & a''_3=0.0000. \end{array}$$

Then from Formula (48) we obtain $\Delta_{1.p.}\delta w'$ (Scheme IV).

Scheme IV

i	i'	$\Delta_{1.p.}\delta w'$					
		sin	cos	$n't$ sin	$n't$ cos	$(n't)^2$ sin	$(n't)^2$ cos
0	0	—	—	—	−0″.0129	—	+0″.0012
	1	−0″.0055	+0″.0010	+0″.0010	+0.0001	+0″.0001	
	2	−0.0008	0.0001	0.0002			
	3	−0.0002					

14. Second-order Perturbations of the Logarithm of the Radius Vector and Longitude in the Orbit of Pluto with Respect to the Perturbing Masses

The constants of integration consist of two parts. The first part is determined by Formulas (37), and combines the terms examined in Section 12 and terms of the group of pronounced covariances which are independent of pronounced covariances (for example, Schemes I and II). The second part is determined by Formulas (43) and (49). For the initial instant t_0 we take the instant of osculation 1930 Sept. 20.0, and we consider the elements obtained in Section 22 of Part One to be the initial elements.

Then, after combining the constants, we finally have /83

$$a'\delta C_1 = 1.5242,$$
$$a'\delta K_1 = -1.5315,$$
$$a'\delta K_2 = -3.0647,$$
$$\delta C_2 = -0.5744,$$
$$\delta C_3 = -3.057.$$

Tables 19, 19', 19", and 20 give perturbations of the logarithm of the radius vector and longitude in the orbit of Pluto of the second order relative to the perturbing masses. The coefficients of the series of perturbations of the logarithm of the radius vector are given to the ninth decimal place of the logarithms and of the longitude in orbit, in thousandths of a second. The mean anomalies of the four major planets and Pluto are the arguments.

The last column of Table 20 contains the values of the integrating factors $\frac{n'}{i_1n_1 + i_2n_2 + i_3n_3 + i_4n_4 + i'n'}$.

Table 19

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i_1	i_2	i_3	i_4	i'	$10^6\delta_2 \log r'$		$\delta_2 w'$	
					$n't$ sin	$n't$ cos	$n't$ sin	$n't$ cos
								"
0	0	0	0	0	—	+ 9	"	+2.427
				1	+7185	−13099	+12.507	5.235
				2	4694	−10762	8.638	3.938
				3	2342	− 5800	4.272	1.839
				4	1041	− 2681	1.872	0.772
				5	434	− 1145	0.769	0.308
				6	170	− 465	0.304	0.118
				7	66	− 180	0.116	0.045
				8	27	− 70	0.044	0.017
				9	8	− 27	0.016	0.006
				10	4	− 10	0.005	0.003
				11	2	− 4	0.002	+0.001
				12		− 1	0.001	
0	0	1	0	−12	2	+ 2	0.001	−0.001
				−11	6	4	0.002	−0.003
				−10	13	8	0.005	−0.008
				− 9	31	20	0.013	−0.020
				− 8	75	48	0.032	−0.050
				− 7	164	104	0.073	−0.115
				− 6	321	202	0.150	−0.237
				− 5	466	+ 289	0.236	−0.374
				− 4	+ 16	− 21	+ 0.003	−0.006
				− 3	− 33	− 42	—	--
				− 2	− 32	+ 167	− 0.176	−0.175
				− 1	+ 548	304	− 0.288	+0.386
				0	369	213	− 0.152	0.271
				1	195	103	− 0.071	0.134
				2	88	45	− 0.030	0.059
				3	39	19	− 0.012	0.024
				4	16	7	− 0.005	0.010
				5	6	4	− 0.002	0.004
				6	2	+ 2	− 0.001	+0.001
0	0	2	0	−16		− 1		
				−15		− 2	− 0.001	
				−14		− 3	− 0.002	
				−13	2	− 6	− 0.004	−0.001
				−12	3	− 16	− 0.010	−0.002
				−11	6	− 36	− 0.024	−0.004
				−10	15	− 79	− 0.056	−0.010
				− 9	29	− 154	− 0.115	−0.021
				− 8	42	− 222	− 0.183	−0.033
				− 7	+ 2	+ 9	− 0.016	−0.003
				− 6	− 4	+ 37	—	—
				− 5	+ 5	− 93	+ 0.077	−0.006
				− 4	51	− 428	0.353	+0.036
				− 3	34	− 286	0.214	0.022
				− 2	18	− 144	0.100	0.011
				− 1	12	− 61	0.040	0.006
				0	5	− 24	0.016	0.003
				1	3	− 10	0.006	0.001
				2	1	− 4	0.002	
				3	0	− 3	+ 0.001	
				4	+ 1	− 1		
0	0	3	0	−16		− 2	− 0.001	
				−15		− 4	− 0.002	
				−14		− 8	− 0.005	

Table 19 (continued)

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i_1	i_2	i_3	i_4	i'	$10^8\delta_2 \log r'$		$\delta_2 w'$	
					$n't$ sin	$n't$ cos	$n't$ sin	$n't$ cos
							"	
0	0	3	0	−13		−17	−0.012	"
				−12	− 1	−35	−0.026	0.001
				−11	− 3	−53	−0.043	0.003
				−10	− 4	− 9	−0.015	0.007
				− 9	− 3	0	—	—
				− 8	+25	+33	−0.032	+0.005
				− 7	−56	−97	+0.074	−0.042
				− 6	−47	−73	0.050	−0.030
				− 5	−30	−39	0.025	−0.018
				− 4	−17	−19	0.012	−0.009
				− 3	− 7	− 8	0.004	−0.003
				− 2	− 1	− 2	+0.001	
				− 1		− 1		
0	0	4	0	−16	− 1			
				−15	− 1	− 1		+0.001
				−14	− 3	− 2	−0.001	0.002
				−13	− 8	− 5	−0.003	0.003
				−12	− 5	− 4	—	—
				−11	+35	+24	−0.010	+0.017
				−10	−33	−12	+0.009	−0.027
				− 9	−28	− 4	+0.002	−0.020
				− 8	−21	+ 3	−0.001	−0.013
				− 7	−15	3	−0.002	−0.008
				− 6	− 9	4	−0.002	−0.005
				− 5	− 5	3	−0.001	−0.002
				− 4	− 2	1		−0.001
0	0	5	0	−16		1	+0.001	
				−15	− 1	+ 1	—	—
				−14	+ 3	−10	+0.002	0.000
				−13	− 6	+13	−0.010	−0.005
				−12	− 7	15	−0.009	−0.005
				−11	− 7	9	−0.005	−0.004
				−10	− 4	6	−0.003	−0.002
				− 9	− 3	5	−0.003	−0.001
				− 8	− 1	4	−0.002	−0.001
				− 7		3	−0.001	
				− 6		+ 1	−0.001	
0	0	6	0	−18			−0.003	+0.002
				−17	− 3	− 4	+0.003	−0.002
				−16	+ 1	+ 1	−0.001	+0.001
				−15	2	3	−0.002	0.001
				−14	3	4	−0.002	0.001
				−13	3	5	−0.002	0.001
				−12	1	4	−0.002	
				−11	0	3	−0.001	
				−10	1	2	−0.001	
				− 9	1	1	−0.001	
				− 8	1	1		
0	0	7	0	−17	1			
				−16	1			0.001
				−15	2			0.001
				−14	3	− 1		0.001
				−13	2			0.001
				−12	1			+0.001
				−11	1			

Table 19 (continued)

i_1	i_2	i_3	i_4	i'	$10^9\delta_2 \log r'$		$\delta_2 w'$	
					$n't$ sin	$n't$ cos	$n't$ sin	$n't$ cos
0	1	0	0	−13		− 1	"	"
				−12	1	− 2	−0.001	−0.001
				−11	4	− 5	−0.004	−0.003
				−10	11	− 15	−0.013	−0.008
				− 9	+ 24	− 34	−0.048	−0.024
				− 8	− 51	+ 47	−0.069	−0.087
				− 7	− 13	+ 7	−0.008	−0.012
				− 6	− 7	− 10	+ 0.001	−0.001
				− 5	− 10	− 18	−0.002	+ 0.004
				− 4	− 22	− 31	−0.010	0.011
				− 3	− 48	− 60	−0.025	0.024
				− 2	− 96	−110	−0.050	0.045
				− 1	−131	−148	−0.070	0.062
				0	+ 1	+ 1	0.000	0.000
				1	− 1	− 4	+ 0.001	0.001
				2	0	− 3	+ 0.001	
				3	0	− 1		
				4	0	0		
				5	+ 1	− 1		
0	2	0	0	− 8		+ 1		
				− 7		1		
				− 6	+ 1	2	−0.001	
				− 5	− 1	2	−0.001	
				− 4	− 1	2	−0.001	
				− 3	− 1	+ 1	−0.002	0.001
				− 2	− 4	− 1	−0.002	0.002
				− 1	− 7	− 6	−0.002	0.003
1	−5	0	0	− 1		+ 1		
				0	− 1	0		
				1	+ 2	− 1		
				2	+ 1			
1	0	0	0	−10	− 1			
				− 9	0			
				− 8	− 3	+ 1		
				− 7	− 4	2	+ 0.001	0.002
				− 6	− 10	6	0.002	0.004
				− 5	− 22	15	0.006	0.011
				− 4	− 54	32	0.014	0.025
				− 3	−118	68	0.032	0.056
				− 2	−228	131	0.062	0.108
				− 1	−310	+178	0.084	+ 0.147
				0	+ 2	0	0.000	0.000
				1	− 9	− 1	0.002	−0.002
				2	− 5	− 1	0.001	−0.002
				3	− 3	− 1		−0.001
				4	− 2	0		−0.001
				5	− 1	+ 1		
2	−6	0	0	− 1	− 1			
				0	− 1			
				1	+ 1	3		
				2		1		
2	−5	0	0	− 1	1	1	0.001	
				0	+ 1	0	+0.002	
				1	− 5	3	−0.003	−0.007

Table 19 (continued)

i_1	i_2	i_3	i_4	i'	$10^6\delta_2 \log r'$		$\delta_2 w'$	
					$n't$ sin	$n't$ cos	$n't$ sin	$n't$ cos
2	−5	0	0	2	−2	1	−0.″001	−0.″002
				3	−1			−0.001
2	−4	0	0	−2	+1			
				−1	+3			−0.002
2	0	0	0	−5	−1			
				−4	−2	1		+0.001
				−3	−3	1	+0.001	0.002
				−2	−7	3	0.002	0.003
				−1	−9	3	+0.002	+0.005
3	−5	0	0	−2		1		
				−1		+2		
4	−10	0	0	0	+1	−1	−0.001	−0.001
				1	−4	+2	−0.003	−0.008
				2	−2	1	−0.001	−0.002
				3	−1			−0.001

Table 19'

i_1	i_2	i_3	i_4	i'	$10^6\delta_2 \log r'$		$\delta_2 w'$	
					$(n't)^2$ sin	$(n't)^2$ cos	$(n't)^2$ sin	$(n't)^2$ cos
0	0	0	0	0	—	+ 101	—	+0.″034
				1	−2	2891	−3.556	0.016
				2	−1	2042	−1.488	0.002
				3	−1	1050	−0.610	0.001
				4		474	−0.241	
				5		200	−0.093	
				6		79	−0.035	
				7		31	−0.013	
				8		12	−0.005	
				9		5	−0.002	
				10		3	−0.001	
				11		1		

Table 19''

i_1	i_2	i_3	i_4	i'	$10^6\delta_2 \log r'$		$\delta_2 w'$	
					$(n't)^3$ sin	$(n't)^3$ cos	$(n't)^3$ sin	$(n't)^3$ cos
0	0	0	0	0	—		—	−0.″299
				1	−12	−6	+0.″006	−0.012
				2	− 4	−2	0.002	−0.003
				3	− 2			−0.001
				4	− 1			

Table 20

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i_1	i_2	i_3	i_4	i'	$10^6\delta_2 \log r'$		$\delta_2 w'$		$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
					sin	cos	sin	cos	
0	0	0	0	0	—	+6735	—	−3.″057	—
				1	+2818	3516	−0.″855	−1.208	+1.0000000
				2	−5221	9426	−7.514	−4.908	0.5000000
				3	−3980	6113	−4.512	−3.094	0.3333333
				4	−2097	3051	−2.135	−1.503	0.2500000
				5	− 959	1361	−0.915	−0.654	0.2000000
				6	− 406	569	−0.372	−0.267	0.1666667
				7	− 165	228	−0.146	−0.106	0.1428571
				8	− 66	88	−0.055	−0.040	0.1250000
				9	− 25	34	−0.021	−0.015	0.1111111
				10	− 8	13	−0.008	−0.005	0.1000000
				11	− 3	5	−0.003	−0.002	0.0909091
				12	− 1	2	−0.001		0.0833333
				13	− 1	1			+0.0769231
0	0	0	1	− 3		1	+0.001		−0.6737056
				− 2	+ 1	+ 4	0.004	−0.002	−2.0647172
				− 1		− 11	0.012		+1.9392165
				0		− 4	0.004		0.6597733
				1		− 1	0.001		+0.3975081
			3	− 7		1	0.001		−0.4076668
				− 6	− 2	3	0.003	+0.002	−0.6882391
				− 5	− 3	+ 7	0.010	0.004	−2.2075862
				− 4	+ 4	− 10	0.014	0.005	+1.8280982
				− 3		− 3	0.003		0.6464055
				− 2			+0.001		+0.3926162
0	0	1	−3	1	− 1		−0.001	0.002	−1.7416361
				2	+ 3			0.004	+2.3483702
				3	2			0.002	0.7013472
				4	1				+0.4122305
0	0	1	0	−13	− 2	− 2		0.002	−0.0997292
				−12	− 9	− 1	−0.001	0.004	−0.1107769
				−11	− 19	− 3	−0.003	0.011	−0.1245771
				−10	− 43	− 12	−0.008	0.028	−0.1423051
				− 9	− 108	− 32	−0.020	0.071	−0.1659157
				− 8	− 255	− 79	−0.050	0.173	−0.1989196
				− 7	− 560	− 183	−0.117	0.397	−0.2483142
				− 6	−1070	− 389	−0.245	0.818	−0.3303430
				− 5	−1435	− 704	−0.380	1.295	−0.4933019
				− 4	+ 558	− 809	+0.436	0.311	−0.9735618
				− 3	+ 258	+ 86	—	—	−36.8239969
				− 2	− 656	+ 679	0.989	+0.168	+1.0279142
				− 1	−1529	−1189	1.209	−1.120	0.5068825
				0	−1006	− 915	0.726	−0.691	0.3363783
				1	− 492	− 485	0.347	−0.336	0.2517089
				2	− 223	− 222	0.150	−0.147	0.2010922
				3	− 95	− 93	0.061	−0.060	0.1674244
				4	− 39	− 39	0.024	−0.024	0.1434135
				5	− 15	− 14	0.009	−0.009	0.1254258
				6	− 7	− 5	0.003	−0.004	0.1114474
				7	− 3	− 1	+0.001	−0.001	+0.1002723
0	0	1	1	− 6	− 1	− 2	−0.001	+0.001	−0.6616014
				− 5	− 3	− 4	−0.004	0.005	−1.9550953
				− 4	+ 4	+ 8	−0.014	0.005	+2.0470159
				− 3	− 1	2	−0.002		0.6718101
0	0	1	3	− 7			+0.001		1.9235935
				− 6			−0.001		+0.6579552

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^6\delta_1 \log r'$ sin	$10^6\delta_1 \log r'$ cos	$\delta_2 w'$ sin	$\delta_2 w'$ cos	$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
							″	″	
0	0	2	—4	0			+0.001	0.004	—8.5469257
				1	+ 1	— 3	0.003	0.002	+1.1325043
				2		— 1	0.001		+0.5310678
0	0	2	—3	— 2			0.002		—1.6629833
				— 1			0.000	+0.001	+2.5083336
				0			0.001		+0.7149644
0	0	2	0	—15	2	+ 2	0.002		—0.1104446
				—14	1	6	0.004	—0.001	—0.1241571
				—13	2	20	0.012	—0.002	—0.1417573
				—12	12	46	0.030	—0.007	—0.1651715
				—11	23	108	0.074	—0.017	—0.1978508
				—10	49	241	0.170	—0.040	—0.2466509
				— 9	79	470	0.353	—0.081	—0.3274059
				— 8	+ 33	+ 668	0.564	—0.127	—0.4867809
				— 7	—435	— 66	0.096	—0.183	—0.9484855
				— 6	— 76	— 58	—	—	—18.4119984
				— 5	+311	— 15	+0.214	—0.504	+1.0574317
				— 4	—110	+1103	—0.862	—0.213	0.5139571
				— 3	—110	729	—0.525	—0.118	0.3394793
				— 2	— 70	336	—0.235	—0.060	0.2534412
				— 1	— 43	141	—0.097	—0.032	0.2021964
				0	— 22	58	—0.038	—0.014	0.1681891
				1	— 10	23	—0.015	—0.006	0.1439742
				2	— 6	7	—0.005	—0.003	0.1258544
				3	— 2	4	—0.002	—0.001	0.1117857
				4		2	—0.001		0.1005461
0	0	2	1	— 7				—0.001	2.1675058
				— 6	— 2			—0.001	0.6842942
				— 5	— 2			—0.001	0.4062795
0	0	2	3	—10			—0.001		+2.0296156
0	0	3	—6	0				+0.002	—5.6979505
				1				0.001	+1.2128588
0	0	3	0	—17		2	+0.001		—0.1237399
				—16		3	0.002		—0.1412136
				—15		7	0.005		—0.1644340
				—14	— 1	16	0.011		—0.1967935
				—13	— 2	36	0.026		—0.2450098
				—12	— 6	71	0.054		—0.3245206
				—11	— 25	+ 104	0.087	—0.002	—0.4804300
				—10	— 75	0	0.026	—0.049	—0.9246685
				— 9	+ 41	— 26	—	—	—12.2746674
				— 8	— 69	+ 10	+0.011	—0.181	+1.0886944
				— 7	+ 11	76	—0.054	—0.010	0.5212320
				— 6	23	40	—0.032	+0.004	0.3426381
				— 5	20	+ 4	—0.007	0.006	0.2551977
				— 4	+ 9	— 8	+0.002	+0.002	0.2033127
				— 3	— 3	— 7	0.003	—0.002	0.1689608
				— 2	— 5	— 3	0.001	—0.002	0.1445393
				— 1	— 3	— 1		—0.002	0.1262860
				0	— 1				+0.1121261
0	0	3	1	—11				—0.001	—1.7674202
				—10				+0.002	+2.3030671
0	0	4	0	—19					—0.1406742
				—18	— 1	+ 1		0.001	—0.1637030
				—17	— 3	1	0.001	0.002	—0.1957474
				—16	— 5	2	0.001	0.004	—0.2433904
				—15	— 9	3	0.002	0.008	—0.3216856

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^5 \log r'$ sin	$10^5 \log r'$ cos	$\delta_2 w'$ sin	$\delta_2 w'$ cos	$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
							"	"	
0	0	4	0	—14	— 8	+ 3	0.004	+ 0.011	—0.4742427
				—13	+24	— 2	+0.004	—0.005	—0.9020184
				—12	+32	—26	—	—	—9.2666005
				—11	—98	+ 7	—0.064	—0.041	+1.1218620
				—10	+ 68	+ 5	—0.010	+0.058	0.5287158
				—09	43	—11	+0.005	0.031	0.3458562
				— 8	21	—23	0.012	0.014	0.2569786
				— 7	9	—22	0.012	0.006	0.2044415
				— 6	+ 2	—16	0.008	+ 0.001	0.1697397
				— 5	— 1	— 9	0.004	—0.001	0.1451089
				— 4	— 3	— 3	0.002	—0.001	0.1267206
				— 3	— 1				+0.1124685
0	0	5	0	—19		+ 1	0.001		—0.2417923
				—18	— 1	3	0.002	+0.001	—0.3188998
				—17	— 2	+ 4	+0.004	0.002	—0.4682128
				—16	— 3	— 5	—0.002	0.001	—0.8804514
				—15	—10	— 5	—	—	—7.3648004
				—14	+21	+22	+0.006	+0.010	+1.1571141
				—13	—24	—26	0.020	—0.021	0.5364177
				—12	—17	—19	0.012	—0.012	0.3491353
				—11	— 9	—13	0.008	—0.006	0.2587845
				—10	— 7	—10	0.005	—0.004	0.2055829
				— 9	— 8	— 8	0.004	—0.004	0.1705257
				— 8	— 7	— 5	0.003	—0.003	0.1456830
				— 7	— 4	— 3	0.001	—0.002	0.1271582
				— 6	— 3	— 1		—0.001	0.1128131
				— 5	— 1	+ 1			+0.1013765
0	0	6	0	—22		1			—0.2402150
				—21		2	0.002		—0.3161618
				—20	+ 1	8	0.006		—0.4623343
				—19	2	19	0.018		—0.8598917
				—18	+ 7	+ 15	0.064	+0.066	—6.1373337
				—17	—50	—39	0.043	—0.058	+1.1946535
				—16	—16	— 6	0.006	—0.016	0.5443472
				—15	— 9	— 2	0.002	—0.007	0.3524772
				—14	— 9	— 3	0.002	—0.005	0.2606160
				—13	— 6	— 4	0.002	—0.003	0.2067370
				—12	— 4	— 4	0.002	—0.002	0.1713190
				—11	— 4	— 3	0.001	—0.002	0.1462616
				—10	— 2			—0.001	0.1275988
				— 9	— 2			—0.001	0.1131598
				— 8	— 1				+0.1016564
0	0	7	0	—21			+0.001	+0.004	—5.2605717
				—20			0.000	—0.001	+1.2347103
				—19			0.000	+0.001	0.5525147
				—18			0.000	+0.001	0.3558837
				—17		+ 1	0.000	0.000	0.2624736
				—16	— 1	2	—0.001	—0.001	0.2079042
				—15	— 3	4	—0.002	—0.001	0.1721198
				—14	— 2	4	—0.002	—0.001	0.1468449
				—13	— 2	2	—0.001	—0.001	0.1280425
				—12	— 1	1	—0.001	—0.001	0.1135086
				—11	— 1	2		—0.001	0.1019378
				—10	— 1	1			0.0925077
0	0	8	0	—17			+0.001		+0.1474328
0	1	—8	0	13			—0.001		—0.4340269
				14			0.000		—0.7668685
				15			—0.001		—3.2894255

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^6 z \log r'$ sin	$10^6 z \log r'$ cos	$z\omega'$ sin	$z\omega'$ cos	$n' \sum_{k=1}^{4} i_k n_k + i' n'$
0	1	—7	0	8		1			—0.2308856
				9	+ 1	2	+ 0.002	—0.001	—0.3001957
				10	3	7	0.005	—0.002	—0.4289709
				11	7	18	0.015	—0.008	—0.7512241
				12	+19	+ 7	0.029	—0.049	—3.0196824
				13	— 8	— 3	0.001	—0.006	+1.4951273
				14	— 3	— 2	0.001	—0.002	0.5992189
				15	— 1			—0.001	+0.3746947
0	1	—6	0	4		1			—0.1866258
				5	— 1	1	0.001	+0.001	—0.2294464
				6	— 3	2	+0.001	0.002	—0.2977682
				7	— 6	+ 1	0.000	0.005	—0.4249312
				8	—14	— 3	—0.004	0.014	—0.7362053
				9	—21	—23	—0.046	0.051	—2.7908260
				10	+16	+15	—0.016	0.016	+1.5584015
				11	7	6	—0.005	0.005	0.6691309
				12	2	3	—0.002	0.002	0.3785465
				13	+ 1	1			0.2745983
0	1	—5	0	2		1	+0.001		—0.2280256
				3		3	0.001		—0.2953797
				4	— 1	+ 3	+0.002	0.001	—0.4192041
				5	— 7	— 1	—0.003	0.007	—0.7217751
				6	—25	—31	—0.056	0.047	—2.5942151
				7	+20	+21	—0.025	0.024	+1.6272679
				8	9	9	—0.007	0.008	0.6193765
				9	4	3	—0.002	0.003	0.3824784
				10	1	+ 1	—0.001	0.001	+0.2766614
0	1	—4	0	— 1				0.001	—0.2266223
				0	— 3	— 1	—0.001	0.001	—0.2930292
				1	— 4	— 3	—0.003	0.006	—0.4144856
				2	—15	— 8	—0.008	0.017	—0.7078998
				3	—56	—26	—0.050	0.101	—2.4234830
				4	+41	+13	—0.013	0.048	+1.7025022
				5	16	7	—0.005	0.014	0.6299726
				6	5	2	—0.002	0.004	0.3864927
				7	+ 1	+ 1		0.001	+0.2787557
0	1	—3	0	— 1	— 2	— 1	—0.001	0.001	—0.6945479
				0	— 2	— 3	—0.004	0.001	—2.2738360
				1	+ 6	+ 6	—0.009	0.006	+1.7850304
				2	3	3	—0.003	+0.002	0.6409375
				3	1	1	—0.001	0.000	0.3905923
				4	0	0	0.000	0.000	0.2808819
				5	0	1	0.000	0.000	0.2192879
				6	1	2	+0.001	—0.001	0.1798492
				7	2	4	0.002	—0.001	0.1524340
				8	+ 2	+ 3	+0.002	—0.001	+0.1322714
0	1	—2	0	— 5	— 1			+0.001	—0.4053602
				— 4	— 2	— 1	—0.002	0.002	—0.6816904
				— 3	— 8	— 2	—0.016	0.014	—2.1415952
				— 2	+ 4	—20	+0.040	+0.002	+1.8759672
				— 1	2	— 4	0.009	0.000	0.6522909
				0	0	— 1	0.000	—0.001	0.3947797
				1	1	+ 2	0.001	—0.001	0.2830409
				2	5	4	0.002	—0.002	0.2206016
				3	10	8	0.004	—0.004	0.1807319
				4	16	15	0.006	—0.008	0.1530677
				5	15	12	0.006	—0.008	+0.1327482

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^7\delta_2 \log r'$		$\delta_2 w'$		$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
					sin	cos	sin	cos	
0	1	—1	0	— 9		1			—0.2861969
				— 8	1	2	0.001	—0.001	—0.4009466
				— 7	3	4	0.004	—0.002	—0.6693002
				— 6	+ 8	+ 8	0.013	—0.004	—2.0238906
				— 5	— 32	— 8	0.010	—0.049	+1.9766668
				— 4	+ 4	— 2	+ 0.001	+0.001	0.6640538
				— 3	7	— 2	0.000	0.003	0.3990579
				— 2	6	— 4	—0.001	+0.001	0.2852333
				— 1	6	— 10	—0.003	—0.001	0.2219311
				0	9	— 21	—0.010	—0.003	0.1816233
				1	13	— 40	—0.019	—0.007	0.1537066
				2	+ 12	— 51	—0.024	—0.006	+0.1332285
0	1	0	0	—13		+ 2	0.001		—0.2211776
				—12	— 3	6	0.004	+0.002	—0.2839897
				—11	— 6	16	0.012	0.005	—0.3966280
				—10	— 16	49	0.038	0.018	—0.6573524
				— 9	— 9	+139	0.118	+ 0.084	—1.9184503
				— 8	+ 43	—266	0.599	—0.042	+2.0887905
				— 7	42	— 12	+ 0.037	+0.032	0.6762487
				— 6	52	+ 25	—0.002	0.020	0.4034298
				— 5	60	35	0.000	+0.001	0.2874599
				— 4	98	53	+ 0.012	—0.026	0.2232768
				— 3	171	83	0.033	—0.068	0.1825235
				— 2	259	+103	+0.048	—0.115	0.1543509
				— 1	130	— 31	—0.012	—0.059	0.1337123
				0	+ 11	+ 83	+0.040	—0.004	0.1179420
				1	0	12	+0.004	—0.002	0.1054992
				2	— 3	6	—0.001	—0.002	0.0954313
				3	— 2	4	—0.001	—0.001	0.0871175
				4		2	—0.001		0.0801363
				5		+ 1			+0.0741909
0	1	1	0	—15		— 1			—0.2818163
				—14	+ 1	— 3	—0.002	—0.001	—0.3924015
				—13	1	— 6	—0.005	—0.002	—0.6458237
				—12	1	— 16	—0.011	—0.007	—1.8234524
				—11	+ 14	+ 38	—0.083	+0.028	+2.2143993
				—10	— 4	28	—0.025	0.000	0.6888999
				— 9	— 3	7	—0.006	0.000	0.4078986
				— 8	— 5	2	—0.001	0.002	0.2897216
				— 7	— 11	1	+0.001	0.004	0.2246389
				— 6	— 22	1	0.002	0.008	0.1834327
				— 5	— 42	6	0.003	0.018	0.1550006
				— 4	— 56	8	0.004	+0.026	0.1341996
				— 3		3	0.001		0.1183209
				— 2			0.001		+0.1058023
0	1	2	0	—15			0.001	—0.001	—1.7374188
				—14			0.000	0.000	+2.3560815
				—13	— 4		0.000	—0.004	0.7020335
				—12	— 3	1	0.000	—0.002	0.4124675
				—11	+ 1	3	0.000	—0.001	0.2920191
				—10	2	4	0.001	—0.001	0.2260176
				— 9	4	8	0.003	—0.002	0.1843510
				— 8	10	15	0.007	—0.004	0.1556558
				— 7	13	21	0.010	—0.006	0.1346904
				— 6	+ 2	+ 1	+0.001	—0.001	+0.1187023
0	1	3	0	—18	— 2	— 2	—0.002	+0.002	—1.6591379
				—17	+ 4	+ 4	—0.009	0.009	+2.5171332
				—16	2	3	—0.002	+0.002	0.7156775
				—15	1	1	—0.001	0.000	0.4171399

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^9\delta_2 \log r'$ sin	$10^9\delta_2 \log r'$ cos	$\delta_2 w'$ sin	$\delta_2 w'$ cos	$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
							"	"	
0	1	3	0	—14	0	0	0.000	0.000	0.2943534
				—13	0	1	0.000	0.000	0.2274135
				—12	1	2	0.000	—0.001	0.1852786
				—11	3	3	+0.001	—0.001	0.1563165
				—10	4	4	0.002	—0.002	0.1351849
0	2	—6	0	— 1	1	1	0.001	—0.001	—0.5320363
				0	+ 3	+ 2	0.002	—0.003	—1.1369180
				1	— 1	— 1	0.000	—0.012	+8.3036414
				2	— 3	— 1	+0.001	—0.003	0.8925152
				3	— 1			—0.001	0.4716027
0	2	—3	0	— 1				+0.001	0.1420664
0	2	—1	0	1			—0.001		0.0667350
				2			—0.001		0.0625600
0	2	0	0	—10		— 1			0.1437300
				— 9		— 2			0.1256677
				— 8		— 3	+0.001		0.1116384
				— 7	— 1	— 5	0.002		0.1004269
				— 6	— 2	— 4	0.002	—0.001	0.0912618
				— 5	— 3	— 5	0.002	—0.002	0.0836296
				— 4	— 2	— 5	0.002	—0.002	0.0771754
				— 3	— 2	— 3	+0.001	—0.003	0.0716461
				— 2	— 1	— 4	—0.002	—0.002	0.0668561
				— 1	—17	— 18	—0.009	+0.006	0.0626665
				0	+ 2	+ 6	+0.002	—0.001	0.0589710
				1		2			0.0556871
				2		+ 1			0.0527496
0	2	1	0	— 5				+0.001	0.0669777
				— 4				0.001	0.0627733
0	3	0	0	— 1				+0.001	0.0409228
0	5	0	0	— 3		— 1			0.0253847
				— 2	1	— 2	—0.001	—0.001	0.0247563
				— 1	+ 4	— 5	—0.003	—0.002	0.0241583
				0	— 1	+ 1			+0.0235884
1	—5	0	0	— 1	+ 4	1		—0.001	—0.0447662
				0	+11	+ 16	—0.008	+0.006	—0.0468641
				1	—83	—115	+0.055	—0.040	—0.0491684
				2	—30	— 42	0.020	—0.015	—0.0517109
				3	—11	— 15	0.007	—0.005	—0.0545307
				4	— 4	— 5	0.002	—0.002	—0.0576758
				5	— 1	— 1	+0.001	—0.001	—0.0612059
1	—4	0	0	1			—0.001	—0.001	—0.0843203
1	—3	0	0	0	— 2	— 2	+0.003		—0.2282692
				1	+12	+ 29	—0.009	+0.006	—0.2957887
				2	+ 4	20	+0.004	0.003	—0.4200283
				3	— 2	35	0.029	0.006	—0.7242219
				4	—23	+ 81	0.159	0.047	—2.6261045
				5	+14	— 66	0.078	0.015	+1.6149666
				6	6	— 27	0.022	0.005	0.6175859
				7	2	— 10	0.007	+0.002	0.3817948
				8		— 3	0.002		0.2763036
				9		— 1	+0.001		0.2164873
1	—2	0	0	— 4		— 1			10.2088962
				— 3	+ 3	+ 2	—0.004		0.9107851
				— 2	— 5	7	—0.004	—0.005	0.4766549
				— 1	— 3	1	—0.002	—0.005	0.3227937

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^5\delta_2 \log r'$ sin	$10^5\delta_2 \log r'$ cos	$\delta_2 w'$ sin	$\delta_2 w'$ cos	$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
							"	"	
1	−2	0	0	0	− 5	+ 3	−0.001	−0.002	0.2440242
				1	+ 19	− 15	+0.007	+0.009	0.1961571
				2	8	− 5	0.003	0.003	0.1639894
				3	3	− 2	0.001	0.001	0.1408857
1	−1	0	0	− 3	1	− 2		0.001	0.1044201
				− 2	4	− 5	0.001	0.002	0.0945474
				− 1	+ 15	− 16	+0.001	0.003	0.0863804
				0	− 2	+ 2	−0.001	0.005	0.0795121
				1				+ 0.001	0.0736556
1	0	−3	0	4	+ 1				0.0619697
				5	2	− 1		−0.001	0.0583536
				6	4	− 2	−0.001	−0.002	0.0551362
				7	7	− 3	−0.001	−0.004	0.0522550
				8	7	− 4	−0.002	−0.003	0.0496600
1	0	−2	0	0	2	− 1			0.0661824
				1	4	− 4	−0.001	−0.002	0.0620742
				2	8	− 7	−0.003	−0.004	0.0584462
				3	17	− 13	−0.007	−0.008	0.0552188
				4	32	− 24	−0.012	−0.014	0.0523293
				5	+ 26	− 23	−0.011	−0.013	0.0497271
1	0	−1	0	− 4	− 1	− 1		+ 0.001	0.0710696
				− 3	− 3	− 3	−0.001	0.001	0.0663016
				− 2	− 7	− 6	−0.002	0.003	0.0621790
				− 1	− 17	− 10	−0.005	0.007	0.0585591
				0	− 35	− 22	−0.010	0.016	0.0553018
				1	− 65	− 40	−0.018	0.031	0.0524038
				2	− 85	− 41	−0.019	+0.041	0.0497943
				3	0	0	0.000		0.0474325
				4	− 1	− 3	+0.001		0.0452845
				5	− 1	− 1			0.0433227
				6	− 2	+ 1			0.0415238
1	0	0	0	− 9	+ 2		−0.001		0.0829501
				− 8	4	− 2	−0.002	−0.001	0.0765964
				− 7	7	− 11	−0.004	−0.004	0.0711468
				− 6	20	− 23	−0.010	−0.009	0.0664212
				− 5	46	− 53	−0.025	−0.021	0.0622842
				− 4	100	−121	−0.057	−0.048	0.0586323
				− 3	197	−243	−0.114	−0.094	0.0553849
				− 2	294	−381	−0.180	−0.139	0.0524784
				− 1	41	−165	−0.078	−0.020	0.0498618
				0	151	+ 3	+0.002	−0.069	0.0474937
				1	19	2	−0.002	−0.008	0.0453403
				2	12	5	−0.003	+0.001	0.0433737
				3	6	5	−0.002	0.002	0.0415706
				4	4	2	−0.001	0.001	0.0399115
				5	+ 2	1			0.0383797
1	0	1	0	−10			+0.071		0.0712845
				− 9		1	0.002		0.0665412
				− 8		6	0.004		0.0623897
				− 7	− 1	18	0.008		0.0587258
				− 6	− 2	39	0.018	0.001	0.0554684
				− 5	− 4	74	0.035	0.002	0.0525533
				− 4	− 6	103	+0.048	+0.003	0.0499294
				− 3	+ 4	1		−0.002	0.0475550
				− 2		+ 2		−0.001	0.0453962
1	0	2	0	−12				−0.001	0.0666616
				−11	3	− .1		−0.001	0.0624956

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^{n}\delta_2 \log r'$ sin	$10^{n}\delta_2 \log r'$ cos	$\delta_2 w'$ sin	$\delta_2 w'$ cos	$\sum_{k=1}^{4} i_k n_k + i' n'$
							"	"	
1	0	2	0	−10	7	− 3	−0.001	−0.003	0.0588196
				− 9	16	− 5	−0.002	−0.007	0.0555521
				− 8	30	−11	−0 005	−0.014	0.0526284
				− 7	42	−15	−0.007	−0.020	0.0499972
				− 6	4	− 1	−0.001	−0.002	0.0476165
1	0	3	0	−14	1				0.0626018
				−13	2	− 1		−0.001	0.0589137
				−12	4	− 2	−0.001	−0.002	0.0556360
				−11	6	− 5	−0.002	−0.003	0.0527038
				−10	9	− 6	−0.003	−0.004	0.0500651
1	1	0	0	− 4			−0.001		0.0391632
				− 3	1	− 2	−0.001		0.0376872
				− 2	1	− 3	−0.001	−0.001	0.0363185
				− 1				−0.001	+0.0350457
2	−7	0	0	0				−0.001	−0.0580035
				1	9	− 4	+ 0.002	+ 0.005	−0.0615751
				2	+ 4		0.001	0.002	−0.0656154
				3			0.001	+0.001	−0.0702231
2	−6	0	0	− 2	− 1	+ 2	0.001		−0.0929231
				− 1	− 2	4	+ 0.001		−0.1024424
				0	− 23	+11	−0.005	−0.011	−0.1141346
				1	+175	−78	+0.037	+0.083	−0.1288397
				2	65	−26	0.014	0.030	−0.1478943
				3	23	− 7	0.006	0.010	−0.1735634
				4	9	0	0.003	0.003	−0.2100141
				5	5	+ 1	+ 0.001	0.000	−0.2658454
				6	+ 1	− 2	−0.001	0.001	−0.3621110
				7	− 6	−13	−0.011	0.006	−0.5676708
				8	− 29	−42	−0.048	0.033	−1.3130522
				9	+ 10	+ 20	−0.047	0.016	+ 4.1943551
				10	7	15	−0.013	0.006	0.8074833
				11	3	6	−0.004	0.002	0.4467445
				12	+ 1	1	−0.001	0.001	+ 0.3087930
2	−6	1	0	4				0.001	−0.5590526
				5	− 4			+0.005	−1.2678441
				6			−0.001	−0.001	+ 4.7335155
2	−6	2	0	3				+0.001	+5.4317338
2	−6	3	0	− 1			−0.002		−1.1861651
				0			+ 0.004	+0.002	+6.3715746
2	−6	4	0	− 6	+ 1	1		−0.001	−0.3484067
				− 5	3	2	0.002	−0.002	−0.5346996
				− 4	+ 8	+ 6	0.005	−0.007	−1.1491491
				− 3	− 6	− 5	0.034	−0.032	+7.7047020
				− 2	− 4	− 7	0.007	−0.005	0.8851196
				− 1	− 1	− 2	0.002	−0.002	0.4695297
				0	− 1	− 1	+0.001		0.3195102
2	−6	5	0	− 6	− 1		−0.006	+0.018	9.7432961
				− 5	+ 9			0.009	0.9069187
				− 4	4			0.003	0.4755938
				− 3	2			+0.001	0.3223067
				− 2	1				+0.2437458
2	−5	0	0	− 3	1			−0.001	−0.3046144
				− 2	3	− 1	−0.001	−0.002	−0.4380512
				− 1	+ 7	− 4	−0.004	−0.006	−0.7795214
				0	− 7	0	0.000	+0.006	−3.5355880
				1	+ 9	− 3	−0.014	−0.013	+1.3943858

Table 20 (continued)

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i_1	i_2	i_3	i_4	i'	$10^9\delta_2 \log r'$ sin	$10^9\delta_2 \log r'$ cos	$\delta_2 w'$ sin	$\delta_2 w'$ cos	$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
							″	″	
2	—5	0	0	2	2	— 4	—0.002	—0.001	0.5823564
				3	1	— 2			+0.3680311
2	—4	—3	0	0				+0.001	—1.3838459
				1				0.002	+3.6052124
				2				0.001	+0.7828547
2	—4	0	0	—11	— 1	1	0.001	0.001	—0.3566216
				—10	— 3	3	0.002	0.002	—0.5542953
				— 9	— 8	+ 7	0.005	0.006	—1.2436381
				— 8	+ 7	— 5	0.048	0.053	+5.1044481
				— 7	23	— 28	0.025	0.021	0.8361850
				— 6	9	— 13	0.009	0.007	0.4553926
				— 5	+ 3	— 8	+0.002	0.003	0.3129002
				— 4	— 1	— 11	—0.002	0.002	0.2383275
				— 3	— 7	— 26	—0.011	0.004	0.1924592
				— 2	— 21	— 71	—0.033	0.010	0.1613968
				— 1	— 59	—190	—0.090	+0.028	0.1389679
				0	+ 7	+ 25	+0.013	—0.004	0.1220121
				1	— 2	7	—0.002	—0.001	0.1087440
				2	— 1	3	—0.001		0.0980786
				3		+ 1			0.0893183
2	—3	0	0	— 4	— 2		—0.001		0.0788976
				— 3	— 3	— 2	—0.002	+0.001	0.0731280
				— 2	— 8	— 8	—0.005	0.003	0.0681447
				— 1	— 21	— 24	—0.012	+0.010	0.0637973
				0	+ 3	+ 4	+0.002	—0.001	0.0599713
				1	1	2			0.0565782
2	—2	0	0	— 3			0.001		0.0451398
				— 2	2	4	0.002	—0.001	0.0431902
				— 1	6	+ 10	+0.005	—0.003	0.0414020
				0			—0.001		0.0397561
2	—1	0	0	— 2	2	— 1		—0.001	0.0316134
				— 1	+ 6	— 3	—0.001	—0.003	0.0306446
2	0	—1	0	1				+0.001	0.0249140
				2	— 3	— 1		+0.001	0.0243084
2	0	0	0	— 5			—0.001		0.0269463
				— 4	+ 1	— 3	—0.002	—0.001	0.0262392
				— 3	3	— 7	—0.004	—0.001	0.0255683
				— 2	+ 4	— 12	—0.006	—0.001	0.0249309
				— 1	— 3	— 5	—0.002	+0.002	0.0243245
				0	+ 4	+ 1		—0.002	0.0237468
2	0	1	0	— 5			+0.001		0.0249478
				— 4		+ 3	0.001		+0.0243405
3	—8	0	0	4			0.001		—1.5068625
				5			+0.001	—0.001	+2.9729215
3	—7	0	0	— 4			—0.003		—5.4087794
				— 3			—0.001		+1.2268201
3	—5	0	0	— 6	1				0.0676929
				— 5	2			—0.001	0.0634011
				— 4	7			—0.003	0.0596210
				— 3	18	— 2	—0.001	—0.009	0.0562664
				— 2	53	— 5	—0.002	—0.026	0.0532691
				— 1	+145	— 12	—0.006	—0.069	0.0505750
				0	— 20	+ 2	+0.001	+0.010	0.0481403
				1	— 3	— 2	+0.001		0.0459293
				2	— 1				0.0439124

Table 20 (continued)

i_1	i_2	i_3	i_4	i'	$10^{6}\delta_2 \log r'$		$\delta_2 w'$		$\frac{n'}{\sum_{k=1}^{4} i_k n_k + i' n'}$
					sin	cos	sin	cos	
								″	
3	−4	0	0	−1				−0.001	0.0353965
3	−3	0	0	−2	+1	− 2	−0.001		0.0279876
				−1	2	− 5	−0.002	−0.001	0.0272256
3	−2	0	0	−3			+0.001		0.0231434
				−2	1	+ 4	0.002		0.0226199
				−1	+1	+11	+0.005	−0.001	0.0221196
				0	−1	− 2	−0.001		0.0216409
3	0	0	0	−1		+ 2			0.0160859
				0	+1	3			0.0158312
				1	1	2			0.0155845
				2	4	1			0.0153454
				3	+2				+0.0151134
4	−11	0	0	1			−0.001		−0.1243097
4	−10	0	0	−2		1	+0.001		−0.3897607
				−1		2	0.001		−0.6387014
				0	−1	+ 6	0.007	+0.002	−1.7677940
				1	+6	− 7	0.030	0.006	+2.3024326
				2	1	− 8	0.008	+0.001	0.6971929
				3		− 2	0.002		0.4107918
				4			+0.001		0.2911782
4	−9	0	0	−1				−0.001	0.1446536
4	−5	0	0	−3			−0.001		0.0257546
				−2	1	− 4	−0.002		0.0251079
				−1	3	−10	−0.004	−0.001	0.0244930
				0			+0.001		0.0239074

PART THREE

RECALCULATION OF FIRST-ORDER PLUTONIAN PERTURBATIONS WITH RESPECT TO THE PERTURBING MASSES

INTRODUCTION

In Part One of the "Theory of Motion of the Planet Pluto", the perturbations of Plutonian coordinates of the first order relative to the masses of Jupiter, Saturn, and Uranus were determined. The determination of these perturbations was based on the XIX system of elements of Pluto given by Bauer. In Part One, these elements were improved and a new system was obtained. Expansions of the perturbing function were brought to the seventh power of eccentricity of Pluto, to the sixth power of $\sigma = \sin \frac{J}{2}$, and to the seventh multiplicity of the difference of mean longitudes. In perturbations of Pluto by Uranus, the long-period terms were not explicitly separated. The perturbations of Pluto by Neptune were obtained by numerical integration in rectangular coordinates. /84

After determining the perturbations of Plutonian coordinates of the second order relative to the perturbing masses of the three major planets, it was decided to redetermine the first-order perturbations with a greater accuracy. For this purpose it was necessary:

1) To base the determination of perturbations on the new elements of Pluto;
2) To increase the accuracy in developing the perturbing function;
3) To express the long-period perturbations of Pluto by Uranus in the form of secular perturbations;
4) In the new determination of the perturbations of Pluto by Neptune, to perform integration in the elements;
5) To perform the calculations with a larger number of decimal places.

The coefficients of the new series of perturbations of Pluto by Jupiter changed very little. The coefficients of perturbations by Uranus were appreciably refined by isolating the long-period terms.

Part Three of the "Theory of Motion of the Planet Pluto" consists of five chapters and appendices.

Chapter I gives an account of the problems of expanding the tables of coefficients of Newcomb operators.

Chapters II, III, and IV are devoted to determining perturbations of Pluto by Jupiter, Saturn, and Uranus, and contain tables of series of perturbations.

Chapter V gives perturbations of the elements and coordinates of Pluto by Neptune.

The appendices contain: I: tables of coefficients of new operators; II: particular values of the operators for calculating corrections for the indirect term of the perturbing function.

CHAPTER I

TABLES OF COEFFICIENTS OF NEWCOMB OPERATORS

1. Expansion of Tables of Coefficients of Newcomb Operators

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In Part One of our work (Sharaf, 1955), we compiled tables of numerical values of the coefficients of the operators for development with respect to mean anomalies. In the present work these tables are expanded. For tables given in Appendix I, we have also calculated:

For operators of the zero class--coefficients for $i = \mp 9$ and ∓ 8;
For operators of the first class--coefficients for $i = \mp 7$ and ∓ 6;
For operators of the second class--coefficients for $i = \mp 5$ and ∓ 4.

The coefficients of the new operators were calculated:

$$P^{nn'}_{mm'} \text{ for values of } \begin{cases} i = \text{from } -9 \text{ to } +9, \\ 2n + n' = 8, \\ 2m + m' = \text{from } 0 \text{ to } +8; \end{cases}$$

$$P^{nn'}_{mm'} \text{ for values of } \begin{cases} i = \text{from } -7 \text{ to } +7, \\ 2n + n' = 6, \\ 2m + m' = \text{from } -6 \text{ to } +6. \end{cases}$$

$$P^{nn'}_{mm'} \text{ for values of } \begin{cases} i = \text{from } -5 \text{ to } +5, \\ 2n + n' = 4, \\ 2m + m' = \text{from } -4 \text{ to } +4; \end{cases}$$

$$P^{nn'}_{mm'} \text{ for values of } \begin{cases} i = \text{from } -3 \text{ to } +3, \\ 2n + n' = 2, \\ 2m + m' = \text{from } -2 \text{ to } +2. \end{cases}$$

The coefficients of simple operators were determined by Newcomb's formulas, and the coefficients of complex operators were obtained by multiplying the numerical coefficients of the simple operators. For operators $P^{0,8}_{0,0}$, $P^{0,8}_{0,2}$, $P^{0,8}_{0,4}$, $P^{0,8}_{0,6}$, $P^{0,8}_{0,8}$ we took into account the corrections indicated in Appendix IV of Part One.

The coefficients of the operators were verified by the first, second, and third methods of verification indicated in Part One.

The tables of the coefficients of the new operators are given in Appendix I.* Appendix II gives particular values of operators for calculating the corrections for the second term of the perturbing function.

* Our calculated, supplementary rows to the tables of operator coefficients given in Part One will be published in the next volume of Transactions of the Institute of Theoretical Astronomy.

CHAPTER II

PEREURBATIONS OF THE FIRST ORDER BY JUPITER

2. Elements

Underlying the new calculations of the perturbations of Pluto by Jupiter /86
were the elements of Pluto, which we obtained in Part One, and the mean elements of Jupiter according to Hill.

The elements were reduced to the ecliptic and equinox 1950.0

Elements of Pluto Osculation 1930 Sept. 20.0	Elements of Jupiter Epoch 1850 Jan. 0 Mean Greenwich noon
$M_0' = 275^\circ.20032,$	$M_0 = 148^\circ.03287,$
$\omega' = 112.71188,$	$\omega = 273.52519,$
$\Omega' = 109.63108,$	$\Omega = 99.77839,$
$i' = 17.11041,$	$i = 1.30807,$
$e' = 0.24707348,$	$e = 0.04825382,$
$n' = 14''.206698,$	$n = 299''.128377,$
$a' = 39.659128,$	$a' = 5.2028029,$
$\frac{1}{m'} = 360000.$	$\frac{1}{m} = 1047.355.$

The elements of the mutual position, obtained by Formulas (63) or (64) of Part One, are numerically equal to:

$$N = 190^\circ.63993, \quad P = 82^\circ.88527, \quad \alpha = 0.13118803,$$
$$N' = 180.82087, \quad P' = 291.89103, \quad \sigma = 0.13764483.$$

3. Development of the Perturbing Function

For developing the perturbing function in recalculating perturbations of Pluto by Jupiter, we used the method of Newcomb operators with respect to multiples of the mean anomalies. However, in the new development, we took into account terms up to the eighth power of eccentricity of Pluto (in comparison, we considered the eccentricity of Jupiter to be a quantity of the second order); up to the eighth power of $\sigma = \sin \frac{J}{2}$, and to the ninth multiplicity of the difference of longitudes. The coefficients of the expansions of the perturbing function and its derivatives were obtained on punched-card machinery (see Section 4 of Part One). The coefficients of the operators were taken from Appendix II of Part One, from Appendix I of Part Three, and from tables which will be published

later. In calculating the Laplace coefficients and their derivatives, we used the tables of Brown and Brouwer (1932) and Innes' formulas (1909) (see Section 3 of Part One).

The coefficients, appearing in the development of circular motion $a'A_i$, /87

$a'B_i$, ..., and their derivatives, were obtained by Formulas (3) of Part One, in which we restricted ourselves to the following terms:

$$a'D^kA_i = D^kc_1^i - \frac{1}{2}\sigma^2D^k(c_3^{i-1}+c_3^{i+1}) + \frac{3}{8}\sigma^4D^k(c_5^{i-2}+4c_5^i+c_5^{i+2}) -$$

$$-\frac{15}{16}\sigma^4D^k(c_7^{i-3}+9c_7^{i-1}+9c_7^{i+1}+c_7^{i+3}),$$

$$(i=0,\ 1,\ldots\ 9;\quad k=0,\ 1,\ldots\ 9),$$

$$a'D^kB_i = \frac{1}{2}D^kc_3^i - \frac{3}{4}\sigma^2D^k(c_5^{i-1}+c_5^{i+1}) + \frac{15}{16}\sigma^4D^k(c_7^{i-2}+3c_7^i+c_7^{i+2}),$$

$$(i=0,\ 1,\ldots\ 7;\ k=0,\ 1,\ldots\ 7),$$

$$a'D^kC_i = \frac{3}{8}D^kc_5^i - \frac{15}{16}\sigma^2D^k(c_7^{i-1}+c_7^{i+1}),$$

$$(i=0,\ 1,\ldots\ 5;\quad k=0,\ 1,\ldots\ 5),$$

$$a'D^kD_i = \frac{5}{16}c_7^i,$$

$$(i=0,\ 1,\ 2,\ 3;\ k=0,\ 1,\ 2,\ 3).$$

Tables 1 and 2 show the values of the coefficients of the expansion of circular motion and their derivatives, with respect to log α and σ. The coefficients $\sigma^6a'D_i$ and their derivatives, in the case of Pluto-Jupiter, proved to be equal to zero, for all practical purposes.

The values of the factors for obtaining corrections in the operators, for the indirect term of the perturbing function in the case of Pluto-Jupiter, are

$$\alpha^{-2}(1-\sigma^2) = 57.003884,$$
$$\alpha^{-2}\sigma^2 = 1.100858.$$

Table 3 gives the coefficients of the expansions of $a'R$, $a'D'R$, $a'\sigma D_\sigma R$, $a'\frac{\partial R}{\partial W'}$, $a'\frac{\partial R}{\partial W}$ and $\frac{1}{n'}a'D_t'R$.

To check the derivatives $a'D'R$, $a'\frac{\partial R}{\partial W'}$ and $a'D_t'R$, we obtain the derivative with respect to time $a'D_t'R$ from the relation

$$D_t'R = \frac{\partial R}{\partial W'}\frac{dw'}{dt} + \frac{\partial R}{\partial p'}\frac{dp'}{dt},$$

where

$$\frac{1}{n'}\frac{dw'}{dt}=1+\sum_{j=1} jH_j\cos jM',$$

$$\frac{1}{n'}\frac{d\varrho'}{dt}=\sum_{j=1}-jK_j\sin jM',$$

and H_j and K_j, known functions of the eccentricity of Pluto, have the following numerical values (e' = 0.24707348):

$$\begin{aligned}
H_0&=1.000000,\\
H_1&=0.490403, & -K_1&=0.241392,\\
2H_2&=0.149224, & -2K_2&=0.088165,\\
3H_3&=0.047188, & -3K_3&=0.030409,\\
4H_4&=0.015155, & -4K_4&=0.010306,\\
5H_5&=0.004907, & -5K_5&=0.003455,\\
6H_6&=0.001591, & -6K_6&=0.001155,\\
7H_7&=0.000519, & -7K_7&=0.000433,\\
8H_8&=0.000192, & -8K_8&=0.000146.\\
9H_9&=0.000064,
\end{aligned}$$

4. Tables of Perturbations of the Logarithm of the Radius Vector, Longitude in Orbit, and Longitude of Node and Inclination

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The calculations were performed by the formulas and schemes in Chapter III of Part One, with an accuracy to the sixth decimal place (up to multiplication by the mass of Jupiter).

The values of the coefficients of the expansions of p, q, $(\frac{a'}{r'})^2$, $p(\frac{a'}{r'})^2$, $q(\frac{a'}{r'})^2$, and of the integrating factors are given in Tables 17 and 20 of Part Two.

Table 1

i	$a'A_i$	$a'DA_i$	$a'D^2A_i$	$a'D^3A_i$	$a'D^4A_i$	$a'D^5A_i$	$a'D^6A_i$	$a'D^7A_i$	$a'D^8A_i$	$a'D^9A_i$
0	2.007700	0.015510	0.031463	0.064716	0.136685	0.302976	0.729013	1.9717	5.582	18.64
1	0.1293850	0.1307594	0.1349210	0.147597	0.186595	0.308476	0.69991	2.0624	6.047	20.64
2	0.01245002	0.02511440	0.05076956	0.1037221	0.2162960	0.468720	1.08711	2.7992	7.811	25.47
3	0.00133922	0.00403137	0.01216281	0.0368339	0.1122424	0.345525	1.08177	3.4750	11.264	38.62
4	0.000150767	0.000604452	0.002426358	0.0097568	0.0393375	0.159220	0.64850	2.6634	10.847	45.35
5	0.000017454	0.000087422	0.000438155	0.00219807	0.01104161	0.055567	0.28052	1.4207	7.097	36.09
6	0.0000020582	0.0000123652	0.0000743161	0.00044689	0.00268930	0.016199	0.09777	0.5908	3.511	21.21
7	0.0000002458	0.0000017226	0.0000120731	0.00008464	0.00059366	0.004166	0.02928	0.2059	1.418	9.95
8	0.0000002965	0.00000023737	0.00000190061	0.000015221	0.000121934	0.000977	0.00784	0.0629	0.492	3.94
9	0.0000000360	0.00000003243	0.00000029211	0.000002631	0.000023700	0.000213	0.00192	0.0173	0.152	1.37

i	$\sigma^2a'B_i$	$\sigma^2a'DB_i$	$\sigma^2a'D^2B_i$	$\sigma^2a'D^3B_i$	$\sigma^2a'D^4B_i$	$\sigma^2a'D^5B_i$	$\sigma^2a'D^6B_i$	$\sigma^2a'D^7B_i$
0	0.002578	0.002767	0.003353	0.005207	0.011250	0.031914	0.10652	0.4019
1	0.0004943	0.0010181	0.0021570	0.001813	0.011719	0.032415	0.10373	0.3870
2	0.00007931	0.00024222	0.00074856	0.0023576	0.0076508	0.025970	0.09331	0.3648
3	0.00001188	0.00004815	0.00019632	0.0008077	0.0033689	0.014329	0.06223	0.2815
4	0.000001719	0.000008679	0.000043986	0.0002241	0.0011503	0.005965	0.03112	0.1662
5	0.000000243	0.000001470	0.000008912	0.00005421	0.00033125	0.002036	0.01249	0.0780
6	0.0000000338	0.0000002386	0.0000016842	0.00001191	0.00008453	0.000602	0.00425	0.0306
7	0.0000000046	0.0000000375	0.0000003023	0.00000243	0.00001972	0.000159	0.00128	0.0104

i	$\sigma^4a'C_i$	$\sigma^4a'DC_i$	$\sigma^4a'D^2C_i$	$\sigma^4a'D^3C_i$	$\sigma^4a'D^4C_i$	$\sigma^4a'D^5C_i$
0	0.000005	0.000011	0.000026	0.000072	0.000227	0.000835
1	0.0000016	0.0000050	0.0000162	0.000054	0.000197	0.000767
2	0.00000036	0.00000147	0.00000613	0.0000261	0.0001147	0.000523
3	0.00000006	0.00000034	0.00000180	0.0000093	0.0000495	0.000267
4	0.000000022	0.000000073	0.000000450	0.0000027	0.0000173	0.000109
5	0.000000002	0.000000014	0.000000101	0.00000072	0.00000521	0.000037

Table 2

i	$\sigma a'D_\sigma A_i$	$\sigma a'DD_\sigma A_i$	$\sigma a'D^2D_\sigma A_i$	$\sigma a'D^3D_\sigma A_i$	$\sigma a'D^4D_\sigma A_i$	$\sigma a'D^5D_\sigma A_i$	$\sigma a'D^6D_\sigma A_i$	$\sigma a'D^7D_\sigma A_i$	$\sigma a'D^8D_\sigma A_i$
0	—0.001936	—0.003982	—0.008413	—0.018676	—0.045080	—0.12310	—0.3839	—1.399	—7.98
1	—0.0053018	—0.0059792	—0.008076	—0.014695	—0.036250	—0.10974	—0.3707	—1.393	—7.85
2	—0.00100954	—0.00212096	—0.0046582	—0.0110354	—0.029274	—0.08939	—0.3098	—1.222	—7.12
3	—0.00016151	—0.00049906	—0.0015709	—0.0050901	—0.017214	—0.06178	—0.2357	—0.985	—5.86
4	—0.000024168	—0.000098684	—0.0004069	—0.0017022	—0.007265	—0.03188	—0.1431	—0.677	—4.29
5	—0.000003490	—0.000017725	—0.00009055	—0.00046644	—0.002429	—0.01284	—0.0682	—0.374	—2.65
6	—0.0000004931	—0.0000029955	—0.00001826	—0.00011198	—0.000691	—0.00430	—0.0266	—0.169	—1.36
7	—0.0000000686	—0.0000004853	—0.00000344	—0.00002448	—0.000175	—0.00125	—0.0089	—0.064	—0.59
8	—0.00000000945	—0.00000007624	—0.000000616	—0.000004995	—0.000040	—0.00033	—0.0026	—0.021	—0.22
9	—0.00000000129	—0.00000001170	—0.000000106	—0.000000966	—0.000008	—0.00008	—0.0007	—0.006	—0.07

i	$\sigma a'D_\sigma\sigma^2B_i$	$\sigma a'DD_\sigma\sigma^2B_i$	$\sigma a'D^2D_\sigma\sigma^2B_i$	$\sigma a'D^3D_\sigma\sigma^2B_i$	$\sigma a'D^4D_\sigma\sigma^2B_i$	$\sigma a'D^5D_\sigma\sigma^2B_i$	$\sigma a'D^6D_\sigma\sigma^2B_i$
0	0.005143	0.005494	0.006579	0.009982	0.020956	0.05783	0.1841
1	0.0009667	0.0019851	0.0041828	0.009236	0.022093	0.05952	0.1809
2	0.00015196	0.00046308	0.00142544	0.0044611	0.014330	0.04788	0.1656
3	0.00002230	0.00009018	0.00036647	0.0015008	0.006216	0.02617	0.1102
4	0.000003158	0.000015912	0.000080428	0.0004082	0.002084	0.01072	0.0543
5	0.000000437	0.000002638	0.000015953	0.00009671	0.000588	0.00359	0.0213
6	0.0000000595	0.0000004189	0.0000029500	0.00002080	0.000147	0.00104	0.0071
7	0.0000000080	0.0000000644	0.0000005181	0.00000416	0.000033	0.00027	0.0020

i	$\sigma a'D_\sigma\sigma^4C_i$	$\sigma a'DD_\sigma\sigma^4C_i$	$\sigma a'D^2D_\sigma\sigma^4C_i$	$\sigma a'D^3D_\sigma\sigma^4C_i$	$\sigma a'D^4D_\sigma\sigma^4C_i$
0	0.000020	0.000044	0.000106	0.000285	0.000885
1	0.0000063	0.0000198	0.000063	0.000214	0.000765
2	0.00000139	0.00000573	0.0000238	0.0001014	0.000442
3	0.00000026	0.00000134	0.0000069	0.0000359	0.000189
4	0.000000045	0.000000279	0.0000017	0.0000105	0.000065
5	0.000000007	0.000000053	0.00000038	0.00000271	0.000019

Table 3

i	i'	$10^6 a' D' R$		$10^6 a' D' R$		$10^6 \sigma a' D_\sigma R$		$10^6 a' \frac{\partial R}{\partial W'}$		$10^6 a' \frac{\partial R}{\partial W'}$		$10^6 \frac{1}{n'} a' D_t' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
0	0		+ 1004268		− 1012897		− 1099		− 10		+ 11		0
	1	− 4371	256190	+ 8848	− 270551	+ 204	− 1436	− 7804	− 4477	+ 8078	4496	− 256191	− 4371
	2	− 1887	64498	3572	− 70157	721	− 1258	− 3241	− 1675	4034	2307	− 128996	− 3773
	3	− 657	18128	1095	− 20064	590	− 800	− 1068	− 438	1695	994	− 54385	− 1970
	4	− 201	5328	253	− 5933	352	− 428	− 298	− 57	665	398	− 21319	− 800
	5	− 57	1604	+ 36	− 1780	179	− 204	− 73	+ 18	255	158	− 8016	− 284
	6	− 17	500	− 8	− 557	86	− 88	− 22	22	102	62	− 3000	− 102
	7	+ 4	163	− 34	− 166	43	− 46	+ 10	28	+ 33	16	− 1139	+ 24
	8	8	38	− 25	− 25	15	− 18	17	+ 16	0	0	− 309	+ 64
1	− 9	152	263	+ 219	+ 435	8	− 5	267	− 165	− 263	152	+ 2367	− 1361
	− 8	518	952	867	1536	+ 13	− 27	938	− 514	− 929	484	7616	− 4140
	− 7	1716	3173	2669	4609	− 14	− 104	3131	− 1665	− 3119	1618	22214	− 12016
	− 6	6891	12583	9338	16413	− 173	− 446	12507	− 6782	− 12480	6702	75503	− 41347
	− 5	28475	51698	34956	62061	− 939	− 1925	51525	− 28215	− 51474	28088	258490	− 142369
	− 4	123155	222836	139899	250163	− 4524	− 8500	222488	− 122566	− 222401	122414	891346	− 492622
	− 3	580813	1048723	622149	1118130	− 22146	− 40488	1048239	− 579691	− 1048021	579621	3146169	− 1742442
	− 2	3225052	5817273	3319829	5982139	− 124045	− 225823	5817829	− 3223388	− 5816565	3223772	11634546	− 6450100
	− 1	+ 26705811	+ 48154671	+ 26888265	+ 48485016	− 1027028	− 1876976	+ 48171195	− 26701687	− 48155341	+ 26706291	48154671	− 26705811
	0	− 10333131	− 18066583	− 10332685	− 18064489	+ 187355	+ 1501709	− 18843609	+ 10131237	+ 18055555	− 10339141	0	0
	1	+ 478047	− 627284	+ 481555	− 631252	534494	− 2071917	+ 1401279	48842	655188	+ 493627	627284	+ 478047
	2	49258	− 91030	50803	− 93656	64847	− 249580	153928	14329	94348	51156	182058	98518
	3	8096	− 17791	8714	− 19038	11703	− 44894	26329	3336	18391	8448	53373	24291
	4	1612	− 3945	1848	− 4473	2483	− 9508	5395	796	4085	1698	15778	6452
	5	355	− 939	447	− 1151	572	− 2197	1217	194	974	378	4695	1777
	6	95	− 270	131	− 362	157	− 600	329	61	271	97	1611	572
	7	24	− 70	+ 33	− 102	+ 43	− 156	85	+ 19	+ 70	24	492	+ 173
2	−10	38	+ 27	− 109	− 81	− 9	− 1	53	− 70	− 52	76	262	− 381
	− 9	121	66	− 380	− 207	− 23	− 3	137	− 254	− 134	266	599	− 1090
	− 8	250	129	− 804	− 396	− 38	− 8	262	− 546	− 258	559	1034	− 2006
	− 7	771	694	− 1517	− 510	− 80	− 39	1004	− 1395	− 1000	1415	4864	− 5400
	− 6	1970	2011	− 3112	− 746	− 175	− 110	2726	− 3335	− 2719	3362	12065	− 11817
	− 5	5506	6750	− 5091	+ 848	− 373	− 337	8297	− 8283	− 8277	8338	33752	− 27533
	− 4	17285	25093	− 2784	13784	− 928	− 1130	28104	− 22572	− 28079	22574	100373	− 69134
	− 3	65136	107756	+ 33178	89463	− 2899	− 4467	112782	− 73467	− 112703	73443	323266	− 195409
	− 2	322225	570582	288770	552243	− 12853	− 22580	576380	− 331046	− 576086	331024	1141165	− 644450
	− 1	+ 2578880	+ 4651547	+ 2588550	+ 4663407	− 99273	− 181731	+ 4652534	− 2577246	− 4650533	+ 2577552	4651547	− 2578880
	0	− 996867	− 1742870	− 996337	− 1740712	+ 17790	+ 143733	− 1817652	+ 977064	+ 1742705	− 997388	0	0
	1	+ 46397	− 61241	+ 46654	− 60539	51533	− 200568	+ 135859	4958	63241	+ 47353	61241	+ 46397
	2	4797	− 8951	4904	− 8723	6252	− 24372	15006	1417	9246	4900	17922	9595
	3	792	− 1778	837	− 1708	1130	− 4448	2592	328	1841	809	5332	2379
	4	163	− 407	181	− 387	238	− 958	536	77	419	162	1607	640
	5	39	− 110	47	− 106	60	− 248	136	24	113	38	551	196
	6	+ 8	− 33	10	− 30	14	− 64	+ 31	8	33	+ 7	+ 196	45
3	−11	− 23	− 2	91	+ 9	4	− 1	− 7	68	+ 6	− 70	− 21	255
	−10	− 62	+ 1	255	− 4	11	− 3	+ 4	193	− 7	− 195	+ 6	616

Table 3 (continued)

i	i'	$10^6 a' R$		$10^6 a' D' R$		$10^6 a a' D_2 R$		$10^6 a' \frac{\partial R}{\partial W'}$		$10^6 a' \frac{\partial R}{\partial W}$		$10^6 \frac{1}{n'} a' D_t' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
3	− 9	− 103	5	430	− 26	16	− 2	21	324	− 24	− 327	37	922
	− 8	− 186	9	778	− 35	26	− 4	26	587	− 29	− 590	64	1484
	− 7	− 353	14	1473	− 51	52	− 2	35	1114	− 40	− 1117	96	2466
	− 6	− 547	142	2628	+ 49	87	− 4	167	1873	− 173	− 1870	861	3283
	− 5	− 645	518	4061	357	120	− 10	541	2623	− 553	− 2612	2585	+ 3224
	− 4	+ 4	1912	5278	1574	+ 122	− 50	1953	+ 2265	− 1975	− 2237	7641	− 17
	− 3	4234	8620	6762	7821	− 58	− 290	8760	− 2943	− 8826	+ 3045	25863	− 12700
	− 2	26302	46373	22929	45124	− 965	− 1718	46677	− 27332	− 46767	27424	92744	− 52606
	− 1	+210020	+378740	210170	+378885	−7957	−14649	+378838	−209905	−378823	+210058	378740	−210020
	0	− 81111	−141799	− 81166	−141781	+1489	+11760	−147916	+ 79497	+141749	− 81109	0	0
	1	+ 3800	− 5006	+ 3783	− 4994	4219	−16348	+ 11109	410	5123	+ 3872	5006	+ 3800
	2	399	− 741	392	− 734	516	− 1985	1227	118	750	406	1470	787
	3	70	− 158	67	− 155	100	− 386	227	30	159	70	473	209
	4	11	− 42	+ 10	− 41	+ 22	− 86	42	+ 11	43	12	170	+ 45
	5				+ 1		− 1			1		+ 1	
4	−12	5	− 2	− 24	10	− 1	+ 1	− 8	− 19	9	20	− 26	− 59
	−11	19	− 11	− 99	58	− 3	3	− 46	− 79	48	79	− 121	− 212
	−10	26	− 16	− 130	85	− 5	3	− 68	− 105	69	105	− 159	− 252
	− 9	35	− 24	− 184	127	− 7	4	− 100	− 148	101	149	− 217	− 311
	− 8	56	− 40	− 300	206	− 12	8	− 165	− 245	167	244	− 322	− 446
	− 7	83	− 59	− 448	303	− 20	9	− 242	− 365	244	362	− 413	− 582
	− 6	93	− 73	− 514	369	− 24	10	− 296	− 425	296	418	− 436	− 564
	− 5	60	− 66	− 351	330	− 24	+ 8	− 260	− 305	+ 263	294	− 322	− 295
	− 4	52	+ 124	+ 179	283	− 18	− 5	+ 42	− 35	− 43	19	+ 501	− 205
	− 3	282	709	837	583	− 25	− 35	766	− 43	− 762	20	2124	− 849
	− 2	1990	3531	1836	3497	− 96	− 138	3537	− 2052	− 3537	2034	7062	− 3978
	− 1	+ 16054	+ 28962	+ 16063	+ 28961	− 614	− 1111	+ 28959	− 16048	− 28967	+ 16055	28962	− 16054
	0	− 6201	− 10836	− 6202	− 10838	+ 114	+ 902	− 11307	+ 6078	+ 10836	− 6202	0	0
	1	+ 294	− 410	+ 292	− 412	334	− 1284	+ 866	42	409	+ 295	410	+ 294
	2	31	− 60	31	− 62	+ 42	− 158	98	11	60	31	104	63
	3	+ 4	− 15	4	− 15	− 8	− 30	15	4	+ 15	+ 4	45	12
5	−13	− 2	+ 3	11	− 15		− 1	12	9	− 13	− 9	33	24
	−12	− 3	6	22	− 36	+ 1	− 1	31	18	− 32	− 18	72	42
	−11	− 3	7	21	− 41	1	− 1	34	17	− 34	− 17	71	38
	−10	− 2	7	17	− 45	0	− 2	38	15	− 38	− 15	70	22
	− 9	− 3	10	21	− 67	2	− 3	55	19	− 55	− 19	92	22
	− 8	− 3	14	26	− 52	2	− 2	74	28	− 72	− 27	110	32
	− 7	− 3	14	+ 25	− 81	2	− 2	72	+ 24	− 71	− 23	91	+ 24
	− 6	+ 1	+ 6	− 9	− 38	2	− 2	+ 32	− 2	− 32	+ 5	+ 30	− 12
	− 5	7	− 4	− 47	+ 25	1	0	− 21	− 36	+ 23	40	− 28	− 44
	− 4	+ 5	− 8	− 37	+ 44	1	− 1	− 35	− 27	+ 36	+ 31	− 32	− 23
	− 3	− 7	+ 4	+ 30	− 11	+ 2	− 2	+ 10	+ 24	− 8	− 23	+ 7	+ 21
	− 2	+ 163	273	162	+ 270	− 8	− 11	271	− 156	− 271	+ 153	544	− 305
	− 1	+ 1219	+ 2199	+ 1218	+ 2198	− 47	− 84	+ 2199	− 1220	− 2200	+ 1220	2199	− 1219
	0	− 455	− 795	− 455	− 795	+ 7	+ 66	− 828	+ 445	+ 794	− 455		

Table 4

i	i'	$10^8 \log r'$ sin	$10^8 \log r'$ cos	$\delta\varpi'$ sin	$\delta\varpi'$ cos	$\delta\Omega'$ sin	$\delta\Omega'$ cos	$\delta i'$ sin	$\delta i'$ cos
0	0	—	−2218568	—	+ 394.″064	—	+ 3.″633	—	+152.″468
	0	—		—	+ 1760.068 $n't$	—	− 2.731 $n't$	—	0.013 $n't$
			2 $n't$	″					
	1	−2075243	− 830577	+831.344	−2001.452	− 3.″562	− 0.528	+ 0.156	− 0.423
	1	+ 893247 $n't$	− 15 $n't$	0.015 $n't$	+ 861.874 $n't$				
	2	− 757953	− 306166	250.756	− 609.111	− 1.550	− 0.913	0.266	− 0.337
	2	+ 326261 $n't$	− 6 $n't$	0.005 $n't$	+ 262.270 $n't$				
	3	− 261428	− 105927	79.011	− 192.632	− 0.656	− 0.497	0.145	− 0.164
	3	+ 112537 $n't$	− 2 $n't$	0.002 $n't$	+ 82.940 $n't$				
	4	− 88596	− 35951	25.327	− 61.872	− 0.263	− 0.222	0.065	− 0.070
	4	+ 38139 $n't$			+ 26.638 $n't$				
	5	− 29786	− 12098	8.191	− 20.035	− 0.100	− 0.090	0.027	− 0.027
	5	+ 12823 $n't$			+ 8.626 $n't$				
	6	− 9967	− 4043	2.662	− 6.517	− 0.036	− 0.036	0.011	− 0.010
	6	+ 4292 $n't$			+ 2.806 $n't$				
	7	− 3305	− 1349	0.870	− 2.122	− 0.016	− 0.015	0.005	− 0.004
	7	+ 1423 $n't$			+ 0.914 $n't$				
	8	− 1100	− 488	0.292	− 0.694	− 0.005	− 0.005	0.001	− 0.001
	8	+ 473 $n't$			+ 0.299 $n't$				
	9	− 381	− 162	0.097	− 0.230				
	9	+ 163 $n't$			+ 0.099 $n't$				
	10	− 130	− 59	0.033	− 0.077				
	10	+ 56 $n't$			+ 0.033 $n't$				
	11	− 47	− 18	0.011	− 0.026				
	11	+ 20 $n't$			+ 0.011 $n't$				
	12	− 16	− 2	0.004	− 0.009				
	12	+ 7 $n't$			+ 0.004 $n't$				
	13	− 5	− 1	0.001	− 0.003				
	13	+ 2 $n't$			+ 0.001 $n't$				
	14	− 2			− 0.001				
1	−11			− 0.001					
	−10		− 3	− 0.001	0.001				
	− 9	− 4	− 10	− 0.004	0.002	− 0.001	− 0.002		
	− 8	− 12	− 24	− 0.011	0.006	− 0.005	− 0.002	− 0.001	0.002
	− 7	− 40	− 72	− 0.033	0.018	− 0.018	+ 0.003	+ 0.001	0.006
	− 6	− 120	− 213	− 0.100	0.055	− 0.073	0.030	0.009	0.022
	− 5	− 353	− 635	− 0.299	0.165	− 0.296	0.150	0.045	0.088
	− 4	− 1049	− 1885	− 0.891	0.493	− 1.229	0.677	0.202	0.362
	− 3	− 3082	− 5550	− 2.630	1.457	− 5.529	3.128	0.927	1.618
	− 2	− 8904	− 16054	− 7.618	4.223	− 29.218	16.595	4.916	8.505
	− 1	− 24245	− 43740	− 20.764	+ 11.515	−230.747	+130.539	+38.751	66.706
	0	+ 3272	+ 5719	+ 2.835	− 1.524	+176.227	− 21.986	−21.695	+ 2.713
	1	− 36	1288	− 0.263	− 0.210	−232.571	− 63.523	+19.252	− 67.444
	2	− 53	399	− 0.132	− 0.058	− 26.800	− 7.372	2.213	− 7.808
	3	− 22	130	− 0.049	− 0.018	− 4.620	− 1.275	0.381	− 1.350
	4	− 7	44	− 0.018	− 0.006	− 0.940	− 0.260	0.077	− 0.275
	5	− 2	14	− 0.006	− 0.002	− 0.209	− 0.058	0.017	− 0.061
	6	− 1	7	− 0.002	− 0.001	− 0.055	− 0.015	0.004	− 0.016
	7	− 1	3	− 0.001		− 0.014	− 0.004	0.001	− 0.004
	8	− 1	0						
	9		+ 1						
2	−10						+ 0.001		
	− 9		− 1				0.002		
	− 8	+ 1	− 1			− 0.001	0.003	0.001	
	− 7	0	− 1	− 0.001	+ 0.001	− 0.003	0.006	0.002	+ 0.001
	− 6	− 1	− 3	− 0.003	0.002	− 0.007	0.012	0.003	0.002
	− 5	− 4	− 13	− 0.008	0.006	− 0.022	0.025	0.008	0.006
	− 4	− 18	− 39	− 0.023	0.015	− 0.073	0.062	0.017	0.020
	− 3	− 64	− 127	− 0.066	0.039	− 0.281	0.188	0.054	0.079
	− 2	− 206	− 381	− 0.186	0.105	− 1.388	0.816	0.239	0.397
	− 1	− 583	− 1054	− 0.501	+ 0.278	− 10.899	+ 6.155	+ 1.822	3.135
	0	+ 80	+ 138	+ 0.068	− 0.036	+ 8.433	− 1.045	− 1.051	+ 0.112

Table 4 (continued)

i	i'	$10^6 \log r'$ sin	$10^6 \log r'$ cos	$\delta\varpi'$ sin	$\delta\varpi'$ cos	$\delta\Omega'$ sin	$\delta\Omega'$ cos	$\delta i'$ sin	$\delta i'$ cos
				"		"	"	"	"
2	1	0	32	—0.006	—0.005	—11.518	—3.134	+0.950	—3.340
	2	— 2	11	—0.003	—0.001	— 1.368	—0.372	0.112	—0.399
	3		4	—0.001		— 0.244	—0.066	0.020	—0.071
	4		+ 2			— 0.051	—0.014	0.004	—0.015
	5					— 0.013	—0.003	+0.001	—0.004
	6					— 0.003	—0.001		—0.001
3	—10						—0.001		
	— 9						—0.001		
	— 8						—0.001		
	— 7						—0.002		
	— 6						—0.004	—0.001	
	— 5						—0.005	—0.001	
	— 4		—0.001	—0.001		— 0.002	—0.005	—0.001	+0.001
	— 3	— 2	—0.004	—0.002	+ 1	— 0.012	+0.003	+0.003	0.005
	— 2	— 7	—0.013	—0.007	4	— 0.069	0.040	0.014	0.023
	— 1	—21	—0.038	—0.018	+ 10	— 0.581	+0.326	+0.100	0.171
	0	+ 3	+0.006	+0.002	— 1	+ 0.460	—0.058	—0.057	+0.007
	1		0.002			— 0.631	—0.172	+0.052	—0.183
	2		+0.001			— 0.075	—0.021	0.006	—0.022
	3					— 0.014	—0.004	0.001	—0.004
	4					— 0.003	—0.001		—0.001
4	— 4						+0.001		
	— 3					— 0.001	0.001		
	— 2					— 0.004	0.003	0.001	+0.001
	— 1	— 1	—0.001	—0.001		— 0.033	+0.019	+0.006	+0.010
	0	0				+ 0.026	—0.003	—0.003	0.000
	1	+ 1				— 0.037	—0.010	+0.003	—0.011
	2					— 0.004	—0.001		—0.001
	3					— 0.001			
5	— 1					— 0.002	+0.001		+0.001
	0					+ 0.002			

For the initial instant in determining the constants of integration, we took the instant of osculation 1930 Sept. 20.0.

The constants of integration are equal to:

$$a'C_1 = -2.303719, \quad C_3 = 394''.064,$$
$$C_2 = -2.782712, \quad C_4 = 3''.633,$$
$$a'K_1 = -0.476595, \quad C_4 = 152''.468.$$
$$a'K_2 = -5.122317,$$

Table 4 gives the new series of perturbations of Pluto by Jupiter. The coefficients of the series of perturbations of the logarithm of the radius vector are given to the ninth decimal place of the common logarithms, and of the longitude in orbit and longitude of the node and inclination, in thousandths of a second of arc. As previously, the mean anomalies of Jupiter and Pluto are the arguments.

CHAPTER III

PERTURBATIONS OF PLUTO BY SATURN

5. Elements

In recalculating perturbations of Pluto by Saturn we used the elements of Pluto given in Chapter II of the present work and the mean elements of Saturn given by Hill. The elements of Saturn, in respect to the ecliptic and equinox 1950.0, are given below: /93

Epoch 1850 Jan. 0 Mean Greenwich Noon

$$\begin{aligned} M_0 &= 284^\circ.71678, \\ \omega &= 338.02621, \\ \Omega &= 113.48103, \\ i &= 2.48787, \\ e &= 0.05606075, \\ n &= 120''.455042, \\ a &= 9.5388433, \\ \frac{1}{m} &= 3501.6. \end{aligned}$$

The elements of the mutual location of the orbits have the following values:

$$\begin{aligned} N &= 175^\circ.51387, \quad P = 162^\circ.51234, \quad \sigma = 0.12731616, \\ N' &= 179.33879, \quad P = 293.37309, \quad \alpha = 0.24052074. \end{aligned}$$

6. Development of the Perturbing Function

The method and accuracy of developing the perturbing function in the case of Pluto-Saturn are the same as in the case of Pluto-Jupiter (see Section 4 of Part Three of the present work).

Tables 5 and 6 contain $\alpha' D^k A_i$, $a'\sigma^2 D^k B_i$, $a'\sigma^4 D^k C_i$; $\sigma a' D^k D_\sigma A_i$, $\sigma a' D^k D_\sigma \sigma^2 B_i$, $\sigma a' D^k D_\sigma \sigma^4 C_i$.

The values of the factors for obtaining the corrections in the operators for the indirect term will be:

$$\begin{aligned} \alpha^{-2}(1-\sigma^2) &= 17.005850, \\ \alpha^{-2}\sigma^2 &= 0.2801959. \end{aligned}$$

Table 7 gives the coefficients of the expansions of:

$$a'R,\ a'D'R,\ a'\sigma D_\sigma R,\ a'\frac{\partial R}{\partial W'},\ a'\frac{\partial R}{\partial W},\ \frac{1}{n'}a'D_t'R.$$

7. Tables of Perturbations of the Logarithm of the Radius Vector, Longitude in Orbit, and Longitude of Node and Inclination

The perturbations were calculated by the formulas and schemes of Chapter III of Part One. The accuracy of the calculations is the same as in the case of Pluto-Jupiter.

In determining the constants of integration, we took 1930 Sept. 20.0 as the initial instant.

The constants of integration are equal to: /101

$$a'C_1 = 1.945381, \qquad C_3 = -58''.778,$$
$$a'K_1 = 0.108825, \qquad C_5 = -4''.368,$$
$$a'K_2 = 2.431093, \qquad C_4 = -30''.816.$$
$$C_2 = 2.150582,$$

Table 8 contains the new series of perturbations of Pluto by Saturn. As in the case of Pluto-Jupiter, the coefficients of the series of perturbations of the logarithm of the radius vector are given to the ninth decimal place of the common logarithms and of the longitude in orbit and longitude of node and inclination in thousandths of a second of arc. As usual, the mean anomalies of Saturn and Pluto serve as the arguments.

Table 5

i	$a'A_i$	$a'DA_i$	$a'D^2A_i$	$a'D^3A_i$	$a'D^4A_i$	$a'D^5A_i$	$a'D^6A_i$	$a'D^7A_i$	$a'D^8A_i$	$a'D^9A_i$
0	2.026531	0.055011	0.115743	0.254908	0.608368	1.64048	5.15137	18.8515	63.009	232.71
1	0.2410550	0.2501485	0.2783570	0.367729	0.660308	1.66536	5.32907	19.9426	67.941	254.10
2	0.04280132	0.08725865	0.18129623	0.3906504	0.8992647	2.30537	6.84876	23.9424	82.923	322.48
3	0.00843437	0.02561764	0.07845117	0.2435239	0.7726774	2.53731	8.76634	32.4812	119.089	478.53
4	0.001744391	0.007038597	0.028524570	0.11635120	0.47917320	2.00124	8.52527	37.3336	159.790	719.40
5	0.000371005	0.001866996	0.009419323	0.04769205	0.24267103	1.24319	6.42634	33.6285	171.545	902.34
6	0.0000803600	0.0001845188	0.0029260126	0.01770786	0.10746815	0.65460	4.00545	24.6585	148.039	908.94
7	0.0000176312	0.0001238820	0.0008713412	0.00613693	0.04329614	0.30610	2.16934	15.4253	107.041	757.08
8	0.00000390543	0.00000133640	0.00025157290	0.002021463	0.016262731	0.13098	1.05639	8.5357	67.350	540.86
9	0.00000087150	0.00000786118	0.00007093756	0.000640530	0.005787824	0.05234	0.47354	4.2897	37.955	341.66

i	$\sigma^2a'B_i$	$\sigma^2a'DB_i$	$\sigma^2a'D^2B_i$	$\sigma^2a'D^3B_i$	$\sigma^2a'D^4B_i$	$\sigma^2a'D^5B_i$	$\sigma^2a'D^6B_i$	$\sigma^2a'D^7B_i$
0	0.001421	0.005556	0.009348	0.022816	0.074908	0.298882	1.38454	7.2899
1	0.0015337	0.0033912	0.0081765	0.022574	0.074221	0.292764	1.35890	7.2041
2	0.00014957	0.0014522	0.0047634	0.0640659	0.271648	0.271648	1.29717	6.9652
3	0.00012343	0.0005607	0.00220157	0.0097120	0.0146484	0.217144	1.13452	6.4223
4	0.000032725	0.000169272	0.00088732	0.0017381	0.02593420	0.146621	0.86541	5.3688
5	0.000008490	0.000052313	0.00032568	0.00205072	0.01311591	0.085592	0.57409	3.9710
6	0.0000021700	0.0000155361	0.00011196	0.000813502	0.00597363	0.044152	0.33706	2.6060
7	0.0000005487	0.0000044731	0.000036662	0.000302203	0.00250959	0.021036	0.17878	1.5386

i	$\sigma^4a'C_i$	$\sigma^4a'DC_i$	$\sigma^4a'D^2C_i$	$\sigma^4a'D^3C_i$	$\sigma^4a'D^4C_i$	$\sigma^4a'D^5C_i$
0	0.000016	0.000043	0.000138	0.000531	0.002432	0.012860
1	0.0000086	0.0000301	0.0001141	0.000481	0.002283	0.012260
2	0.00000343	0.00001520	0.00007031	0.0003450	0.0018206	0.010419
3	0.00000120	0.00000641	0.00003563	0.0002015	0.0012278	0.007785
4	0.000000384	0.000002442	0.0000158 22	0.0001050	0.0007128	0.005083
5	0.000000117	0.000000913	0.000006393	0.00004835	0.00037324	0.002954

i	$\sigma^6a'D_i$	$\sigma^6a'DD_i$	$\sigma^6a'D^2D_i$	$\sigma^6a'D^3D_i$
0	0.000000	0.000000	0.000001	0.000008
1	0.000000	0.0000002	0.0000012	0.000007
2	0.00000002	0.00000013	0.00000080	0.0000050
3	0.00000001	0.00000006	0.00000044	0.0000031

Table 6

i	$\sigma a' D_0 A_i$	$\sigma a' D D_3 A_i$	$\sigma a' D^2 D_3 A_i$	$\sigma a' D^3 D_3 A_i$	$\sigma a' D^4 D_3 A_i$	$\sigma a' D^5 D_0 A_i$	$\sigma a' D^6 D_0 A_i$	$\sigma a' D^7 D_0 A_i$	$\sigma a' D^8 D_3 A_i$
0	—0.006007	—0.013222	— 0.031619	—0.086128	— 0.277933	— 1.07159	—4.8654	—25.124	—121.93
1	— 0.0096732	— 0.0137449	— 0.027323	—0.075393	— 0.260152	— 1.04621	—4.9226	—24.926	—121.72
2	—0.00328722	— 0.00769431	— 0.0202095	— 0.0618705	— 0.223562	— 0.93898	—4.4988	—23.982	—120.62
3	— 0.00095518	— 0.00315814	— 0.0110226	—0.0414118	— 0.170158	— 0.77636	—3.9342	—21.905	— 115.70
4	— 0.000260832	—0.001117726	— 0.00493708	— 0.02270886	— 0.109969	—0.56626	—3.1382	— 18.663	— 105.20
5	— 0.000068877	—0.000362926	—0.00194908	—0.01072924	—0.060932	—0.35941	—2.2270	— 14.474	— 88.57
6	—0.0000178097	—0.0001114135	— 0.00070606	—0.00454766	— 0.029888	—0.20131	—1.4030	—10.089	— 67.77
7	—0.0000045392	—0.0000328838	— 0.00024042	—0.00177776	— 0.013328	— 0.10160	—0.7942	— 6.335	— 46.86
8	—0.00000114486	—0.00000942044	—0.000078156	— 0.000653278	—0.005513	— 0.04709	—0.4103	— 3.620	— 29.41
9	—0.00000028643	—0.00000264137	— 0.000024504	—0.000228566	— 0.002148	—0.02036	—0.1963	— 1.906	— 16.91

i	$\sigma a' D_3 \sigma^2 B_i$	$\sigma a' D D_3 \sigma^2 B_i$	$\sigma a' D^2 D_3 \sigma^2 E_i$	$\sigma a' D^3 D_3 \sigma^2 B_i$	$\sigma a' D^4 D_3 \sigma^2 B_i$	$\sigma a' D^5 D_3 \sigma^2 B_i$	$\sigma a' D^6 D_3 \sigma^2 B_i$
0	0.008774	0.010876	0.017800	0.041878	0.132095	0.50331	0.2223
1	0.0029900	0.0065519	0.015535	0.041725	0.131938	0.49572	0.2191
2	0.00086022	0.00272594	0.0089373	0.0308498	0.114486	0.46596	0.2126
3	0.00023171	0.00096234	0.0040698	0.0176609	0.079416	0.37425	0.1886
4	0.000060242	0.000309681	0.00160922	0.00848539	0.045626	0.25165	0.1446
5	0.000015320	0.000093898	0.00057954	0.00360924	0.022741	0.14543	0.0956
6	0.0000038371	0.0000273142	0.00019535	0.00140537	0.010185	0.07449	0.0557
7	0.0000009503	0.0000077027	0.000062688	0.000511976	0.004201	0.03468	0.0292

i	$\sigma a' D_0 \sigma^4 C_i$	$\sigma a' D D_0 \sigma^4 C_i$	$\sigma a' D^2 D_3 \sigma^4 C_i$	$\sigma a' D^3 D_3 \sigma^4 C_i$	$\sigma a' D^4 D_3 \sigma^4 C_i$
0	0.000064	0.000169	0.000537	0.002042	0.009224
1	0.0000339	0.0001177	0.000443	0.001847	0.008662
2	0.00001340	0.00005903	0.0002722	0.0013209	0.006895
3	0.00000462	0.00002479	0.0001366	0.0007783	0.004631
4	0.000001471	0.000009325	0.00006014	0.00039660	0.002690
5	0.000000444	0.000003682	0.00002409	0.00018124	0.001390

Table 7

i	i'	$10^6 a' R$		$10^6 a' D' R$		$10^6 a' D_a R$		$10^6 a' \frac{\partial R}{\partial W'}$		$10^6 a' \frac{\partial R}{\partial W}$		$10^8 \frac{1}{n'} a' D_t' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
0	0	—	+ 1015046	—	— 1046241	—	— 3451	—	+ 186	—	— 103	—	0
	1	+ 15222	269574	— 31571	— 307227	— 130	— 2997	— 13195	15449	+ 12919	— 14798	— 269574	+ 15219
	2	8913	69453	— 20321	— 83217	+ 1903	— 3094	— 4557	10307	6334	— 7892	— 138907	17826
	3	4348	19652	— 10673	— 23732	1702	— 2239	— 790	5537	2541	— 3563	— 58959	13042
	4	1956	5703	— 5077	— 6563	1030	— 1324	+ 245	2679	911	— 1501	— 22807	7829
	5	840	1646	— 2277	— 1645	506	— 694	350	1196	292	— 633	— 8225	4199
	6	356	473	— 974	— 328	214	— 358	241	510	101	— 268	— 2837	2133
	7	134	135	— 367	— 9	108	— 185	147	187	+ 34	— 68	— 948	932
	8	28	24	— 92	+ 31	+ 47	— 58	59	+ 49			— 196	+ 221
1	— 9	16	26	— 287	83	— 12	+ 80	106	— 7	— 26	+ 16	+ 228	— 142
	— 8	138	133	— 1322	259	— 76	188	367	— 242	— 172	293	1070	— 1108
	— 7	+ 158	479	— 3644	1007	— 131	275	967	— 530	— 672	603	3357	— 1104
	— 6	— 848	1906	— 10175	4719	— 136	451	2758	+ 43	— 2252	+ 10	11433	+ 5088
	— 5	— 7235	8512	— 30668	18915	— 32	+ 574	9822	5494	— 9024	— 5464	42557	36185
	— 4	— 42391	40674	— 99651	73085	+ 917	— 204	42438	38950	— 41460	— 38975	162718	169604
	— 3	— 234719	209014	— 368577	299981	6940	— 5737	210776	229095	— 210023	— 229183	627045	704152
	— 2	— 1421083	1235274	— 1712520	1463854	45775	— 39862	1236222	1415200	— 1236216	— 1415064	2470549	2842162
	— 1	—12270337	+10614554	—12805660	+11084734	+406702	—348017	+ 10614689	+12275328	—10614876	—12271090	+ 10614554	12270337
	0	+ 4654834	— 4131895	+ 4651273	— 4119136	—364066	+ 114756	— 4105385	— 4865385	+ 4127178	+ 4658344	0	0
	1	171842	+ 115832	180923	+ 125875	+ 533706	51903	+ 53535	+ 368537	— 108044	162143	— 115832	171842
	2	23388	10006	29757	13582	61177	6240	2847	38597	— 9198	22331	— 20010	46777
	3	4082	1376	7317	2489	9938	1160	88	5933	— 1240	3959	— 4128	12239
	4	737	236	2188	524	1750	293	—91	992	— 197	742	— 947	2950
	5	115	52	726	90	288	105	—64	146	— 35	132	— 263	580
	6	+ 13	16	265	2	44	45	—34	18	— 10	+ 23	— 94	75
	7		+ 2	77	1	6	+ 11	— 8	5	— 2		— 20	3
2	—10	— 143	— 44	390	178	65	— 31	— 124	235	+ 88	— 285	— 438	1429
	— 9	— 456	— 252	1369	1005	171	— 31	— 720	876	658	— 999	— 2263	4113
	— 8	— 947	— 540	2929	2182	244	0	— 1583	1894	1512	— 2043	— 4314	7570
	— 7	— 2267	— 835	6584	3783	398	— 15	— 2542	4475	2410	— 4657	— 5844	15864
	— 6	— 5306	— 1373	15028	6825	710	+ 16	— 4242	10519	4048	— 10738	— 8239	31829
	— 5	— 11982	— 1655	31185	12218	1232	137	— 6268	23137	6041	— 23333	— 8273	59910
	— 4	— 26556	+ 993	55668	21928	2095	+ 282	— 5400	47863	+ 5189	— 47848	+ 3973	106231
	— 3	— 62704	20305	+ 68092	46643	3781	— 65	+ 13314	96309	— 13559	— 96180	60919	188112
	— 2	— 202919	140264	— 65830	163758	8698	— 3834	135615	238523	— 136197	— 238253	280526	405831
	— 1	— 1384413	+ 1203722	— 1419235	+ 1209951	+ 46780	— 38329	+ 1203809	+ 1380397	— 1203226	— 1379186	+ 1203722	1384413
	0	+ 522387	— 461085	+ 518660	— 467323	— 38706	+ 15619	— 459856	— 544728	+ 459349	+ 523583	0	0
	1	19672	+ 13641	18290	+ 11295	+ 61386	6958	+ 6257	+ 41750	— 13578	19293	— 13641	19672
	2	2923	1394	2349	+ 538	7625	1078	410	4640	— 1515	2971	— 2790	5844
	3	600	247	356	— 40	1441	229	62	807	— 302	637	— 742	1792
	4	141	52	+ 42	— 32	328	45	24	167	— 72	161	— 206	557
	5	40	14	0	— 7	90	6	12	44	— 18	48	— 62	200
	6	13	3	— 1	— 4	26	+ 1	5	+ 11	— 3	17	— 5	+ 77

Table 7 (continued)

i	i'	$10^6 a'R$		$10^6 a'D'R$		$10^6 a' D_0 R$		$10^6 a' \frac{\partial R}{\partial W'}$		$10^6 a' \frac{\partial R}{\partial W}$		$10^6 \frac{1}{n'} a' \dot{D}_t' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
3	−11	62	125	− 213	− 502	− 46	− 26	367	− 148	− 375	187	+ 1375	− 688
	−10	103	415	− 300	− 1795	− 85	− 89	1348	− 192	− 1378	266	4149	− 1047
	− 9	148	720	− 379	− 3189	− 77	− 137	2412	− 221	− 2443	288	6487	− 1330
	− 8	385	1251	− 1357	− 5419	− 120	− 196	4068	− 939	− 4086	1014	10012	− 3079
	− 7	881	2295	− 3447	− 9811	− 194	− 346	7361	− 2492	− 7361	2584	16076	− 6168
	− 6	1667	3928	− 6995	− 16401	− 239	− 583	12373	− 5043	− 12335	5133	25571	− 9995
	− 5	2556	5842	− 11604	− 25617	− 299	− 836	17974	− 8295	− 17871	8368	29209	− 12775
	− 4	+ 2519	7028	− 14496	− 26106	− 287	−1022	20601	− 9714	− 20386	9808	28115	− 10075
	− 3	− 1552	6495	− 10483	− 13609	+ 21	−1006	14965	− 2820	− 14541	+ 3016	19481	+ 4599
	− 2	− 18760	11578	− 6957	+ 22674	997	− 991	8192	+ 22697	− 7441	− 22421	23154	− 37733
	− 1	−131129	+113474	−133819	+116107	+4659	−4675	+113600	+ 130824	−112638	−130481	+113474	+131129
	0	+ 49374	− 43788	+ 49008	− 43323	−3670	+1064	− 43624	− 51545	+ 43982	+ 49497	0	0
	1	1846	+ 1217	1772	+ 1377	+5781	530	+ 545	+ 3924	− 1105	1814	− 1217	1846
	2	264	105	260	153	688	65	+ 18	420	− 83	263	− 207	529
	3	56	15	61	31	126	14	− 4	74	− 9	54	− 46	166
	4	+ 15	+ 2	20	4	+ 25	5	− 4	+ 12	0	+ 12	− 5	+ 57
	5	− 1	− 1	+ 3	2	− 3	1	− 1	− 1	+ 1	− 2	+ 2	− 3
4	−12	+ 38	− 96	− 203	468	+ 5	36	− 366	− 162	386	+ 150	− 1158	− 450
	−11	157	− 236	− 899	1187	− 17	80	− 938	− 729	977	715	− 2589	− 1734
	−10	247	− 313	− 1440	1602	− 43	80	− 1269	− 1174	1294	1169	− 3137	− 2469
	− 9	324	− 478	− 1785	2485	− 51	102	− 1972	− 1439	1990	1431	− 4310	− 2916
	− 8	472	− 805	− 2491	4220	− 87	169	− 3375	− 2005	3385	1981	− 6443	− 3778
	− 7	638	− 1211	− 3293	6572	− 147	231	− 5080	− 2683	5087	2627	− 8489	− 4476
	− 6	742	− 1456	− 3692	7712	− 200	269	− 6181	− 3015	6166	2931	− 8743	− 4448
	− 5	667	− 1156	− 3123	6287	− 239	235	− 5095	− 2588	5061	2452	− 5771	− 3324
	− 4	+ 277	− 44	− 1447	+ 1074	− 233	+ 94	− 897	− 1208	+ 838	+ 188	− 179	− 1108
	− 3	− 400	+ 1132	+ 433	− 3908	− 245	− 25	+ 3376	+ 559	− 3472	− 852	+ 3400	+ 1197
	− 2	− 1709	1067	− 241	+ 1840	− 157	+ 56	703	2066	− 771	− 2280	2131	3416
	− 1	− 11691	+ 10105	− 11757	+ 10220	+ 388	− 386	+ 10125	+ 11691	− 10073	− 11691	+ 10106	11691
	0	+ 4381	− 3886	+ 4583	− 3888	− 332	+ 109	− 3866	− 4561	+ 3886	+ 4367	0	0
	1	171	+ 107	165	+ 101	+ 536	53	+ 53	+ 357	− 109	174	− 107	171
	2	32	3	+ 30	− 1	69	+ 9	− 3	34	− 6	+ 34	− 8	67
	3	+ 1		0	− 1	1			1	− 1			
	4			− 1	− 1	1							
5	−13	− 61	23	+ 365	− 127	16	− 16	+ 104	301	− 115	− 305	+ 298	794
	−12	− 161	23	1014	− 94	42	− 20	+ 70	838	− 87	− 847	264	1922
	−11	− 185	4	1198	+ 43	44	− 5	− 47	994	+ 41	− 999	33	2032
	−10	− 208	5	1325	− 36	49	− 4	+ 30	1104	− 31	− 1105	58	2116
	− 9	− 292	27	1827	− 222	78	− 8	191	1536	− 198	− 1525	242	2628
	− 8	− 372	60	2328	− 452	105	+ 2	358	1963	− 382	− 1939	482	2986
	− 7	− 358	+ 60	2240	− 469	110	18	388	1910	− 414	− 1870	+ 424	2501
	− 6	− 188	− 10	+ 1224	− 104	93	35	+ 51	+ 1073	− 95	− 1023	− 59	+ 1131
	− 5	+ 62	− 92	− 305	+ 432	60	57	− 394	− 181	+ 345	+ 243	− 468	− 315
	− 4	+ 180	− 67	− 1035	+ 391	36	58	− 372	− 778	+ 328	+ 841	− 263	− 715
	− 3	− 14	+ 104	+ 42	− 519	28	+ 15	+ 380	+ 74	− 408	− 49	+ 311	+ 39
	− 2	− 150	98	− 33	+ 166	7	− 26	+ 65	177	− 73	− 203	195	302

Table 7 (continued)

i	i'	$10^7 a' R$		$10^8 a' D' R$		$10^8 a' D_\sigma R$		$10^8 a' \frac{\partial R}{\partial W'}$		$10^7 a' \frac{\partial R}{\partial W}$		$10^8 \frac{1}{n'^2} a' \dot{D}'_t R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
5	— 1	—1032	+891	—1036	+912	+33	—35	+894	+1030	—889	—1031	891	+1032
	0	+383	—332	+410	—286	—98	+84	—330	— 384	+332	+ 383		
	1			0	+ 2		— 2		— 1	1	1		
	2			— 1	1		— 1						
6	—14	34	+ 17	— 231	—122	—11	— 2	+104	— 188	—102	+ 193	239	— 450
	—13	64	54	— 459	—412	—19	—12	354	— 382	— 352	388	707	— 824
	—12	48	58	— 343	—458	—12	—16	398	— 292	—398	292	705	— 576
	—11	31	53	— 243	—383	—10	—14	327	— 209	—328	208	591	— 338
	—10	42	62	— 342	—430	—16	—23	372	— 302	—364	298	631	— 417
	— 9	66	64	— 491	—441	—18	—34	388	— 438	—371	434	572	— 583
	— 8	+49	46	— 390	—321	—13	—33	302	— 344	—278	341	366	— 395
	— 7	— 3	+ 14	— 36	—110	— 5	—32	+119	— 30	— 94	+ 30	+104	+ 16
	— 6	— 41	— 12	+262	+ 96	+ 5	—20	— 56	+ 213	+ 79	— 213	— 85	239
	— 5	— 22	— 28	+167	178	+ 5	—15	—131	+ 150	155	— 150	—141	+ 114
	— 4	+28	— 12	— 165	+ 56	— 4	— 6	— 49	— 132	+ 56	+ 138	— 41	— 111
	— 3	0	+ 12	+ 6	— 62	+ 5	+ 4	+ 39	+ 10	— 46	— 4	+ 32	+ 1
	— 2	— 1	0	7	+ 4	— 2	+ 1	— 3	3	+ 2	— 5	— 1	4
	— 1		— 1	2	— 1		— 1	— 1		3	— 3	— 1	
7	—15	— 5	— 19	43	+149	+ 1	+ 5	—130	37	130	— 37	—278	80
	—14	— 1	— 39	+ 7	330	0	11	—289	+ 5	289	— 5	—557	+ 23
	—13	+ 7	— 26	— 70	227	— 3	8	—194	— 64	194	+ 64	—337	— 93
	—12	13	— 10	— 96	88	— 4	3	— 73	— 81	73	82	—111	— 161
	—11	14	— 12	— 82	104	— 5	9	— 97	— 71	93	69	—129	— 149
	—10	+ 4	— 19	— 11	153	— 7	12	—144	— 17	136	+ 10	—184	— 33
	— 9	— 3	— 10	+ 18	+ 87	— 6	9	— 86	+ 9	+ 79	— 17	— 86	+ 19
	— 8	0	+ 7	— 1	— 35	— 6	+ 2	+ 33	— 13	— 37	+ 6	+ 55	— 3
	— 7	0	9	— 13	— 88	— 8	— 6	54	— 10	— 54	+ 6	58	— 5
	— 6	— 5	+ 4	+ 25	— 37	— 1	0	+ 26	+ 16	— 26	— 20	+ 18	+ 23
	— 5	— 2	— 9	+ 34	+ 36	+ 3	— 4	— 38	+ 25	+ 43	— 24	— 37	+ 17
	— 4	+ 3	— 1	— 24	+ 8	3	— 4	— 4	— 16	+ 5	+ 17	— 5	— 12
	— 3			+ 2	— 7	2		+ 4	+ 2	— 7	— 1	+ 2	
8	—16	— 5	+ 8	45	— 69	2	— 2	61	40	— 61	— 40	122	+ 80
	—15	— 12	12	120	— 109	4	— 4	83	107	— 83	— 107	157	190
	—14	— 8	+ 3	77	— 31	3	— 1	+ 11	70	— 11	— 70	+ 22	111
	—13	— 2	— 7	23	+ 49	2	+ 1	— 42	19	+ 42	— 19	— 81	37
	—12	— 4	— 6	33	+ 46	4	1	— 34	31	+ 35	— 29	— 72	51
	—11	— 3	+ 2	+ 29	— 14	4		+ 11	+ 29	— 11	— 26	+ 11	+ 35
	—10	+ 2	+ 3	— 4	— 25	+ 2		+ 20	— 3	— 19	+ 6	+ 26	— 15
	— 9	2	0	— 28	+ 2	0	3	— 4	— 20	+ 3	21	0	— 32
	— 8	+ 5	— 3	— 18	+ 17	0	4	— 40	— 30	+ 58	+ 30	— 38	— 31
	— 7	— 2	+ 2	— 3	— 8	— 1	2	+ 13	+ 10	— 15	— 13	+ 16	+ 13
	— 6	— 1	2	+ 13	— 16	0	0	+ 14	11	— 13	— 13	+ 11	11
	— 5			+ 6	+ 6	— 3	1	— 5	+ 5	+ 5	— 6	— 3	+ 5
	— 4			— 2	3			+ 1	— 2	1	+ 2		— 1
	— 3			— 1									

Table 8

i	i'	$10^8 \log r'$ sin	$10^8 \log r'$ cos	$\delta w'$ sin	$\delta w'$ cos	$\delta\Omega'$ sin	$\delta\Omega'$ cos	$\delta i'$ sin	$\delta i'$ cos
					"		"		"
0	0	—	+336522	—	— 58.778	—	— 4.368	—	—30.816
	0	—	— $12n't$	—	—220.385$n't$	—	— 2.778$n't$	—	+ 0.009$n't$
					"		"		"
	1	+294545	+120750	—118.125	+283.193	— 2.413	+ 0.106	+ 0.028	— 0.036
	1	—113596$n't$	85$n't$	— 0.085$n't$	—109.606$n't$				
	2	+107683	44548	— 35.997	+ 86.250	— 1.255	— 0.757	0.246	— 0.234
	2	— 41491$n't$	32$n't$	— 0.026$n't$	— 33.353$n't$				
	3	+ 37168	15408	— 11.389	+ 27.288	— 0.606	— 0.451	0.142	— 0.144
	3	— 14312$n't$	11$n't$	— 0.008$n't$	— 10.547$n't$				
	4	+ 12604	5226	— 3.659	+ 8.767	— 0.269	— 0.204	0.065	— 0.070
	4	— 4850$n't$	4$n't$	— 0.003$n't$	— 3.388$n't$				
	5	+ 4240	1757	— 1.185	+ 2.840	— 0.113	— 0.080	0.025	— 0.031
	5	— 1631$n't$	1$n't$	— 0.001$n't$	— 1.097$n't$				
	6	+ 1420	586	— 0.386	+ 0.924	— 0.048	— 0.028	0.009	— 0.013
	6	— 545$n't$			— 0.357$n't$				
	7	+ 471	195	— 0.126	+ 0.301	— 0.021	— 0.012	0.004	— 0.006
	7	— 181$n't$			— 0.116$n't$				
	8	+ 157	71	— 0.042	+ 0.098	— 0.006	— 0.005	+ 0.001	— 0.002
	8	— 60$n't$			— 0.038$n't$				
	9	+ 54	24	— 0.014	+ 0.033				
	9	— 21$n't$			— 0.013$n't$				
	10	+ 18	9	— 0.005	+ 0.011				
	10	— 7$n't$			— 0.004$n't$				
	11	+ 7	3	— 0.002	+ 0.004				
	11	— 2$n't$			— 0.001$n't$				
	12	+ 2	1	— 0.001	+ 0.001				
	12	— 1$n't$							
	13	+ 1							
1	—14	1	2	+ 0.001	— 0.001				
	—13	4	6	0.004	— 0.003				
	—12	12	17	0.012	— 0.008				
	—11	38	50	0.037	— 0.025				
	—10	109	143	0.116	— 0.073				
	— 9	+ 301	+ 338	0.359	— 0.144	— 0.123	— 0.020	— 0.005	+ 0.035
	— 8	— 1131	— 757	1.190	— 1.892	+ 0.315	+ 0.131	+ 0.031	— 0.092
	— 7	+ 155	— 67	+ 0.064	— 0.002	0.149	0.073	0.015	— 0.043
	— 6	355	— 27	— 0.069	+ 0.098	0.146	0.046	+ 0.006	— 0.040
	— 5	557	— 207	— 0.153	— 0.004	+ 0.133	+ 0.009	— 0.010	— 0.033
	— 4	1139	— 748	— 0.384	— 0.304	— 0.034	— 0.166	— 0.069	+ 0.021
	— 3	2861	— 2289	— 1.086	— 1.152	— 0.830	— 1.030	— 0.333	0.264
	— 2	7806	— 6642	— 3.127	— 3.555	— 4.885	— 5.745	— 1.716	1.469
	— 1	+ 20883	— 18046	— 8.533	— 9.849	—36.956	—44.200	—12.798	+10.932
	0	— 2692	+ 2380	+ 1.146	+ 1.361	+10.947	+34.729	— 1.363	— 4.279
	1	— 627	+ 83	0.100	— 0.131	3.890	—45.370	+13.186	+ 1.274
	2	— 197	0	0.023	— 0.065	0.425	— 4.704	1.372	0.132
	3	— 65	— 3	0.006	— 0.024	0.073	— 0.698	0.204	0.022
	4	— 22	— 1	0.002	— 0.009	0.018	— 0.113	0.033	0.005
	5	— 7		0.001	— 0.003	0.006	— 0.017	0.005	0.002
	6	— 2			— 0.001	+ 0.002	— 0.002	+ 0.001	0.001
2	—11	— 1			— 0.001				
	—10	— 4	— 2	0.001	— 0.002	— 0.004	— 0.008	— 0.002	0.001
	— 9	— 9	— 5	0.002	— 0.004	— 0.003	— 0.017	— 0.005	0.001
	— 8	— 15	— 9	0.004	— 0.007	0.000	— 0.022	— 0.006	0.000
	— 7	— 25	— 12	0.004	— 0.011	— 0.001	— 0.032	— 0.008	0.001
	— 6	— 40	— 18	0.005	— 0.018	+ 0.002	— 0.052	— 0.012	0.001
	— 5	— 58	— 27	+ 0.004	— 0.030	+ 0.010	— 0.083	— 0.019	0.000
	— 4	— 68	— 43	— 0.002	— 0.047	+ 0.019	— 0.130	— 0.030	0.000
	— 3	— 36	— 84	— 0.023	— 0.076	— 0.002	— 0.218	— 0.052	0.011
	— 2	+ 124	— 197	— 0.083	— 0.134	— 0.202	— 0.471	— 0.121	0.079
	— 1	+ 567	— 504	— 0.239	— 0.283	— 1.907	— 2.383	— 0.663	+ 0.602

Table 8 (continued)

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i	i'	$10^6 \log r'$ sin	$10^6 \log r'$ cos	$\delta w'$ sin	$\delta w'$ cos	$\delta u'$ sin	$\delta u'$ cos	$\delta i'$ sin	$\delta i'$ cos
				"	"	"	"	"	"
2	0	—80	+70	+0.032	+0.036	+0.743	+1.845	—0.047	—0.198
	1	—18	4	0.003	—0.004	0.280	—2.755	+0.801	+0.091
	2	— 5	1	+0.001	—0.002	0.042	—0.324	0.095	0.013
	3	— 2			—0.001	0.009	—0.058	0.017	0.003
	4	— 1				+0.002	—0.013	0.004	
	5						—0.003	0.001	
	6						—0.001		
3	—11		1			—0.001	+0.003	0.001	
	—10		2	—0.001		—0.005	0.004	0.001	0.001
	— 9	+ 1	3	—0.001		—0.007	0.004	0.001	0.002
	— 8	1	5	—0.002	+0.001	—0.009	0.005	0.001	0.002
	— 7	3	7	—0.003	0.001	—0.015	0.008	0.002	0.003
	— 6	5	10	—0.004	0.002	—0.024	0.010	0.003	0.005
	— 5	6	12	—0.005	0.002	—0.033	0.012	0.004	0.006
	— 4	7	11	—0.005	+0.002	—0.039	+0.011	0.004	0.005
	— 3	6	+ 4	—0.004	0.000	—0.036	—0.001	+0.003	+0.001
	— 2	8	— 8	—0.004	—0.005	—0.034	—0.034	—0.005	—0.005
	— 1	+25	—21	—0.010	—0.012	—0.152	—0.154	—0.039	+0.026
	0	— 3	+ 3	+0.001	+0.002	+0.034	+0 117	—0.003	—0.016
	1	— 1				0.014	—0.176	+0.051	+0.005
	2	— 1				0.002	—0.020	0.006	
	3						—0.004	+0.001	
	4						—0.001		
4	—12					0.001			
	—11					0.003	+0.001		—0.001
	—10	+ 1	— 1			0.003	0.001		—0.001
	— 9	1	— 1			0.003	0.002		—0.001
	— 8	1	— 1	+0.001		0.005	0.003		—0.001
	— 7	1	— 2	0.001		0.007	0.004		—0.002
	— 6	1	— 2	0.001		0.008	0.006		—0.002
	— 5	1	— 1	0.001		0.007	0.007		—0.001
	— 4					+0.003	0.006	—0.008	0.000
	— 3					—0.001	0.006	—0.002	+0.002
	— 2					+0.002	+0.004	—0.002	0.001
	— 1					—0.009	—0.010	—0.003	+0.002
	0					+0.003	+0.008	0.000	—0.001
	1					+0.001	—0.012	+0.004	
	2						—0.002		
5	—12						—0.001		
	—11						—0.001		
	—10						—0.001		
	— 9						—0.002		
	— 8						—0.002		
	— 7						—0.002		
	— 6					+0.001	—0.002		
	— 5					0.001	—0.001		
	— 4					+0.001	—0.001	0.001	
	— 3					0.000	—0.001		
	— 2					0.000	0.000		
	— 1					—0.001	—0.001		
	0					+0.002	0.002		

CHAPTER IV

PERTURBATIONS OF PLUTO BY URANUS

8. Elements

In the determination of the perturbations of Pluto in the first order, relative to the mass of Uranus, we used the elements of Pluto which we obtained in Part One and the elements of Uranus given by Newcomb. /102

Elements of Uranus. Ecliptic and Equinox 1950.0.
Epoch 1900 Jan. 0 Mean Greenwich Noon

$$
\begin{aligned}
M_0 &= 74^\circ.31362,\\
\omega &= 96.03563,\\
\Omega &= 73.71127,\\
i &= 0.77384,\\
e &= 0.04704,\\
n &= 42''.23434,\\
a &= 19.190979,\\
\frac{1}{m} &= 22869.
\end{aligned}
$$

The elements of the mutual position of the orbits are:

$$
\begin{aligned}
N &= 217^\circ.4510, \quad \text{P} = 238^\circ.5846, \quad \alpha = 0.48389816,\\
N' &= 181.5994, \quad \text{P}' = 291.1125, \quad \sigma = 0.1434036.
\end{aligned}
$$

9. Development of the Perturbing Function

Development of the perturbing function, as in previous cases, was carried out by the method of Newcomb operators. In calculating the coefficients of the development of circular motion and their derivatives in Formulas (3) of Part One, it was necessary to use, in view of the large value of α, the following terms:

$$
\begin{aligned}
a'D^k A_i = {} & D^k c_7^i - \frac{1}{2}\sigma^2 D^k (c_3^{i-1} + c_3^{i+1}) + \frac{3}{8}\sigma^4 D^k (c_5^{i-2} + 4c_5^i + c_5^{i+2}) -\\
& - \frac{5}{16}\sigma^6 D^k (c_7^{i-3} + 9c_7^{i-1} + 9c_7^{i+1} + c_7^{i+3}) +\\
& - \frac{35}{128}\sigma^8 D^k (c_9^{i-4} + 16c_9^{i-2} + 36c_9^i + 16c_9^{i+2} + c_9^{i+4}) -\\
& - \frac{63}{256}\sigma^{10} D^k (c_{11}^{i-5} + 25c_{11}^{i-3} + 100c_{11}^{i-1} + 100c_{11}^{i+1} + 25c_{11}^{i+3} + c_{11}^{i+5}),
\end{aligned}
$$

$$
(i = 0,\ 1,\ldots\ 9;\ k = 0,\ 1,\ldots\ 9),
$$

Table 9

i	$a'A_i$	$a'DA_i$	$a'D^2A_i$	$a'D^3A_i$	$a'D^4A_i$	$a'D^5A_i$	$a'D^6A_i$	$a'D^7A_i$	$a'D^8A_i$	$a'D^9A_i$
0	2.11326714	0.24854488	0.58889645	1.558506	4.60311	13.4692	14.106	— 507.16	9076	275710
1	0.51037496	0.58951899	0.85164552	1.752418	4.89358	14.6815	20.203	— 473.84	9126	273976
2	0.18127484	0.38984507	0.89466822	2.274963	6.53736	19.9043	41.269	— 364.32	9333	269255
3	0.07119741	0.22327015	0.71896022	2.402177	8.35903	29.3971	86.398	— 131.43	9951	263231
4	0.02930993	0.12068750	0.50310478	2.128690	9.12402	38.8669	150.547	+ 263.46	11425	259458
5	0.01240164	0.06322231	0.32420732	1.672412	8.65080	44.2888	214.609	775.75	13992	262315
6	0.00534297	0.03247304	0.19788047	1.207984	7.36446	44.4211	257.668	1290.51	17344	274031
7	0.00233162	0.01645557	0.11623187	0.820779	5.77845	40.2774	272.808	1690.03	20727	292528
8	0.00102733	0.00825737	0.06635606	0.532531	4.25790	33.7355	261.384	1905.72	23315	312156
9	0.00045605	0.00411285	0.03705997	0.333311	2.98586	26.5274	231.408	1929.85	24549	326198

i	$\sigma^1a'B_i$	$\sigma^2a'DB_i$	$\sigma^2a'D^2B_i$	$\sigma^2a'D^3B_i$	$\sigma^2a'D^4B_i$	$\sigma^2a'D^5B_i$	$\sigma^2a'D^6B_i$	$\sigma^2a'D^7B_i$
0	0.01693952	0.03645025	0.12382920	0.580471	3.37334	23.1533	183.884	1694.15
1	0.01095774	0.03244496	0.12114943	0.575043	3.35279	23.0994	183.947	1697.00
2	0.00622864	0.02409928	0.10611191	0.544842	3.27971	22.9193	183.985	1704.59
3	0.00335292	0.01611794	0.08386937	0.481654	3.09554	22.4066	183.248	1713.44
4	0.00175121	0.01008430	0.06117953	0.396330	2.77670	21.2680	180.024	1714.91
5	0.00089724	0.00602685	0.04197667	0.305942	2.35645	19.3836	172.452	1694.51
6	0.00045363	0.00348404	0.02746963	0.223748	1.89671	16.8805	159.653	1637.82
7	0.00022712	0.00196394	0.01731807	0.156438	1.45553	14.0499	142.187	1537.13

i	$\sigma^4a'C_i$	$\sigma^4a'DC_i$	$\sigma^4a'D^2C_i$	$\sigma^4a'D^3C_i$	$\sigma^4a'D^4C_i$	$\sigma^4a'D^5C_i$
0	0.00029140	0.00138799	0.00830100	0.059458	0.49065	4.5177
1	0.00024595	0.00128450	0.00796999	0.057957	0.48207	4.4618
2	0.00017643	0.00104801	0.00703561	0.053500	0.45659	4.2941
3	0.00011530	0.00077960	0.00575387	0.046666	0.41539	4.0171
4	0.00007097	0.00054174	0.00440960	0.038532	0.36212	3.6397
5	0.00004190	0.00035755	0.00320180	0.030256	0.30238	3.1862

i	$\sigma^6a'D_i$	$\sigma^6a'DD_i$	$\sigma^6a'D^2D_i$	$\sigma^6a'D^3D_i$
0	0.00000667	0.00005030	0.00044106	0.004432
1	0.00000613	0.00004776	0.00042600	0.004324
2	0.00000496	0.00004127	0.00038447	0.004012
3	0.00000372	0.00003300	0.00032601	0.003547

Table 10

i	$a'D_0A_i$	$a'DD_0A_i$	$a'D^2D_0A_i$	$a'D^3D_0A_i$	$a'D^4D_0A_i$	$a'D^5D_0A_i$	$a'D^6D_0A_i$	$a'D^7D_0A_i$	$a'D^8D_0A_i$
0	−0.04157070	−0.11922572	−0.42306360	−1.873874	−10.05536	−63.7396	−489.422	−5078.34	126958
1	−0.04443612	−0.11135012	−0.40084490	−1.835190	−10.00876	−63.8328	−491.262	−5090.90	125362
2	−0.02726654	−0.08920658	−0.35805544	−1.730342	− 9.77404	−63.7386	−495.746	−5125.56	120642
3	−0.01508062	−0.06251034	−0.29224926	−1.549068	− 9.27476	−62.8412	−499.482	−5168.74	113660
4	−0.00796332	−0.04024714	−0.21942728	−1.301100	− 8.43542	−60.4484	−497.568	−5193.14	103072
5	−0.00409514	−0.02448850	−0.15402386	−1.026086	− 7.29326	−56.0824	−484.826	−5160.54	91360
6	−0.00207016	−0.01431590	−0.10254842	−0.765082	− 5.98736	−49.8536	−457.816	−5031.84	78764
7	−0.00103404	−0.00812454	−0.06548788	−0.543826	− 4.68288	−42.3830	−416.656	−4780.84	66176
8	−0.00051196	−0.00450740	−0.04044972	−0.371310	− 3.50684	−34.5176	−364.716	−4405.56	54374
9	−0.00025184	−0.00245618	−0.02431778	−0.245076	− 2.52848	−27.0194	−307.336	−3929.96	43904

i	$\sigma a'D_0\sigma^2B_i$	$\sigma a'DD_0\sigma^2B_i$	$\sigma a'D^2D_0\sigma^2B_i$	$\sigma a'D^3D_0\sigma^2B_i$	$\sigma a'D^4D_0\sigma^2B_i$	$\sigma a'D^5D_0\sigma^2B_i$	$\sigma a'D^6D_0\sigma^2B_i$
0	0.03198916	0.06320586	0.18896182	0.748016	3.47156	18.2160	125.936
1	0.02011540	0.05569760	0.18584172	0.747764	3.48732	18.4496	127.930
2	0.01106976	0.04041810	0.16173818	0.717126	3.50936	19.1046	133.792
3	0.00575854	0.02625674	0.12569354	0.635792	3.40791	19.7618	142.230
4	0.00290278	0.01590458	0.08953058	0.519262	3.11401	19.7608	149.990
5	0.00143382	0.00918268	0.05971062	0.394648	2.66070	18.7400	153.174
6	0.00069810	0.00511962	0.03786230	0.282496	2.13586	16.7488	149.350
7	0.00033626	0.00277960	0.02307696	0.192556	1.62266	14.1500	138.318

i	$\sigma a'D_0\sigma^1C_i$	$\sigma a'DD_0\sigma^1C_i$	$\sigma a'D^2D_0\sigma^1C_i$	$\sigma a'D^3D_0\sigma^1C_i$	$\sigma a'D^4D_0\sigma^1C_i$
0	0.00109450	0.00500214	0.02833028	0.188480	0.139330
1	0.00091626	0.00461092	0.02715736	0.183626	0.136898
2	0.00061892	0.00372726	0.02383976	0.169050	0.129622
3	0.00041784	0.00273840	0.01931804	0.146618	0.117650
4	0.00025310	0.00187546	0.01462638	0.119968	0.102016
5	0.00014694	0.00121812	0.01046730	0.093058	0.080788

i	$\sigma a'D_0\sigma^0D_i$	$\sigma a'DD_0\sigma^0D_i$	$\sigma a'D^2D_0\sigma^0D_i$
0	0.00005770	0.00027956	0.00241702
1	0.00003458	0.00026514	0.00233332
2	0.00002784	0.00022838	0.00210292
3	0.00002102	0.00018186	0.00177958

Table 11

i	i'	$10^6 a'R$		$10^6 a'D'R$		$10^6 a' D_g R$		$10^6 a' \frac{\partial R}{\partial W'}$		$10^6 a' \frac{\partial R}{\partial W}$		$10^6 \frac{1}{n'} a' D_t' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
0	0	—	—1063140	—	—1203210	—	— 24898	—	+ 1922	—	— 1288	—	0
	1	+ 29759	272931	— 71421	— 360200	— 3583	— 16938	+ 20960	31719	— 24133	— 30400	— 272931	+ 29759
	2	21954	59538	— 67061	— 64376	+ 10696	— 20302	27193	29389	— 13458	— 13706	— 119076	43907
	3	12380	10770	— 43109	+ 5812	10025	— 15822	20878	17455	— 6254	— 4211	— 32312	37137
	4	5951	+ 27	— 22098	16593	5509	— 9854	12794	7926	— 2571	— 654	— 107	23793
	5	2398	— 1366	— 8644	11861	2057	— 5125	6616	+ 2439	— 919	+ 385	+ 6828	11994
	6	746	— 1088	— 2019	7015	+ 149	— 2351	3092	— 89	— 286	453	6528	4481
	7	196	— 633	+ 375	3626	— 489	— 1096	1459	— 684	— 117	+ 242	4426	1366
	8	+ 134	— 267	— 224	1686	— 190	— 519	+ 631	— 145			2142	1071
1	— 9	— 431	+ 79	+ 2888	637	— 830	— 623	— 536	+ 1459	— 79	— 431	699	3884
	— 8	— 1040	76	5345	2334	— 1192	— 1520	— 1837	2929	+ 272	— 1323	601	8325
	— 7	— 1580	594	9097	+ 912	— 2528	— 1610	— 1822	5349	+ 335	— 2176	4157	11052
	— 6	— 2084	2926	13966	— 7717	— 5837	— 1662	+ 1045	10620	— 1835	— 3904	17550	12505
	— 5	— 1999	8148	+ 13503	— 29283	—10266	— 2506	7521	18276	— 7480	— 7286	40743	+ 9995
	— 4	+ 390	17156	— 24191	— 84381	—13976	— 4050	19265	24460	— 17798	— 11087	68619	— 1559
	— 3	— 9788	+ 16274	— 183529	— 226967	—13927	— 7477	+ 22651	39054	— 20211	— 29775	+ 48821	+ 29367
	— 2	— 210479	+ 140350	— 780847	— 667137	— 2384	— 7605	— 131399	231242	+ 132854	— 233032	— 280098	420958
	— 1	—2813441	—2162380	—4093969	—3115062	+ 96047	+ 74469	—2159260	+ 2812104	+ 2159690	—2810383	—2162380	+ 2813141
	0	+ 1233386	+ 908796	+ 1177349	+ 915003	—69214	—109666	+ 984296	—1244784	— 916033	+ 1236000	0	0
	1	— 6265	+ 47462	— 21294	119432	+ 23333	+ 134269	— 79769	+ 32102	— 51685	— 6592	— 47462	— 6265
	2	— 415	— 1568	+ 2536	48461	— 1563	+ 1030	+ 161	— 1470	+ 999	+ 657	+ 3136	+ 829
	3	— 922	— 3284	6049	22679	— 2858	— 4762	2545	— 2747	3566	— 176	9854	— 2767
	4	— 758	— 1632	4879	8927	— 2015	— 2123	+ 762	— 1851	2002	— 357	6530	— 3037
	5	— 503	— 612	3214	2906	— 1174	— 387	— 203	— 1000	855	— 367	3057	— 2518
	6	— 299	— 150	1846	362	— 629	+ 196	— 426	— 591	282	— 211	906	— 1799
	7	— 121	— 79	809	+ 206	— 316	151	— 206	— 284	+ 79	— 121	553	— 845
2	—10	+ 100	+ 526	1961	— 3463	— 1410	716	+ 2015	+ 1242	— 1052	+ 202	5261	— 1005
	— 9	— 28	1423	5089	— 6508	— 2548	163	4088	3599	— 3355	— 846	12813	+ 255
	— 8	+ 785	1907	+ 3912	— 9584	— 2988	764	5804	+ 3332	— 4257	— 344	15252	— 6278
	— 7	4975	2816	— 8632	— 17537	— 4866	3078	11466	— 3573	— 7095	+ 7330	19710	— 34824
	— 6	14516	3991	— 37780	— 27836	— 8269	4579	19625	— 20536	— 13026	25063	23955	— 87079
	— 5	35809	+ 1696	— 108416	— 24432	—12044	4651	19781	— 62253	— 12865	66130	+ 8471	— 179047
	— 4	75432	— 8553	— 242992	+ 7017	—17559	3395	+ 654	— 144838	+ 3333	145809	— 34211	— 301732
	— 3	127038	— 32347	— 432560	77593	—24509	2404	— 50219	— 258231	50650	256263	— 97044	— 381117
	— 2	+ 117713	— 70912	— 522506	+ 118795	—27362	6454	— 115743	— 273821	119613	+ 271199	— 141817	— 235426
	— 1	— 316394	— 224715	— 248168	— 236436	+ 4380	+ 4662	— 216965	+ 328195	+ 224328	— 336150	— 224715	+ 316394
	0	+ 103850	+ 78713	+ 161930	+ 115563	—27747	— 20272	+ 90394	— 118074	— 71619	+ 93627	0	0
	1	— 6280	+ 2971	24867	19274	— 7054	+ 11247	— 10105	+ 3038	+ 280	— 11545	— 2971	— 6280
	2	— 2403	— 80	12581	+ 4250	— 3603	1299	— 2186	358	1232	— 4821	+ 162	— 4808
	3	— 931	+ 68	5116	— 188	— 1250	925	— 1135	282	+ 206	— 1932	— 197	— 2788
	4	— 331	137	1812	— 1064	— 227	762	— 637	310	— 181	— 709	— 545	— 1321
	5	— 81	103	356	— 821	+ 67	432	— 276	219	— 169	— 186	— 517	— 399
	6	— 28	+ 52	+ 123	— 448	+ 64	248	— 151	+ 109	— 105	— 58	— 314	— 173

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Table 11 (continued)

i	i'	$10^5 a' R$		$10^5 a' D' R$		$10^5 a' D_2 R$		$10^5 a' \frac{\partial R}{\partial W'}$		$10^5 a' \frac{\partial R}{\partial W'}$		$10^5 \frac{1}{n'} a' D_1' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
3	−11	+ 788	− 94	− 3609	− 2992	− 225	2079	+ 1871	− 2462	+ 282	+ 2363	− 1035	− 8663
	−10	1861	+ 678	− 6325	− 9012	− 1871	2521	6672	− 4524	− 3619	5745	+ 6787	− 18603
	− 9	2542	+ 82	− 9154	− 7171	− 1664	2314	+ 5530	− 6093	− 2902	6844	+ 747	− 22882
	− 8	5611	− 3926	− 25052	+ 7768	− 908	4845	− 4707	−17418	+ 8700	16263	− 31418	− 44894
	− 7	11384	−10462	− 51791	31746	− 2545	7766	− 21104	−37676	25840	35645	− 73231	− 79690
	− 6	18630	−23708	− 82714	88166	− 5014	9590	− 59703	−60559	63595	58554	−142253	−111769
	− 5	26289	−43674	−112461	175024	− 7956	12230	−122603	−83294	123627	81427	−218376	−131441
	− 4	29320	−60481	−119148	250507	−10225	14423	−178777	−89938	177966	87347	−241920	−117285
	− 3	+19113	−47506	− 70423	+196763	− 9489	12894	−144048	− 57867	+143525	+ 51789	−142518	− 57338
	− 2	− 7305	+ 11495	+ 28809	− 59995	− 8778	1794	+ 36443	+ 11577	− 39430	− 22389	+ 22987	+ 14611
	− 1	−31746	−15287	15365	− 33828	−11414	6832	− 16660	+ 32244	+ 10090	−46820	− 15287	+ 31746
	0	+ 6354	+ 7921	24346	− 595	− 4840	1337	+ 8104	− 6413	− 10181	+ 919	0	0
	1	− 891	1096	4371	− 3963	− 309	2482	− 447	+ 1834	− 2365	− 2530	− 1096	− 894
	2	− 146	376	+ 569	− 2194	+ 407	811	+ 12	799	− 1012	− 474	− 755	− 294
	3	+ 34	155	− 473	+ 1038	479	301	57	394	− 445	+ 93	− 462	+ 98
	4	39	40	− 393	− 304	244	63	65	136	− 128	115	− 160	159
	5	27	+ 18	− 246	− 134	137	14	+ 32	+ 64	− 54	82	− 91	+ 136
4	−12	402	− 1247	− 6274	+ 5150	2424	1387	− 4108	− 4367	+ 4987	1608	− 14960	− 4824
	−11	1961	− 1877	− 15621	5471	+ 1351	2942	− 4284	−12234	6623	9789	− 20650	− 21582
	−10	+ 1579	− 2136	− 11821	6338	− 41	2381	− 4502	− 9225	6117	+ 8423	− 21361	− 15791
	− 9	− 600	− 6619	− 1870	31012	+ 2247	3255	− 23465	− 1404	24224	− 974	− 59568	+ 5392
	− 8	− 1881	−12967	+ 1945	63749	3137	6225	− 49957	+ 632	50216	− 3760	−103747	15051
	− 7	− 5694	−20470	23372	103057	2420	8171	− 80131	15672	80155	−18205	−143302	39861
	− 6	−10944	−27325	54368	137794	2620	10143	−108473	39633	107218	−40912	−163952	65662
	− 5	−13273	−26791	71455	133515	2302	10907	−107476	52665	104598	−54108	−133948	66374
	− 4	− 7104	−12457	+ 42140	+ 57327	+ 546	8670	− 51658	+29566	+ 46597	−32373	− 49827	+ 28418
	− 3	+ 4345	+ 8154	− 18725	− 48767	− 3242	5601	+ 27298	−18541	− 35208	+15633	+ 24455	− 13032
	− 2	− 1821	+ 4165	+ 8686	− 25796	− 1387	6798	7983	+ 4768	− 16207	− 6697	+ 8331	+ 3643
	− 1	− 2208	− 211	− 56	− 9239	− 262	1702	912	+ 2720	− 3154	− 3164	− 211	2208
	0	+ 768	+ 837	− 433	− 1727	+ 619	+ 46	1496	− 381	− 1786	+ 1109	0	0
	1	118	+ 102	− 1114	− 389	558	− 75	312	+ 128	− 305	510	− 102	118
	2	81	− 13	− 735	+ 154	302	− 196	152	+ 7	+ 47	333	+ 22	164
	3	23	− 14	− 242	140	94	− 92	47	− 22	55	98	41	71
	4	+ 12	− 10	− 110	101	26	− 55	+ 21	− 18	42	+ 49	+ 42	49
	−13	− 1207	− 1098	+ 4024	9100	2742	− 959	− 7083	+ 3753	5493	− 6035	− 14281	15693
	−12	− 740	− 2920	− 1722	19809	3005	+ 1022	− 16352	− 1343	15899	− 1617	− 35033	8877

Table 11 (continued)

i	i'	$10^7 a' R$		$10^7 a' D' R$		$10^7 \sigma a' D_0 R$		$10^7 a' \frac{\partial R}{\partial W'}$		$10^7 a' \frac{\partial R}{\partial W}$		$10^6 \frac{1}{n'} a' D_t' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
5	−11	− 378	−2231	− 2711	12971	1053	1909	−10556	− 2533	11389	+ 1527	−24533	4163
	−10	− 4933	−2416	+26806	15014	3014	474	−11860	+ 21695	11115	−22688	−24145	49327
	− 9	− 8793	−4483	49437	29501	5515	1537	−25214	41000	23353	−41809	−40340	79133
	− 8	−12346	−4838	72636	30525	5482	2940	−27069	58765	25407	−59450	− 38697	98768
	− 7	−14816	−4078	88936	23023	5965	3225	−21455	72927	19821	−72531	−28548	103722
	− 6	−11796	−2299	71186	+ 9482	5557	3556	−11805	59764	+ 8976	−58481	−13790	70785
	− 5	− 2722	− 135	+15301	− 4423	3291	3476	− 1713	+15475	− 2358	−13556	− 669	+ 13606
	− 4	+ 5000	+1438	−32279	−12286	1388	3647	+ 3861	−22264	− 8344	+ 25199	+ 5754	− 20000
	− 3	1002	2084	− 7140	−13574	2056	2286	6775	− 1972	− 9850	4515	6250	− 3004
	− 2	+ 161	+ 353	− 1869	− 2220	275	+ 185	1105	− 478	− 1487	950	+ 704	− 323
	− 1	− 32	− 157	− 1144	+ 508	146	− 553	154	− 178	+ 425	425	− 157	+ 32
	0	+ 48	− 68	− 429	656	+ 83	− 372	+ 49	− 126	351	248	0	0
	1	15	− 54	− 100	− 473	− 7	− 188	− 20	− 51	+ 269	+ 76	+ 54	+ 15
	2	+ 4	− 18	− 2	+ 157	− 14	− 53	− 15	− 16	+ 95	+ 11	+ 38	+ 6
	3	− 2	− 8	+ 26	+ 66	− 22	− 15	− 11	− 10	+ 37	− 9	+ 22	− 6
6	−14	− 1759	+ 583	+12302	− 508	+1049	−2654	+ 1011	+ 10444	− 3499	−10557	+ 8163	+ 24633
	−13	− 3430	−1379	24499	+16989	1923	−1997	−14309	20843	+ 11214	−20982	−17939	44593
	−12	− 1839	−1602	11850	+15213	1040	+ 532	−13087	9857	+12694	− 9875	−19223	22063
	−11	− 3283	+2294	21087	−15904	1849	−1217	+13684	17461	−13864	−17640	+25227	36100
	−10	− 5902	3293	40159	−21403	4054	−2390	18243	35506	−19055	−34639	32929	59015
	− 9	− 6366	3876	44245	−26351	4078	−1254	20879	39052	−22048	−37983	34870	57293
	− 8	− 5821	4442	39245	−32214	3783	−1215	26137	35182	−26775	−33847	35527	46566
	− 7	− 3128	+2871	+19173	−21881	3693	− 775	17682	+19692	−18284	−17198	20089	+ 21896
	− 6	+ 531	− 1	− 6509	− 1100	2605	+ 231	+ 333	− 1381	− 939	+ 4357	+ 4	− 3193
	− 5	2331	−1320	−18244	+ 9134	1958	+ 610	− 7942	−11167	+ 7752	14171	− 6605	− 11654
	− 4	994	+ 370	− 7407	− 2307	+1174	− 189	+ 2235	− 3802	− 2120	5468	+ 1478	− 3978
	− 3	+ 39	+ 36	− 434	+ 177	− 136	− 137	+ 162	− 225	+ 2	+ 174	+ 108	− 117
	− 2	− 47	− 78	+ 410	829	− 304	− 294	− 155	− 37	514	− 269	− 161	+ 92
	− 1	− 27	− 46	259	446	− 138	− 164	− 84	+ 8	274	− 159	− 46	+ 27
	0	− 19	− 23	204	182	− 72	− 38	− 49	+ 24	135	− 120	0	0
	1	− 4	− 4	60	+ 38	− 22	− 5	− 13	8	+ 23	− 29	+ 4	− 4
	2	− 3		+ 32	− 5	− 8	+ 11	− 8	5	− 1	− 18		− 6
7	−15	+ 344	+2548	− 2769	−20493	− 200	−1484	+17835	− 2410	−17835	+ 2410	+38218	− 5164
	−14	− 1842	+2669	+17121	−20171	+1178	−1422	+17321	+15278	−17321	−15278	+37375	+ 25783
	−13	− 2008	− 250	+17630	+ 4376	+1149	+ 32	− 4121	+15553	+ 4121	−15553	− 3260	+ 26120
	−12	+ 146	+2424	− 3083	−18019	− 98	−1636	+15475	− 2932	−15723	+ 2921	+29084	− 1741

Table 11 (continued)

i	i'	$10^5 a' R$		$10^5 a' D' R$		$10^5 a' D_3 R$		$10^5 a' \frac{\partial R}{\partial W'}$		$10^5 a' \frac{\partial R}{\partial W}$		$10^5 \frac{1}{n'} a' D_t' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
7	−11	− 248	4521	+ 921	−35060	+ 744	−3500	+ 31780	+ 1700	−31063	− 1051	49732	+ 2735
	−10	− 335	3838	+ 2515	−30508	1415	−2406	27065	+ 4175	−26664	− 2850	38370	+ 3351
	− 9	+ 100	3168	− 2473	−25138	1693	−1835	22331	− 305	−22030	+ 1624	28506	− 900
	− 8	259	+ 1209	− 4252	− 8840	1262	−1745	+ 9318	− 1550	− 8201	3156	+ 9568	− 2082
	− 7	273	− 844	− 3388	+ 7574	1074	− 849	− 4790	− 1117	+ 6031	2464	− 5906	− 1910
	− 6	307	−1206	− 2806	10181	673	− 421	− 7334	− 1164	8324	2040	− 7238	− 1841
	− 5	+ 401	− 117	− 3038	+ 1062	+ 56	− 265	− 261	− 2320	+ 637	+ 2535	− 587	− 2010
	− 4	− 50	+ 72	+ 612	− 453	− 265	+ 155	+ 276	+ 164	− 430	− 465	+ 279	+ 193
	− 3	− 74	+ 2	778	− 19	− 289	+ 44	− 22	190	− 12	− 516	+ 8	220
	− 2	− 24	− 2	358	+ 19	− 128	− 3	− 11	83	+ 15	− 233	− 5	67
	− 1	− 18	− 1	189	− 3	− 55	+ 1	− 1	48	− 1	− 123	+ 1	+ 18
	0	− 6	− 1	55	0	− 18	0	− 4	15	+ 6	− 39		0
	1	− 2		+ 23	− 4	− 6	+ 1	+ 1	+ 6	− 3	− 14		− 2
8	−16	+ 2076	+ 1188	−18704	−10702	−1306	− 747	9503	−16669	− 9503	+ 16669	+ 19007	−33219
	−15	+ 880	2672	− 5821	−24976	− 413	−1686	22297	− 4852	−22297	+ 4852	40086	−13199
	−14	−1913	1449	+ 19818	−14071	+ 1095	−1028	12671	+ 17868	−12671	−17868	20299	+ 26780
	−13	+ 1141	1149	−10559	− 9192	− 746	− 729	8047	− 9408	− 8047	+ 9408	14944	−14838
	−12	2524	1540	−23148	−11729	−1765	−1849	11766	−21068	−10688	20717	18482	−30291
	−11	922	691	− 7601	− 5307	−1048	−1858	6791	− 7389	− 5324	6911	7660	− 10145
	−10	520	+ 499	− 4537	− 3434	− 709	−1283	4615	− 4545	− 3560	4198	+ 5008	− 5203
	− 9	+ 134	0	− 1291	+ 1453	− 286	−1256	+ 659	− 1517	+ 715	+ 1342	− 14	− 1200
	− 8	− 364	− 617	+ 3444	6304	+ 51	− 893	− 3795	+ 2871	5053	− 2946	− 4924	+ 2924
	− 7	− 254	− 541	+ 2468	5132	118	− 435	− 3371	+ 2046	4132	− 2069	− 3795	+ 1782
	− 6	+ 151	− 192	− 1359	+ 1728	50	− 187	− 1045	− 1092	+ 1560	− 1190	− 1155	− 913
	− 5	+ 30	+ 19	− 207	− 91	+ 42	+ 14	+ 154	− 114	− 155	+ 164	+ 91	− 153
	− 4	− 11	− 3	+ 106	+ 63	− 34	8	− 33	+ 38	+ 35	− 80	− 14	+ 40
	− 3	− 8	− 2	68	23	− 29	6	− 17	24	15	− 56	− 6	+ 24
	− 2	− 3	− 1	24	11	− 8	1	0	8	+ 1	− 19		6
	− 1	− 1		+ 8	3	− 3	1	+ 1	+ 3	− 2	− 7		+ 1
9	−17	+ 1938	− 813	−19330	8106	−1311	550	− 7313	−17438	+ 7313	+ 17438	−13813	−32938
	−16	+ 2682	−1125	−26773	+ 11228	− 1769	742	−10121	−24136	+ 10121	+ 24136	−17991	−42908
	−15	− 15	+ 6	+ 225	− 94	− 157	+ 66	+ 56	+ 133	− 56	− 133	+ 93	+ 221
	−14	+ 538	561	− 4526	− 5219	− 442	− 215	4595	− 4048	− 4595	+ 4048	7855	− 7531
	−13	2015	+ 148	−19178	− 938	−1205	+ 9	+ 752	−17136	− 752	17136	+ 1914	−26197
	−12	837	− 296	− 8281	+ 2969	− 590	192	− 2694	− 7474	+ 2694	7474	− 3550	−10040
	−11	346	− 25	− 3513	+ 551	− 283	+ 46	− 530	− 3139	+ 530	3139	− 269	− 3818

Table 11 (continued)

i	i'	$10^6 a'R$		$10^6 a'D'R$		$10^6 \sigma a' D_\sigma R$		$10^6 a' \frac{\partial R}{\partial W'}$		$10^6 a' \frac{\partial R}{\partial W}$		$10^6 \frac{1}{n'} a' D_l' R$	
		sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos
9	−10	+ 204	+145	−1791	−1044	−225	−340	+ 1274	−1730	− 971	+1632	+ 1458	− 2047
	− 9	+ 180	− 40	+1862	+ 356	− 9	−317	+ 89	+1535	+ 240	−1642	− 358	+ 1627
	− 8	−151	−100	+1381	949	+ 22	− 96	− 629	+1179	775	−1226	− 892	+ 1202
	− 7	+ 19	− 93	− 260	898	− 2	− 63	− 658	− 225	767	+ 222	− 652	− 135
	− 6	15	− 12	− 98	+ 99	+ 8	− 36	− 16	− 77	+ 60	88	− 67	− 86
	− 5	+ 4	+ 4	− 23	− 25	+ 10	+ 9	+ 16	− 15	− 23	+ 22	+ 14	− 14
	− 4	− 2	− 1	+ 9	+ 14	− 1		− 10	+ 8	+ 12	− 8	− 6	+ 4
	− 3			+ 5	− 1	− 1							
10	−15	+730	−306	−7322	+3071	−413	173	−2757	−6573	2757	+6573	−4594	−10956
	−14	+856	−359	−8579	+3598	−476	+200	−3230	−7702	+3230	+7702	−5024	−11981
	−13	−134	+ 56	+1342	− 563	+ 45	− 19	+ 505	+1203	− 505	−1203	+ 729	+ 1738
	−12	−100	172	+1136	−1657	+ 39	− 75	1481	+1037	−1481	−1037	2075	+ 1206
	−11	+256	+ 21	−2434	− 141	−124	− 5	+ 112	−2180	− 112	+2180	+ 227	− 2823
	−10	+ 31	− 37	− 345	+ 351	− 19	+ 15	− 320	− 304	+ 320	+ 304	− 371	− 310
	− 9	− 70	+ 9	+ 703	− 80	+ 31	− 5	+ 65	+ 600	− 65	− 600	+ 74	+ 633
	− 8	− 14	− 3	+ 146	+ 40	− 2	− 32	+ 3	+ 115	+ 23	− 125	− 24	+ 113
	− 7	+ 9	− 19	− 81	+ 169	− 12	− 14	− 133	− 96	+ 156	+ 88	− 135	− 66
	− 6	0	− 1	+ 16	− 6	+ 1	0	+ 1	+ 7	− 1	− 7	− 2	+ 3
	− 5	1	+ 1	1	− 11	+ 2	+ 1	+ 2	− 7	− 2	+ 8	+ 2	− 7
	− 4			1	+ 2								
	− 3			+ 1									
11	−13	114	− 48	−1149	482	− 58	24	− 432	−1029	+ 432	1029	− 623	− 1487
	−12	+103	− 43	−1038	+ 435	− 52	+ 22	− 390	− 930	+ 390	+ 930	− 520	− 1239
	−11	− 39	+ 16	+ 395	− 166	+ 18	− 7	+ 146	+ 349	− 145	− 349	+ 179	+ 426
	−10	− 31	22	+ 323	− 224	+ 14	− 10	201	+ 297	− 201	− 297	229	+ 320
	− 9	+ 10	+ 1	− 91	− 14	− 5	0	+ 14	− 82	− 14	+ 82	+ 17	− 88
	− 8	3	− 4	− 22	+ 39	− 1	+ 1	− 37	− 22	+ 37	22	− 35	− 16
	− 7		− 2	+ 3	+ 11		0	− 15	− 3	+ 15	+ 3	− 12	− 1
	− 6			+ 4	− 3		− 1	+ 7	+ 1	− 7	− 1	+ 5	+ 1
	− 5			− 1									
12	−11	7	− 3	− 77	+ 32	− 4	+ 2	− 28	− 66	+ 28	+ 66	− 34	− 81
	−10	+ 4	− 2	− 41	+ 17	− 2	+ 1	− 14	− 33	+ 14	+ 33	− 15	− 37
	− 9	− 4	+ 2	+ 30	− 13	+ 1		+ 14	+ 33	− 14	− 33	+ 14	+ 33
	− 8	− 1		+ 10	− 5	+ 1		3	+ 8	− 3	− 8	3	7
	− 7			− 4	+ 2								

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$$a'D^kB_i = \frac{1}{2} D^k c_3^i - \frac{3}{4} \sigma^2 D^k (c_5^{i-1} + c_5^{i+1}) + \frac{15}{16} \sigma^4 D^k (c_7^{i-2} + 3c_7^i + c_7^{i+2}) -$$
$$- \frac{35}{32} \sigma^6 D^k (c_9^{i-3} + 6c_9^{i-1} + 6c_9^{i+1} + c_9^{i+3}) +$$
$$+ \frac{315}{296} \sigma^8 D^k (c_{11}^{i-4} + 10c_{11}^{i-2} + 20c_{11}^i + 10c_{11}^{i+2} + c_{11}^{i+4}),$$
$$(i = 0, 1, \ldots 7; \; k = 0, 1, \ldots 7),$$
$$a'D^kC_i = \frac{3}{8} D^k c_5^i - \frac{15}{16} \sigma^2 D^k (c_7^{i-1} + c_7^{i+1}) + \frac{35}{64} \sigma^4 D^k (3c_9^{i-2} + 8c_9^i + 3c_9^{i+2}) -$$
$$- \frac{315}{128} \sigma^6 D^k (c_{11}^{i-3} + 5c_{11}^{i-1} + 5c_{11}^{i+1} + c_{11}^{i+3}),$$
$$(i = 0, 1, \ldots 5; \; k = 0, 1, \ldots 5),$$
$$a'D^kD_i = \frac{5}{16} D^k c_7^i - \frac{35}{32} \sigma^2 D^k (c_9^{i-1} + c_9^{i+1}) + \frac{315}{256} \sigma^4 D^k (2c_{11}^{i-2} + 5c_{11}^i + 2c_{11}^{i+2}),$$
$$(i = 0, 1, 2, 3; \quad k = 0, 1, 2, 3).$$

Tables 9 and 10 give the values of $a'D^kA_i$, $\sigma^2 a'D^kB_i$, $\sigma^4 a'D^kC_i$, $\sigma^6 a'D^kD_i$; $\sigma a'D^kD_\sigma A_i$, $\sigma a'D^kD_\sigma \sigma^2 B_i$, $\sigma a'D^kD_\sigma \sigma^4 C_i$, $\sigma a'D^kD_\sigma \sigma^6 D_i$.

The numerical values of the corrections for the indirect term of the perturbing function are given below:

$$\alpha^{-2}(1 - \sigma^2) = 4.182807,$$
$$\alpha^{-2}\sigma^2 = 0.087824.$$

Table 11 contains the coefficients of the expansions of $a'R$, $a'D'R$, $\sigma a'D_\sigma R$, $a' \frac{\partial R}{\partial W'}$, $a' \frac{\partial R}{\partial W}$, $\frac{1}{n'} a'D_i'R$.

10. Perturbations of the Logarithm of the Radius Vector, Longitude in Orbit, and Longitude of Node and Inclination

In determining the perturbations of Pluto from Uranus we must pay particular attention to pronounced covariances of the mean motions, the multiples $n_3 - 3n'$. In the new determination of the perturbations of Pluto by Uranus, the long-period perturbations for these covariances $i(n_3 - 3n')$ (i = 1, 2, 3, 4) were expressed in terms of secular perturbations.

In this case the calculations are somewhat simpler than similar calculations in Section 13 of Part Two, since $\frac{1}{n'} a'D_i'R$, $a' \frac{\partial R}{\partial i'}$, $a' \frac{\partial R}{\partial W'}$ and other

quantities entering the second members of equations, determining perturbations of the first order, do not contain mixed terms in groups with pronounced covariances. Therefore, in Schemes I and II which we used, having appropriately replaced the quantity $a'\delta D'_t R$ by $a'D'_t R$, and $a'\delta\frac{\partial R}{\partial W}$, by $a'\frac{\partial R}{\partial W}$, etc., there will be no mixed terms.

Formulas (38), (40), (45), and (47) are also simplified. In them all quantities C and D with all superscripts and subscripts will be equal to zero.

The constants of integration, just as in Section 13 of Part Two, are formed from two parts. The first part is determined by formulas in Section 8 of Part One, and the second part by Formulas (43) and (49) in Section 6 of Part Two. For the initial instant, we will take the instant of osculation 1930 Sept. 20.0 and, after combining the constants, we have:

$$\begin{aligned} a'C_1 &= 0.76427, \\ a'K_1 &= -1.41604, & C_3 &= 25''.752, \\ a'K_2 &= 0.19520, & C_5 &= -0''.648, \\ C_2 &= 0.28916, & C_4 &= -1''.541. \end{aligned}$$

Table 12 shows the perturbations of Pluto by Uranus. The coefficients of the series of perturbations of the logarithm of the radius vector are given to the tenth decimal place of the common logarithms, and of the longitude in orbit and longitude of node and inclination, in thousandths of a second of arc. As previously, the mean anomalies of Uranus and Pluto serve as the arguments.

Table 12

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i	i'	$10^5 \log r'$ sin	$10^5 \log r'$ cos	$\delta w'$ sin	$\delta w'$ cos	$\delta\Omega'$ sin	$\delta\Omega'$ cos	$\delta i'$ sin	$\delta i'$ cos
					"		"		"
0	0	—	+3978$n't$	—	−2.952$n't$		−3.669$n't$		−0.272$n't$
	1	+3650$n't$	−5124$n't$	+5.335$n't$	3.158$n't$				
	2	1169$n't$	−1889$n't$	1.577$n't$	0.925$n't$				
	3	382$n't$	− 654$n't$	0.493$n't$	0.289$n't$				
	4	129$n't$	− 221$n't$	0.158$n't$	0.093$n't$				
	5	44$n't$	− 74$n't$	0.051$n't$	0.029$n't$				
	6	14$n't$	− 25$n't$	0.017$n't$	0.009$n't$				
	7	5$n't$	− 8$n't$	0.006$n't$	+0.003$n't$				
	8	1$n't$	− 3$n't$	0.002$n't$					
	9	+1$n't$	− 1$n't$	0.001$n't$					
	10		− 1$n't$						
0	0	—	+134$(n't)^2$	—	−1.134$(n't)^2$		−0.054$(n't)^2$		
	1	− 695$(n't)^2$	− 103$(n't)^2$	0.108$(n't)^2$	−0.674$(n't)^2$				
	2	− 254$(n't)^2$	− 38$(n't)^2$	0.032$(n't)^2$	−0.204$(n't)^2$				
	3	− 88$(n't)^2$	− 13$(n't)^2$	0.010$(n't)^2$	−0.065$(n't)^2$				
	4	− 30$(n't)^2$	− 5$(n't)^2$	0.003$(n't)^2$	−0.021$(n't)^2$				
	5	− 10$(n't)^2$	− 2$(n't)^2$	+0.001$(n't)^2$	−0.007$(n't)^2$				
	6	− 3$(n't)^2$	− 1$(n't)^2$		−0.002$(n't)^2$				
	7	− 1$(n't)^2$			−0.001$(n't)^2$				
	0	—		—	−0.030$(n't)^3$		−0.001$(n't)^3$		
0	1	− 17$(n't)^3$	+ 1$(n't)^3$	−0.001$(n't)^3$	−0.016$(n't)^3$				
	2	− 6$(n't)^3$			−0.005$(n't)^3$				
	3	− 2$(n't)^3$			−0.002$(n't)^3$				
	4	− 1$(n't)^3$							
0	0	—	+7209	—	+25.752	—	−0.648	—	−1.541
	1	+4993	−23069	+20.661	9.860	−1.848	+0.418	+0.016	+0.129
	2	1881	− 8558	6.580	2.181	−1.086	−0.603	0.242	−0.202
	3	679	− 2997	2.113	0.614	−0.563	−0.379	0.142	−0.147
	4	240	− 1052	0.684	0.187	−0.263	−0.158	0.059	−0.078
	5	82	− 353	0.224	0.059	−0.110	−0.048	0.019	−0.035
	6	27	− 119	0.074	0.019	−0.043	−0.004	+0.002	−0.015
	7	9	− 42	0.024	0.007	−0.017	+0.007	−0.002	−0.006
	8	3	− 14	0.009	0.002	−0.007	+0.002	−0.001	−0.003
	9	1	− 5	0.003					
	10		− 2	0.001					
1	−10	− 1							
	− 9	− 4		0.001	0.001	0.011	−0.015	−0.005	−0.003
	− 8	− 9	+ 2	0.003	0.002	0.032	−0.027	−0.010	−0.009
	− 7	− 20	12	0.007	0.005	0.041	−0.070	−0.024	−0.011
	− 6	− 47	44	0.016	0.013	0.053	−0.211	−0.069	−0.007
	− 5	− 96	136	+0.026	0.042	0.119	−0.553	−0.167	0.000
	− 4	− 110	+ 359	− 0.096	+0.201	+0.393	−1.483	−0.398	+0.034
	− 3	− 146	− 26	—	—	—	—	—	—
	− 2	− 277	− 1023	+ 3.621	− 4.605	−0.889	+0.246	−0.374	−0.233
	− 1	+6839	+ 4474	+ 3.039	− 3.354	+3.940	−5.453	−1.927	−1.439
	0	− 803	− 972	− 0.132	+0.423	−3.981	+2.512	+0.501	−0.327
	1	− 63	− 377	+ 0.138	0.017	+3.707	−0.541	+0.169	+1.063
	2	− 17	− 144	0.056	0.002	+0.022	+0.034	−0.005	−0.008
	3	− 8	− 55	0.021		−0.088	0.049	−0.015	−0.033
	4	− 4	− 20	0.008		−0.034	0.030	−0.010	−0.013
	5	− 2	− 6	0.003		−0.006	0.016	−0.005	−0.003
	6	− 1	− 2	0.001		+0.002	0.008	−0.003	
	7		− 1			0.001	0.004	−0.001	
2	−14	+ 1							
	−13	1			−0.001				
	−12	3	+ 1		−0.002				
	−11	10	2	0.001	−0.007				
	−10	29	6	0.001	−0.022	−0.020	−0.037	−0.011	0.007
	− 9	88	15	0.002	−0.069	−0.008	−0.090	−0.028	0.007
	− 8	269	44	0.008	−0.209	−0.045	−0.157	−0.045	0.022

Table 12 (continued)

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i	i'	$10^8 \log r'$ sin	$10^8 \log r'$ cos	$\delta\pi'$ sin	$\delta\pi'$ cos	$\delta\Omega'$ sin	$\delta\Omega'$ cos	$\delta i'$ sin	$\delta i'$ cos
				″	″	″	″	″	″
2	— 7	695	130	0.008	—0.602	—0.330	—0.492	—0.118	+0.125
	— 6	+ 301	+ 38	—	—	—	—	—	—
	— 5	—3013	—181	0.188	—1.245	+0.558	+1.370	+0.219	—0.223
	— 4	+2418	—568	0.392	+1.739	0.202	0.978	0.115	—0.075
	— 3	2466	—583	0.349	1.438	0.098	0.905	0.096	—0.035
	— 2	1516	—385	0.241	0.765	0.185	+0.750	+0.070	—0.078
	— 1	506	— 13	0.127	0.147	+0.089	—0.107	—0.145	—0.107
	0	69	— 75	0.019	0.079	—0.382	+0.501	—0.105	—0.087
	1	19	— 28	0.011	0.021	+0.173	0.115	—0.042	+0.045
	2	4	— 9	0.004	0.006	0.06	0.050	—0.019	0.003
	3	1	— 3	+0.002	0.002	0.011	0.016	—0.006	0.003
	4	1			+0.001	0.008	+0.003	—0.001	0.003
	5					0.004	—0.001		0.001
	6					+0.002	—0.001		0.001
3	—16	1							
	—15	2			—0.001				
	—14	4			—0.003				
	—13	13		—0.001	—0.008				
	—12	38	+ 2	—0.002	—0.024				
	—11	101	+ 8	—0.012	—0.066	—0.109	—0.009		0.033
	—10	245	— 52	—0.095	—0.187	—0.258	—0.181	—0.041	+0.086
	— 9	+ 88	— 55	—	—	—	—	—	—
	— 8	— 775	+501	—0.158	—0.307	+0.574	+0.094	—0.020	—0.150
	— 7	+ 602	—654	+0.489	+0.454	0.441	0.136	—0.012	—0.097
	— 6	613	—765	0.467	0.380	0.359	0.181	+0.003	—0.073
	— 5	441	—668	0.365	0.247	0.342	0.217	0.010	—0.053
	— 4	268	—496	0.251	0.142	0.321	0.223	+0.005	—0.046
	— 3	121	—264	0.130	0.064	0.236	0.172	—0.022	—0.031
	— 2	+ 21	— 46	0.030	+0.014	0.023	0.140	—0.054	+0.020
	— 1	0	— 8	0.011	—0.001	0.088	0.160	—0.067	0.023
	0	— 6	— 3	0.002		0.015	0.050	—0.019	0.008
	1	— 2	0	0.001		0.027	+0.004	—0.003	0.009
	2	— 1	0			0.008	—0.004	+0.001	0.003
	3	0	+ 1			0.003	—0.004	0.001	0.001
	4	+ 1				0.001	—0.002	0.001	
	5						—0.001		
4	—17	1	— 1		—0.001				
	—16	2	— 2	—0.001	—0.001				
	—15	7	— 5	—0.004	—0.004				
	—14	20	— 16	—0.011	—0.013				
	—13	54	— 45	—0.032	—0.037				
	—12	+ 66	— 49	—	—	—	—	—	—
	—11	— 355	+305	—0.199	—0.237	0.353	—0.174	—0.074	—0.091
	—10	— 52	— 97	+0.092	—0.037	0.137	—0.001	—0.008	—0.031
	— 9	— 48	—258	0.159	—0.027	0.120	—0.087	—0.027	—0.019
	— 8	— 43	—268	0.149	—0.023	0.172	—0.090	—0.028	—0.019
	— 7	— 59	—237	0.123	—0.029	0.180	—0.056	—0.023	—0.022
	— 6	— 68	—190	0.094	—0.032	0.186	—0.050	—0.018	—0.018
	— 5	— 57	—124	0.060	—0.026	0.171	—0.037	—0.019	—0.007
	— 4	— 30	— 51	0.026	—0.013	0.118	—0.006	—0.018	+0.012
	— 3	— 3	— 1	0.003	—0.002	0.058	+0.043	—0.008	0.033
	— 2	— 3	+ 5		—0.001	0.074	0.018	—0.008	0.028
	— 1	— 1	2		—0.001	0.017	+0.003	—0.002	0.007
	0	0	1			+0.001	—0.006	+0.002	0.001
	1	+ 1	1			0.000	—0.005	0.002	
	2	0	+ 1			—0.001	—0.002	+0.001	
	3	+ 1				—0.001	—0.001		
5	—17	— 1	— 2						
	—16	— 4	— 6	0.002	—0.002				
	—15	— 21	— 32	+0.144	—0.112				
	—14	+ 196	+296	—0.252	+0.175				
	—13	— 89	—121	+0.077	—0.059	—0.060	—0.158	—0.042	+0.024

Table 12 (continued)

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i	i'	$10^8 \log r'$		$\delta w'$		$\delta\Omega'$		$\delta i'$	
		sin	cos	sin	cos	sin	cos	sin	cos
				″	″	″	″	″	″
5	−12	−40	−156	0.088	−0.022	+0.036	−0.115	−0.034	− 0.002
	−11	−27	− 86	0.048	−0.018	0.053	−0.031	−0.008	−0.011
	−10	−62	− 53	0.029	−0.032	0.009	−0.067	−0.013	+0.002
	− 9	−72	− 44	0.023	−0.036	0.027	−0.102	−0.014	0.005
	− 8	−70	− 31	0.016	−0.033	0.045	−0.087	−0.015	0.003
	− 7	−58	− 19	0.009	−0.027	0.044	−0.082	−0.011	0.004
	− 6	−37	− 8	0 004	−0.017	0.043	−0.068	−0.005	0.009
	− 5	−12	− 1	+0.001	−0.006	0.039	−0.035	+0.004	0.014
	− 4	+ 2	+ 2	0.000		0.038	−0.013	0.011	0.014
	− 3	2	3	−0.001		0.022	−0.018	0.007	0.009
	− 2	1	+ 1			+0.002	−0.002	0.001	+0.001
	− 1		0			- -0.004	−0.001	0.001	−0.001
	0		0			−0.003	−0.001		−0.001
	1		− 1			−0.001			−0.001
6	−19	− 1			+0.001				
	−18	− 1			0.002				
	−17	+ 9			+0.005				
	−16	− 4	− 1	+0.001	−0.007				
	−15	−15	0	0.001	−0.014				
	−14	−50	0	0.003	−0.027	−0.076	− 0.028	−0.004	+0.022
	−13	−53	− 23	0.011	−0.026	−0.046	−0.042	−0.006	+0.017
	−12	−30	− 14	+0.006	−0.016	+ 0.010	−0.020	−0.002	−0.001
	−11	−27	+ 9	−0.004	−0.013	−0.020	−0.029	−0.004	+0.004
	−10	−28	13	−0.006	−0.014	−0.034	−0.056	−0.002	0.007
	− 9	−23	14	−0.006	−0.011	−0.016.	−0.050	−0.002	0.008
	− 8	−16	12	−0.005	−0.008	−0.013	−0.041	0.000	0.006
	− 7	− 8	7	−0.003	−0.004	−0.007	−0.037	+0.005	0.004
	− 6	0	2	−0.001	−0.001	+0.003	−0.024	0.008	+0.002
	− 5	+ 2			+0.001	+0.006	−0.017	0.009	0.000
	− 4	+ 1				−0.001	−0.009	+0.004	0.000
	− 3					−0.001	+0.001	0.000	0.000
	− 2					−0.002	0.002	−0.001	−0.001
	− 1					−0.001	0.001		
7	−21			+0.002					
	−20			−0.001					
	−19			−0.001					
	−18		1	−0.001					
	−17		2	−0.002					
	−16		6	−0.005					
	−15	− 1	24	−0.011	−0.001	−0.028	+0.004	+0.001	+0.004
	−14	−12	19	−0.009	−0.006	−0.023	−0.019	−0.003	+0.004
	−13	−10	6	−0.003	−0.005	0.000	−0.016	−0.003	−0.001
	−12	− 2	10	−0.005	−0.001	−0.020	+0.001	+0.001	+0.003
	−11	− 1	13	−0.006	−0.001	−0.039	−0.008	0.002	0.002
	−10	− 1	10	−0.005	−0.001	−0.024	−0.014	0.004	0.002
	− 9	+ 1	7	−0.003		−0.017	−0.010	0.004	+0.002
	− 8	1	+ 3	−0.001		−0.014	−0.011	0.004	−0.002
	− 7	1	0			−0.006	−0.009	0.003	−0.003
	− 6		− 1			−0.003	−0.005	0.002	−0.003
	− 5					−0.002	0.000	+0.001	−0.001
	− 4					+0.001	+0.002	−0.001	
	− 3						0.002	−0.001	
	− 2						0.001		
8	−18	1		−0.001	+0.001				
	−17	2	+ 2	−0.001	0.001				
	−16	9	8	−0.004	0.004	−0.010	0.018	+0.003	+0.001
	−15	+ 4	11	−0.005	+0.001	−0.021	+0.005	+0.001	0.003
	−14	− 3	7	−0.003	−0.001	−0.012	−0.012	−0.002	0.002
	−13	+ 2	4	−0.002	+0.001	−0.007	+0.008	+0.001	+0.001
	−12	5	3	−0.002	0.002	−0.017	0.016	0.002	−0.002
	−11	2	1	−0.001	0.001	−0.016	0.009		−0.003
	−10	1	+ 1		0.001	−0.010	0.005		−0.002

Table 12 (continued)

i	i'	$10^6 \log r'$		$\delta\varpi'$		$\delta\Omega'$		$\delta i'$	
		sin	cos	sin	cos	sin	cos	sin	cos
						"	"		"
8	— 9	+1	0			—0.009	+0.002		—0.003
	— 8	0	—1			—0.006	—0.001		—0.003
	— 7	—1	—1			—0.003	—0.001		—0.002
	— 6		—1		"	—0.001			—0.001
9	—18	+2	—1	"	0.001			"	
	—17	7	—3	+0.001	0.003	+0.006	+0.015	0.002	—0.001
	—16	8	—3	0.001	0.003	0.008	0.018	0.003	—0.001
	—15	2	—1		0.001	+0.001	0.001	0.000	
	—14	2	+1		0.001	—0.002	0.004	0.000	
	—13	3			0.001	0.000	0.010	0.002	
	—12	2			+0.001	+0.001	0.004	0.001	
	—11	1				0.000	0.002		
	—10					—0.002	0.001		
10	—15	1				+0.001	0.003	0.001	
	—14					0.001	0.003	0.001	

CHAPTER V

PERTURBATIONS OF PLUTO BY NEPTUNE

In Part One, the calculations of the perturbations of Pluto by Neptune were based on the XIX system of elements of Pluto according to Bauer, and as the method of determining perturbations of rectangular coordinates we used Cowell's method, integration of the equations of motion. /115

Recalculating the perturbations of Pluto by Neptune, we use the system of elements of Pluto obtained after correction with consideration of second-order perturbations of the coordinates, and we will discuss the method of calculating perturbations in the elements.

The main formulas are given below in the form in which they were used.

The differential equations determining the osculating elements are as follows:

$$
\begin{aligned}
w\delta n' &= w^2 \frac{d^2 n'}{dt} = \frac{1}{10}[(5)S' + (6)T'],\\
\delta e' &= w\frac{de'}{dt} = (7)S' + (8)T',\\
\delta\Omega' &= w\frac{d\Omega}{dt} = (9)W',\\
\delta i' &= w\frac{di'}{dt} = (11)W',\\
\delta\pi' &= w\frac{d\pi'}{dt} = (1)S' + (2)T' + (10)W'\\
\delta\varepsilon' &= w\frac{d\varepsilon'}{dt} = [(3) + (12)]S' + (4)T' + (10)W',
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
(1) &= -p'\cos v' : \sin\varphi',\\
(2) &= (p' + r')\sin v' : \sin\varphi',\\
(3) &= -p'\cos v'\tan\frac{\varphi'}{2},\\
(4) &= (p' + r')\sin v'\tan\frac{\varphi'}{2},\\
(5) &= -g'\sin\varphi'\sin v',\\
(6) &= -g'p' : r',
\end{aligned}
\qquad
\begin{aligned}
(7) &= a'\cos^2\varphi'\sin v',\\
(8) &= a'\cos^2\varphi'(\cos v' + \cos E'),\\
(9) &= r'\sin u' : \sin i',\\
(10) &= r'\sin u'\tan\frac{i'}{2},\\
(11) &= r'\cos u',\\
(12) &= -2r'\cos\varphi',
\end{aligned}
$$

wherein /116

$$
\begin{aligned}
u' &= \omega' + v',\\
g' &= 30\frac{kw}{\sqrt{a'}},
\end{aligned}
$$

The components of the perturbing acceleration are found as

$$
\begin{aligned}
S' &= NK\xi_N - N\Delta^{-3}r',\\
T' &= NK\eta_N,\\
W' &= NK\zeta_N,
\end{aligned}
$$

where

$$K=\frac{1}{\Delta^3}-\frac{1}{r^3}_{\mathrm{N}},$$

$$N=\frac{\omega k''}{\sqrt{p'}}m_{\mathrm{N}}$$

and the coordinates of Neptune ξ_{N}, η_{N}, ζ_{N}, in respect to a movable system in the orbit plane of Pluto, are determined in relation to rectangular equatorial coordinates of Neptune:

$$\begin{Bmatrix}\xi_{\mathrm{N}}\\ \eta_{\mathrm{N}}\\ \zeta_{\mathrm{N}}\end{Bmatrix}=\begin{Bmatrix}x_{\mathrm{N}}\\ y_{\mathrm{N}}\\ z_{\mathrm{N}}\end{Bmatrix}\begin{Bmatrix}P_x & Q_x & R_x\\ P_y & Q_y & R_y\\ P_z & Q_z & R_z\end{Bmatrix}\begin{Bmatrix}\cos v' & -\sin v' & 0\\ \sin v' & \cos v' & 0\\ 0 & 0 & 0\end{Bmatrix}$$

To obtain x_{N}, y_{N}, z_{N}, and also r_{N}^{-3} from the calculated value of r_{N}^2, we use Comry's tables.

Let us turn to the method of quadratures.

The equations subject to integration have the form:

$$\frac{dx}{dt}=f(x,\ t),\quad \frac{d^2y}{dt^2}=f(y,\ t);$$

$$\text{for } t=t_0,\ x=0,\ y=0.$$

For the initial instant of integration we take 1930 Sept. 21.0. The interval of integration w = 320^{d}.

The first columns of the sums, and the unknown values, were determined by the formulas:

$$f^{-1}_{-\frac{1}{2}}=-\frac{1}{2}f_0+\frac{1}{12}f_0^1-\frac{11}{720}f_0^3+\cdots,$$

$$f_0^{-2}=-\frac{1}{12}f_0+\frac{1}{240}f_0^2-\cdots,$$

$$x_n=f_n^{-1}-\frac{1}{12}f_n^1+\frac{11}{720}f_n^3-\cdots,$$

$$y_n=f_n^{-2}+\frac{1}{12}f_n-\frac{1}{240}f_n^2+\cdots,$$

wherein the second difference no longer affects the result.

Having performed integration of all Equations (1), we obtain the perturbations of elements $w\delta n'$, $\delta\varepsilon'$, $\delta\pi'$, $\delta i'$, $\delta\Omega'$, $\delta e'$. Integrating the equation $w\frac{dn'}{dt} = w^2\frac{d^2\delta'\lambda'}{dt^2} = f(y, t)$, we find the quantity $\delta'\lambda$. Further, we calculate

$$\delta \log a' = -\frac{2}{3}\frac{w\delta n'}{wn},$$
$$\delta\lambda' = \delta\varepsilon' + \delta'\lambda'$$

and finally,

$$\delta M' = \delta\lambda' - \delta\pi'.$$

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The change to perturbations of coordinates is accomplished by the formulas

$$\delta w' = \delta\lambda' + \sum jH_j \cos jM'\delta M' + \sum \frac{\partial H_j}{\partial e'} \sin jM'\delta e',$$
$$\delta \log r' = \delta \log a' + \sum -jK_j \sin jM'\delta M' + \sum \frac{\partial K_j}{\partial e'} \cos jM'\delta e'.$$

Table 13 gives perturbations of the elements of Pluto due to Neptune for the period from 1913 to 1980 every 320 days.

Table 13

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*	Date	$10^3 \log a'$	$10^3 a'$	$\delta x'$	$\delta v'$	$\delta \Omega'$	$\delta i'$	$10^3 \log r'$	$r a'$
−20	1913 March 13.5	−50417	−45100	−24.831	+ 53.661	−0.418	+0.596	− 5859	−0.295
−19	1914 Jan. 27.5	−52591	−43245	−32.453	49.520	−0.385	0.570	− 5252	−0.168
−18	1914 Dec. 13.5	−48859	−41338	−30.121	45.532	−0.350	0.541	− 4684	−0.136
−17	1915 Oct. 29.5	−45225	−39377	−27.891	41.700	−0.314	0.509	− 4151	−0.109
−16	1916 Sept. 13.5	−41695	−37365	−25.740	38.026	−0.278	0.474	− 3656	−0.085
−15	1917 Aug. 30.5	−38271	−35301	−23.667	34.509	−0.242	0.439	− 3195	−0.064
−14	1918 June 15.5	−34955	−33188	−21.667	31.151	−0.208	0.402	− 2768	−0.047
−13	1919 May 1.5	−31751	−31028	−19.740	27.951	−0.176	0.365	− 2.74	−0.033
−12	1920 March 16.5	−28658	−28823	−17.880	24.909	−0.146	0.328	− 2013	−0.021
−11	1921 Jan. 30.5	−25678	−26576	−16.087	22.024	−0.119	0.292	− 1685	−0.012
−10	1921 Dec. 16.5	−22811	−24288	−14.357	19.294	−0.095	0.256	− 1387	−0.005
− 9	1922 Nov. 1.5	−20055	−21964	−12.688	16.718	−0.074	0.222	− 1119	0.000
− 8	1923 Sept. 17.5	−17410	−19606	−11.076	14.292	−0.056	0.189	− 881	+0.003
− 7	1924 Aug. 2.5	−14873	−17219	− 9.520	12.016	−0.040	0.158	− 673	0.004
− 6	1925 June 19	−12444	−14866	− 8.017	9.885	−0.028	0.129	− 493	0.005
− 5	1926 May 5	−10120	−12371	− 6.565	7.897	−0.018	0.101	− 342	0.005
− 4	1927 March 21	− 7898	− 9917	− 5.162	6.049	−0.011	0.076	− 219	0.004
− 3	1928 Feb. 4	− 5778	− 7450	− 3.806	4.338	−0.006	0.054	− 124	0.002
− 2	1928 Dec. 20	− 3756	− 4972	− 2.495	2.762	−0.002	0.033	− 56	0.001
− 1	1929 Nov. 5	− 1830	− 2487	− 1.227	+ 1.317	0.000	+0.015	− 14	0.000
0	1930 Sept. 21	0	0	0.000	0.000	0.000	0.000	0	0.000
1	1931 Aug. 7	+ 1737	+ 2487	+ 1.187	− 1.191	0.000	−0.013	− 13	0.000
2	1932 June 22	3383	4968	2.336	− 2.259	−0.002	−0.023	− 52	0.003
3	1933 May 8	4939	7443	3.448	− 3.207	−0.003	−0.031	− 118	0.006
4	1934 March 24	6408	9907	4.526	− 4.038	−0.004	−0.037	− 210	0.012
5	1935 Feb. 7	7792	12356	5.570	− 4.755	−0.005	−0.040	− 328	0.020
6	1935 Dec. 24	9094	14790	6.583	− 5.362	−0.005	−0.040	− 473	0.031
7	1936 Nov. 8	10316	17204	7.566	− 5.862	−0.005	−0.039	− 643	0.045
8	1937 Sept. 24	11460	19594	8.519	− 6.258	−0.003	−0.035	− 840	0.063
9	1938 Aug. 10	12530	21960	9.445	− 6.555	+ 0.001	−0.029	− 1063	0.085
10	1939 June 26	13526	24297	10.345	− 6.754	0.006	−0.022	− 1312	0.112
11	1940 May 11	14453	26604	11.219	− 6.861	0.013	−0.012	− 1586	0.144
12	1941 March 27	15311	28876	12.069	− 6.877	0.023	−0.001	− 1888	0.182
13	1942 Feb. 10	16103	31114	12.896	− 6.806	0.034	0.012	− 2215	0.226
14	1942 Dec. 27	16832	33314	13.701	− 6.652	0.049	0.026	− 2567	0.277
15	1943 Nov. 12	17498	35474	14.484	− 6.416	0.065	0.041	− 2945	0.336
16	1944 Sept. 27	18103	37592	15.248	− 6.103	0.085	0.058	− 3350	0.404
17	1945 Aug. 13	18649	39666	15.992	− 5.715	0.107	0.075	− 3780	0.480
18	1946 June 29	19138	41696	16.718	− 5.254	0.132	0.094	− 4236	0.566
19	1947 May 15	19573	43679	17.427	− 4.724	0.160	0.113	− 4717	0.663
20	1948 March 30	19954	45614	18.120	− 4.128	0.191	0.132	− 5224	0.771
21	1949 Feb. 13	20285	47501	18.798	− 3.469	0.225	0.153	− 5757	0.891
22	1949 Dec. 30	20567	49338	19.461	− 2.750	0.262	0.173	− 6314	1.024
23	1950 Nov. 15	20803	51126	20.110	− 1.974	0.301	0.194	− 6897	1.170
24	1951 Oct. 1	20995	52863	20.747	− 1.144	0.343	0.215	− 7504	1.332
25	1952 Aug. 16	21146	54548	21.372	− 0.263	0.387	0.236	− 8137	1.509
26	1953 July 2	21257	56181	21.985	0.666	0.434	0.257	− 8794	1.703
27	1954 May 18	21331	57762	22.588	1.639	0.482	0.277	− 9475	1.913
28	1955 April 3	21370	59290	23.181	2.655	0.533	0.297	−10180	2.143
29	1956 Feb. 17	21375	60766	23.764	3.710	0.586	0.317	−10910	2.392
30	1957 Jan. 2	21349	62188	24.338	4.803	0.639	0.336	−11662	2.662
31	1957 Nov. 18	21294	63559	24.905	5.931	0.694	0.354	−12436	2.953
32	1958 Oct. 4	21211	64878	25.464	7.092	0.750	0.372	−13234	3.266
33	1959 Aug. 20	21102	66145	26.016	8.282	0.808	0.389	−14052	3.604
34	1960 July 5	20971	67361	26.563	9.501	0.865	0.405	−14892	3.966
35	1961 May 21	20818	68527	27.103	10.746	0.923	0.421	−15751	4.354
36	1962 April 6	20648	69645	27.639	12.014	0.981	0.435	−16629	4.769
37	1963 Feb. 20	20462	70715	28.171	13.304	1.038	0.449	−17525	5.212
38	1964 Jan. 6	20263	71739	28.698	14.614	1.095	0.461	−18438	5.684
39	1964 Nov. 21	20053	72718	29.222	15.941	1.151	0.473	−19366	6.186
40	1965 Oct. 7	19837	73653	29.744	17.283	1.206	0.483	−20307	6.720
41	1966 Aug. 23	19615	74547	30.262	18.640	1.259	0.493	−21261	7.285
42	1967 Oct. 9	19392	75401	30.779	20.008	1.310	0.502	−22226	7.884

*Number of integration step.

Table 13 (continued)

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*	Date	10^6 log a'	10^4 e'	δL'	δπ'	δΩ'	δi'	10^4 log r'	δw'
43	1968 May 24	19168	76215	31.293	21.387	1.360	0.509	−23199	8.517
44	1969 April 9	18946	76992	31.806	22.777	1.408	0.516	−24179	9.185
45	1970 Feb. 23	18730	77733	32.318	24.174	1.453	0.522	−25163	9.888
46	1971 Jan. 9	18520	78441	32.830	25.581	1.495	0.527	−26149	10.627
47	1971 Nov. 25	18320	79117	33.340	26.994	1.535	0.532	−27135	11.403
48	1972 Oct. 10	18132	79762	33.851	28.414	1.572	0.535	−28117	12.216
49	1973 Aug. 26	17957	80379	34.361	29.841	1.606	0.538	−29094	13.067
50	1974 July 12	17800	80970	34.873	31.274	1.637	0.540	−30062	13.956
51	1975 May 28	17662	81537	35.385	32.712	1.666	0.542	−31019	14.882
52	1976 April 12	17547	82081	35.898	34.158	1.691	0.544	−31961	15.845
53	1977 Feb. 26	17456	82605	36.413	35.609	1.713	0.544	−32887	16.848
54	1978 Jan. 12	17393	83112	36.929	37.066	1.731	0.545	−33792	17.886
55	1978 Nov. 28	17359	83602	37.447	38.530	1.747	0.546	−34674	18.961
56	1979 Oct. 14	17357	84076	37.966	40.002	1.760	0.546	−35529	20.071
57	1980 Aug. 29	17388	84536	38.487	41.482	1.770	0.546	−36354	21.216

*Number of integration step.

$P^{0,8}_{0,0}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$	$10^{11} D^8$
−9	+ 64361.314270	−3297.725342	−735.451538	+38.222601	+ 3.131965	−1449110	−660536	+ 1898872	+ 678168
−8	24085.637153	−1577.102973	−339.891412	23.135932	1.775940	−1117079	−476074	1898872	678168
−7	7794.345954	− 675.990099	−138.107354	12.949164	0.891947	− 824110	−313314	1898872	678168
−6	2067.234375	− 249.230469	− 46.447021	6.490424	0.362800	− 570204	−172255	1898872	678168
−5	409.743415	− 73.522000	− 11.370049	2.744086	+ 0.086934	− 355360	−052897	1898872	678168
−4	50.695746	− 14.916775	− 1.323052	0.850776	−0.021586	− 179579	+ 44759	1898872	678168
−3	2.686340	− 1.321045	+ 0.135376	+ 0.107368	−0.033074	− 42860	120714	1898872	678168
−2	+ 0.129340	+ 0.004774	− 0.040337	− 0.033013	−0.002218	+ 54796	174967	1898872	678168
−1	− 0.038812	− 0.053901	− 0.007745	+ 0.039008	+ 0.031921	113389	207520	1898872	678168
0	0	+ 0.034180	+ 0.088623	0.089057	0.045905	132921	218370	1898872	678168
1	− 0.038812	− 0.053901	− 0.007745	+ 0.039008	+ 0.031921	113389	207520	1898872	678168
2	+ 0.129340	+ 0.004774	− 0.040337	− 0.033013	−0.002218	+ 54796	174967	1898872	678168
3	2.686340	− 1.321045	+ 0.135376	+ 0.107368	−0.033074	− 42860	120714	1898872	678168
4	50.695746	− 14.916775	− 1.323052	0.850776	−0.021586	− 179579	+ 44759	1898872	678168
5	409.743415	− 73.522000	− 11.370049	2.744086	+ 0.086934	− 355360	− 52897	1898872	678168
6	2067.234375	− 249.230469	− 46.447021	6.490424	0.362800	− 570204	−172255	1898872	678168
7	7794.345954	− 675.990099	−138.107354	12.949164	0.891947	− 824110	−313314	1898872	678168
8	24085.637153	−1577.102973	−339.891412	23.135932	1.775940	−1117079	−476074	1898872	678168
9	64361.314270	−3297.725342	−735.451538	38.222601	3.131965	−1449110	−660536	1898872	678168

$P^{0,8}_{0,2}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$	$10^{11} D^8$
−9	− 14798.004026	+ 2849.675722	− 40.311914	−24.390695	+ 1.360346	+ 550727	−501844	− 217013	+ 542534
−8	− 4540.143750	1063.817969	− 29.230230	−11.140820	0.851286	275391	−387369	0	542534
−7	− 1108.112250	330.539122	− 16.588031	− 4.237625	0.483122	+ 93804	−281575	+ 217013	542534
−6	− 189.169097	77.354644	− 7.229232	− 1.151812	0.234370	− 9657	−184161	434027	542534
−5	− 16.488964	10.707080	− 2.112326	− 0.104085	0.083545	− 50618	− 96028	651041	542534
−4	− 0.221528	0.236892	− 0.216645	+ 0.059852	+ 0.009163	− 44705	− 16276	868055	542534
−3	− 0.113856	0.053293	+ 0.051400	− 0.005702	−0.010259	− 7541	+ 54796	1085069	542534
−2	+ 0.054688	+ 0.005078	− 0.047938	− 0.021452	+ 0.003792	+ 45247	117187	1302083	542534
−1	− 0.024946	− 0.048541	+ 0.006011	+ 0.041900	0.029834	98036	170898	1519097	542534
0	− 0.001389	+ 0.032899	0.088086	0.088650	0.046381	135200	215928	1736111	542534
1	− 0.235254	− 0.058057	0.021039	+ 0.023096	+ 0.031949	141113	252278	1953125	542534
2	− 5.984027	+ 0.194054	0.669292	− 0.125467	−0.034945	+ 100152	279947	2170138	542534
3	− 74.649468	− 1.601557	5.532682	− 0.077740	−0.175787	− 3309	298936	2387152	542534
4	− 502.337500	− 26.471093	22.840212	+ 0.820573	−0.412061	− 184896	309244	2604166	542534
5	− 2301.980610	− 148.805344	68.143804	3.723768	−0.765251	− 460232	310872	2821180	542534
6	− 8181.272743	− 557.088845	167.412044	10.411143	−1.256841	− 844944	303819	3038194	542534
7	− 24284.666699	−1647.629053	360.623937	23.411995	−1.908317	−1354655	288085	3255208	542534
8	− 63019.684028	−4158.285503	705.862652	46.130620	−2.741162	−2004991	263671	3472222	542534
9	−147343.787424	−9349.198985	1283.909277	82.971316	−3.776861	−2811578	230577	3689236	542534

$\varkappa^{9,8}_{0,4}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$	$10^{11} D^8$
−9	+1588.590156	−600.168018	+79.698204	− 2.382058	−0.428605	+413357	−53168	−759548	+271267
−8	348.371181	−165.287283	28.457281	− 1.432302	−0.168487	245009	−60221	−542534	271267
−7	47.834155	− 31.312516	7.691411	− 0.686865	−0.040584	123535	−58593	−325520	271267
−6	+ 1.835330	− 2.449805	1.155317	− 0.224523	+0.005233	43728	−48285	−108506	271267
−5	− 0.092418	+ 0.178098	− 0.047325	− 0.019884	+0.008678	+ 380	−29296	+108506	271267
−4	+ 0.054688	− 0.072396	+ 0.004872	+ 0.014941	−0.000952	− 11719	− 1627	325520	271267
−3	− 0.016908	+ 0.030127	+ 0.019754	− 0.011323	−0.004777	+ 2224	+34722	542534	271267
−2	+ 0.021181	− 0.012088	− 0.039627	− 0.010786	+0.005667	3701	79752	759548	271267
−1	− 0.023950	− 0.030127	+ 0.013742	+ 0.041943	0.028426	87402	133463	976562	271267
0	+ 0.177778	+ 0.267422	+ 0.059581	+ 0.068088	0.051131	148220	195855	1193576	271267
1	4.961195	2.390305	− 0.360932	− 0.156961	+0.050996	214247	266927	1410590	271267
2	54.835156	20.062434	− 1.576410	− 1.045312	−0.005184	280273	346679	1627604	271267
3	341.589309	105.017725	− 2.816509	− 3.238243	−0.161027	341092	435112	1844618	271267
4	1488.588368	396.695920	− 0.743175	− 7.647862	−0.470570	391493	532226	2061631	271267
5	5106.503882	1201.578434	+12.518104	− 15.498779	−0.998267	426270	638020	2278645	271267
6	14762.733507	3109.584180	51.847048	− 28.369770	−1.818986	440213	752495	2495659	271267
7	37530.757777	7154.465403	142.097336	− 48.235444	−3.018014	428114	875651	2712673	271267
8	86313.684375	15025.203516	321.565354	− 77.507910	−4.691056	384766	1007486	2929687	271267
9	183164.229902	29335.404932	645.958946	−119.078445	−6.944230	304959	1148003	3146701	271267

$P^{9,8}_{0,6}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$	$10^{11} D^8$
−9	− 60.148953	+ 39.065450	− 10.683523	+ 1.528879	−0.107058	+ 4937	+49370	−341021	+77504
−8	− 6.239186	5.793936	− 2.237181	0.454102	−0.047597	13672	22786	−248015	77504
−7	+ 0.111311	+ 0.099939	− 0.156939	+ 0.069754	−0.014448	11990	+ 4882	−155009	77504
−6	0.023438	− 0.050167	+ 0.035649	− 0.008930	−0.000450	+ 5100	− 4340	− 62003	77504
−5	+ 0.002987	+ 0.007908	− 0.009284	− 0.000049	+0.001557	− 1790	− 4882	+ 31001	77504
−4	− 0.004663	− 0.009462	+ 0.007828	+ 0.003299	−0.001264	− 3472	+ 3255	124007	77504
−3	+ 0.002637	+ 0.020347	+ 0.000233	− 0.008653	−0.001752	+ 5262	20073	217013	77504
−2	+ 0.013492	− 0.019624	− 0.035488	− 0.004004	+0.007254	29622	45572	310019	77504
−1	− 0.004844	− 0.131753	− 0.050668	+ 0.049148	+0.032916	74816	79752	403025	77504
0	− 1.301786	− 1.849665	− 0.475809	0.141037	0.082395	146050	122612	496031	77504
1	− 17.976226	− 14.884070	− 3.251829	+ 0.178564	0.162853	248535	174153	589037	77504
2	− 116.296850	− 71.768594	− 13.361312	− 0.056369	0.281451	387478	234375	682043	77504
3	− 506.959947	− 253.505615	− 40.859760	− 0.948530	0.445350	568088	303276	775049	77504
4	− 1723.534722	− 729.212091	− 103.406842	− 3.091016	0.661713	795573	380859	868055	77504
5	− 4928.346111	− 1812.765398	− 229.297641	− 7.326926	0.937701	1075141	467122	961061	77504
6	− 12395.045089	− 4039.449163	− 460 993908	−14.791026	1.280474	1412001	562065	1054067	77504
7	− 28225.116488	− 8265.599096	− 859.155312	−26.951416	1.697195	1811361	665690	1147073	77504
8	− 59363.574306	−15794.248822	−1507.170688	−45.651193	2.195025	2278429	777991	1240079	77504
9	−116987.094517	−28529.775718	−2516.189286	−73.150124	2.781125	2818414	898980	1333085	77504

APPENDIX I. (continued)

$P^{0,8}_{0,8}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$	$10^{11} D^8$
−9	+0.445320	−0.487970	+0.240227	−0.069208	+0.012720	−15218	+11528	−50378	+9688
−8	0.003906	−0.010617	+0.011444	−0.006519	+0.002175	− 4394	5269	−34877	9688
−7	+0.000760	−0.000977	−0.000322	+0.000643	−0.000192	− 81	+1220	−19376	9688
−6	−0.000570	+0.001285	+0.000094	− 0.000524	+0.000086	+326	− 678	− 3875	9688
−5	+0.000950	−0.001829	−0.000735	+0.000724	+0.000079	− 570	− 406	+11625	9688
−4	−0.002778	+0.002097	+0.003295	−0.000496	−0.000538	− 163	+2034	27126	9688
−3	+0.008780	+0.004652	−0.009057	−0.003860	+0.000513	+4150	6646	42627	9688
−2	−0.018080	−0.054664	−0.008250	+0.017000	0.008116	14974	13427	58128	9688
−1	−0.079786	+0.180318	+0.289163	0.135327	0.029756	34912	22379	73629	9688
0	+1.625397	2.951183	1.748662	0.492071	0.075526	66569	33501	89130	9688
1	14.422622	15.353683	6.270880	1.316724	0.158120	112549	46793	104631	9688
2	68.245101	54.306487	17.369733	2.948153	0.292858	175456	62255	120132	9688
3	2[illegible]8.453113	154.019930	40.875545	5.855433	0.497584	257894	79888	135633	9688
4	687.762388	376.464765	85.763169	10.658683	0.792865	362467	99690	151134	9688
5	1731.656870	825.340909	165.105114	18.149894	1.201794	491780	121663	166635	9688
6	3934.660851	1664.046197	297.149671	29.313766	1.750086	648438	145806	182136	9688
7	8245.845481	3138.145132	506.524038	45.348543	2.466062	835042	172119	197637	9688
8	16182.944655	5602.837635	825.562442	67.686840	3.380647	1054199	200602	213138	9688
9	30075.455274	9555.927787	1295.759270	98.016485	4.527369	1308512	231255	228639	9688

$P^{1,6}_{-1,6}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$
−9	−217.650582	+107.273735	−21.459216	+2.069290	−0.069825	− 39495	+42318	−108507
−8	− 46.944444	29.529860	− 7.633159	0.978580	−0.051272	− 15732	31467	−108507
−7	− 5.716816	5.130742	− 1.915505	0.364320	−0.032066	+ 212	20616	−108507
−6	− 0.093750	+0.221875	− 0.192318	+0.078078	−0.014813	8352	+9765	−108507
−5	− 0.016385	− 0.004353	+0.025338	−0.007749	−0.002116	8680	− 1085	−108507
−4	+0.019444	+0.003818	− 0.023610	+0.030067	+0.003420	+1196	− 11936	−108507
−3	− 0.026367	− 0.029982	+0.024772	+0.015582	−0.000814	− 14105	− 22786	−108507
−2	+0.022222	+0.102258	+0.034420	−0.026301	−0.017416	− 37218	− 33637	−108507
−1	0.067817	− 0.094585	− 0.255740	−0.169856	−0.018991	− 68114	− 44488	−108507
0	0.000000	− 0.506250	− 0.856771	−0.438520	−0.098144	−105880	− 55339	−108507
1	7.705751	+3.403396	− 1.154742	−0.834896	−0.167480	−153428	− 66189	−108507
2	61.829862	26.904860	+0.464280	−1.340756	−0.259602	−207793	− 77040	−108507
3	276.285351	104.815526	6.989224	−1.917035	−0.377116	−269965	− 87891	−108507
4	913.513888	302.812152	23.159026	−2.503841	−0.522624	−339954	− 98741	−108507
5	2493.496200	731.243370	55.837621	−3.020142	−0.698750	−417750	−109592	−108507
6	5945.512500	1561.442187	114.388936	−3.365276	−0.908040	−503362	−12.443	−108507
7	12809.652884	3044.538485	211.051898	−3.415919	−1.153160	−596786	−131293	−108507
8	25495.077777	5532.771527	361.315448	−3.029228	−1.436684	−698127	−142111	−108507
9	47602.028322	9503.302437	584.293523	−2.040244	−1.761233	−807075	−152995	−108507

APPENDIX I. (continued)

$P^{1,6}_{-1,4}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^9 D^7$
−9	+ 5143.828125	− 1394.654819	+103.343555	+ 4.171940	− 0.861650	+ 210285	+ 149740	− 651042
−8	1579.133333	− 524.745833	50.666144	1.450913	− 0.514648	184247	110677	− 651042
−7	367.609375	− 158.765488	21.356574	+ 0.168034	− 0.269206	152994	71615	−651042
−6	51.950000	− 32.983333	7.004687	−0.239194	− 0.109700	116535	+ 32552	−651042
−5	+ 1.286460	− 2.574479	+ 1.325326	−0.208270	−0.020509	74870	− 6510	−651042
−4	− 0.250000	+ 0.377082	− 0.091667	−0.051694	+ 0.013996	+ 27993	− 45573	−651042
−3	+ 0.034375	− 0.114973	− 0.031445	+ 0.043034	+ 0.009440	− 24090	− 84635	−651042
−2	− 0.041667	+ 0.054168	0.095834	+ 0.013412	−0.018555	− 81382	−123698	− 651042
−1	+ 0.028125	0.083724	0.005012	−0.078060	−0.051362	− 143878	−162760	−651042
0	0.000000	+ 0.266667	0.285938	−0.013880	−0.082357	− 211590	−201823	−651042
1	− 7.557291	− 1.010286	2.153450	+ 0.428451	−0.086914	− 284504	−240885	−651042
2	− 88.700000	− 19.266667	7.197396	1.776433	− 0.052410	− 362629	−279948	−651042
3	− 506.546875	− 104.928250	17.132617	4.562565	+0.036782	− 445960	−319010	−651042
4	− 1988.716667	− 373.327083	33.548960	9.474348	0.196289	− 534506	−358073	−651042
5	− 6165.765625	− 1043.701432	57.661264	17.324283	0.441732	− 628255	−397135	−651042
6	− 16222.625000	− 2487.195833	90.059383	29.049866	0.788736	− 727210	−436198	−651042
7	− 37805.038544	− 5283.861069	130.458135	45.713609	1.252931	− 831383	−475260	− 651042
8	− 80223.000000	−10288.654164	177.447395	68.502998	1.849937	− 940756	−514323	−651042
9	−157993.190625	−18706.438415	228.241992	98.730533	2.595378	−1055335	−553385	−651042

$P^{1,6}_{-1,2}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^9 D^7$
−9	−38257.842771	+ 4006.819177	+ 293.237471	−40.402343	−0.527833	+1370116	− 16276	−1627601
−8	−14454.500000	1845.385414	131.242191	−23.797854	−0.206547	1040039	− 48828	−1627601
−7	− 4621.857093	749.779138	48.562339	−12.793946	−0.006998	0718741	− 81380	−1627604
−6	− 1159.062500	254.606770	12.508462	− 6.007814	+ 0.094240	0496421	−113932	−1627601
−5	− 195.131835	64.990396	+ 0.641113	− 2.241140	0.120604	0283200	−146484	−1627601
−4	− 13.791667	+ 9.380210	− 1.101167	− 0.495116	0.095539	+ 0109046	−179036	−1627601
−3	+ 0.426759	− 0.132975	− 0.291828	+ 0.059572	+ 0.042482	−0026038	−211589	−1627601
−2	− 0.156250	− 0.005208	+ 0.138672	+ 0.052734	− 0.015136	−0122069	−244141	−1627601
−1	+ 0.037435	+ 0.073080	− 0.002116	− 0.070312	− 0.053871	−0179038	−276693	−1627601
0	0.000000	− 0.010417	− 0.028646	− 0.051758	−0.050293	−0196940	−309245	−1627601
1	1.043945	− 0.477376	− 0.255371	+ 0.178710	+ 0.019043	−0175780	−341797	−1627601
2	29.458333	− 3.846354	− 3.871746	0.503907	0.177571	−0115558	−374349	−1627601
3	267.915036	− 9.620279	− 18.817220	0.619142	0.448730	−0016277	−406901	−1627601
4	1400.625000	− 1.473961	− 59.656250	+ 0.032226	0.855958	+ 0122068	−439453	−1627601
5	5277.244465	+ 73.058434	− 149.453288	− 1.936525	1.422688	0299180	−472005	−1627604
6	16000.531250	330.395833	− 321.647786	− 6.154298	2.172366	0515949	−504557	−1627601
7	41542.750977	986.222817	− 621.929197	−13.675780	3.128419	0771485	−537109	−1627601
8	95995.833333	2398.302080	−1110.111980	−25.743164	4.314292	1066083	−569661	−1627604
9	202560.278333	5116.786944	−1862.010582	−43.786131	5.753420	1399734	−602214	−1627604

APPENDIX I. (continued)

$\dot{P}^{1,6}_{1,0}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$
−9	−125491.781250	+10530.855477	+ 935.761064	−92.467667	−1.826175	+2510851	+ 65104	−2170139
−8	− 54182.222222	5342.138889	492.717017	−58.986542	−1.087023	2003037	+ 21701	−2170139
−7	− 20800.329858	2474.460505	234.353947	−35.392141	−0.556207	1547308	− 21701	−2170139
−6	− 6824.625000	1015.937500	96.476565	−19.559464	−0.202476	1143664	− 65104	−2170139
−5	− 1793.836805	352.332900	31.639538	− 9.613496	+0.005425	0792100	−108507	−2170139
−4	− 335.444444	94.555556	6.647571	− 3.929250	0.098741	0492621	−151910	−2170139
−3	− 34.218750	16.160156	+ 0.055338	− 1.131728	0.108723	0245227	−195312	−2170139
−2	− 0.763888	+ 0.847222	− 0.332464	− 0.095920	0.066623	+0049913	−238715	−2170139
−1	− 0.059028	− 0.036892	+ 0.038845	+ 0.053169	+0.003689	−0093316	−282118	−2170139
0	0.000000	0.000000	− 0.026042	− 0.059462	−0.048828	−0184460	−325521	−2170139
1	+ 0.059028	+ 0.095920	+ 0.027561	− 0.058811	−0.059679	−0223524	−368924	−2170139
2	0.763888	− 0.465278	+ 0.004340	+ 0.180120	+0.002387	−0210503	−412326	−2170139
3	34.218750	− 4.753906	− 3.541014	0.532336	0.168621	−0145397	−455729	−2170139
4	335.444444	− 10.694444	− 19.803821	+ 0.622828	0.470269	−0028211	−499132	−2170139
5	1793.836805	+ 6.434460	− 66.229382	− 0.173396	0.938585	+0141060	−542535	−2170139
6	6824.625000	121.500000	− 171.013023	− 2.731335	1.604820	0362416	−585938	−2170139
7	20800.329858	497.015189	− 375.600041	− 8.176001	2.500217	0635852	−629340	−2170139
8	54182.222222	1430.638889	− 737.185767	−17.882382	3.656033	0961373	−672743	−2170139
9	125491.781250	3412.675773	−1331.215492	−33.475475	5.103519	1338979	−716145	−2170139

$P^{1,6}_{1,2}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$	$10^{10} D^7$
−9	+ 38257.842771	− 8257.690596	+388.124183	+35.130859	−3.668455	+ 374344	+ 569661	−1627604
−8	14454.500000	− 3652.197914	212.356767	18.728194	−2.451331	362951	472005	−1627604
−7	4621.857093	− 1410.044433	105.710773	8.711916	−1.529134	338199	374349	−1627604
−6	1159.062500	− 447.783853	46.024090	3.214846	−0.862796	301109	276693	−1627604
−5	195.131835	− 104.016764	16.259603	+ 0.682290	−0.413246	250650	179035	−1627604
−4	+ 13.791667	− 12.828125	3.880209	− 0.127932	−0.141437	187174	+ 81380	−1627604
−3	− 0.426759	+ 0.275228	+ 0.223794	− 0.145508	−0.008302	110678	− 16275	−1627604
−2	+ 0.156250	− 0.072917	− 0.121744	+ 0.012370	+ 0.025228	+ 21159	− 113732	−1627604
−1	− 0.037435	− 0.035644	+ 0.056478	+ 0.011016	−0.001790	− 81382	− 211589	−1627604
0	0.000000	− 0.010417	− 0.028646	− 0.051758	−0.050293	− 196910	− 309214	−1627604
1	− 1.043945	− 0.566568	+ 0.210775	+ 0.054362	−0.081217	− 325520	− 406901	−1627604
2	− 29.458333	− 10.882814	2.112630	0.992187	−0.055501	− 467122	− 501557	−1627604
3	− 267.915036	− 79.684733	7.139812	3.707030	+0.065918	− 621743	− 602214	−1627604
4	− 1400.625000	− 348.682289	16.255208	9.456706	0.322102	− 789388	− 699870	−1627604
5	− 5277.244465	− 1128.507282	29.296712	19.811525	0.752118	− 970050	− 797526	−1627604
6	− 16000.531250	− 2997.151042	44.352214	36.654278	1.395018	−1163739	− 895182	−1627604
7	− 41542.750977	− 6920.901528	57.134601	62.180336	2.289875	−1370115	− 992839	−1627604
8	− 95995.833333	−14397.781248	60.356772	98.897460	3.475748	−1590173	−1090495	−1627604
9	−202560.278333	−27623.484534	43.106610	149.625975	4.991696	−1822914	−1188151	−1627604

APPENDIX I. (continued)

$P^{1,6}_{1,4}$

i		D	D^2	D^3	D^4	10^2D^5	10^3D^6	$10\ D^7$
—9	—5143.828125	+1966.191277	—290.057227	+17.683658	+0.110078	— 750645	+384115	—651042
—8	—1579.133333	722.137498	—128.596352	9.752993	—0.094232	— 503257	319010	—651042
—7	— 367.607375	211.281112	— 47.788474	4.770878	—0.059570	— 302734	253006	—651042
—6	— 51.950000	41.641657	— 13.223437	1.924875	—0.070640	— 149085	188802	—651042
—5	— 01.286460	+ 2.831771	— 1.866081	0.527410	—0.053059	— 42320	123698	—651042
—4	+ 0.250000	— 0.439582	+ 0.193750	+ 0.016018	—0.022460	+ 17577	+ 58594	—651042
—3	— 0.034375	+ 0.126431	— 0.008789	— 0.046810	+0.005534	+ 30600	— 6510	—651042
—2	+ 0.041667	— 0.075000	— 0.063542	+ 0.026432	+0.015300	— .3258	— 71615	—651042
—1	— 0.028125	— 0.055598	+ 0.064648	+ 0.048242	—0.008790	— 83982	—136719	—651042
0	0.000000	+ 0.266667	0.285938	— 0.043880	—0.082357	— 211590	—201823	—651042
1	+ 7.557291	8.567578	2.635482	— 0.187435	—0.221028	— 386056	—265927	—651042
2	88.700000	63.616567	13.523436	— 0.194923	—0.440130	— 607421	—332031	—651042
3	506.546875	273.777210	45.984960	+ 0.246159	—0.756184	— 875650	—397135	—651042
4	1988.716667	870.506250	121.930208	1.573308	—1.183919	—1190754	—462240	—651042
5	6165.765625	2276.854558	274.391334	4.349024	—1.739258	—1552735	—527344	—651042
6	16222.625000	5190.966667	549.787500	9.260810	—2.437824	—1961590	—592448	—651042
7	37805.038544	10684.580861	1010.144860	17.121157	—3.295250	—2417317	—657552	—651042
8	80223.000000	20316.529167	1735.376563	28.867575	—4.327151	—2919924	—722656	—651042
9	157993.190625	36261.237373	2825.517777	45.562565	—5.549154	—3469105	—787760	—651042

$P^{1,6}_{1,6}$

i		D	D^2	D^3	D^4	10^2D^5	10^3D^6	10^4D^7
—9	+ 217.650582	— 131.451133	+ 34.722042	— 5.190470	+0.473145	— 262155	+ 81380	—108507
—8	46.944444	— 35.337716	11.691145	— 2.186348	0.249080	— 171988	66189	—108507
—7	5.716816	— 5.947430	2.706803	— 0.694186	0.107596	— 100042	50998	—108507
—6	0.073750	— 0.237498	+ 0.230598	— 0.113322	0.030763	— 46332	35807	—108507
—5	+ 0.016385	+ 0.001076	— 0.025882	+ 0.012871	+0.000054	— 10850	20616	—108507
—4	— 0.019144	0.001012	+ 0.023958	— 0.005013	—0.002660	+ 6401	+ 5425	—108507
—3	+ 0.026367	+ 0.021192	— 0.033302	— 0.005904	+0.001394	+ 5425	— 9766	—108507
—2	— 0.022222	— 0.091146	+ 0.013932	+ 0.031423	+0.002084	— 13782	— 24957	—108507
—1	— 0.067817	+ 0.162102	+ 0.127246	— 0.021635	—0.025119	— 51215	— 40148	—108507
0	0.000000	— 0.505250	— 0.856771	— 0.438520	—0.098141	— 106880	— 55338	—108507
1	— 7.705751	— 11.109148	— 6.101530	— 1.638198	—0.231320	— 180772	— 70530	—108507
2	— 61.829862	— 57.819792	— 21.645144	— 4.186675	—0.451878	— 272897	— 85720	—108507
3	— 276.285351	— 196.910641	— 57.276920	— 8.793989	—0.769012	— 383245	—100911	—108507
4	— 913.513888	— 531.190525	— 127.402373	— 16.317209	—1.201048	— 511826	—116102	—108507
5	— 2493.496200	— 1229.912610	— 251.956219	— 27.758942	—1.775120	— 658640	—131293	—108507
6	— 5945.512500	— 2552.360937	— 457.205864	— 44.267621	—2.500488	— 823678	—146484	—108507
7	—12809.652884	— 4874.488897	— 776.696710	— 67.137523	—3.398382	—1006944	—161675	—108507
8	—25495.077777	— 8719.656250	—1252.092184	— 97.808750	—4.487035	—1208443	—176866	—108507
9	—47602.028322	—14792.416695	—1934.055697	—137.858045	—5.784665	—1428165	—192057	—108507

APPENDIX I. (continued)

$P^{2,4}_{-2,4}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$
—9	+2636.422852	—461.954102	+5.567708	+3.051758	—0.132161	—55339	+32552
—8	1036.125000	—219.343750	4.694661	1.834961	—0.103841	—42318	32552
—7	338.823242	— 90.568685	3.402995	0.982717	—0.078125	—29297	32552
—6	83.421875	— 30.277344	2.083333	0.432617	—0.055013	—16276	32552
—5	12.231445	— 6.993164	1.001302	+0.122070	—0.034505	— 3255	32552
—4	0.343750	— 0.614583	+0.297526	— 0.011393	—0.016602	+9766	32552
—3	0.006836	+0.084961	— 0.012370	— 0.030273	—0.001302	22786	32552
—2	0.000000	— 0.042969	— 0.037760	+0.002930	+0.011393	35807	32552
—1	0.008789	— 0.021810	— 0.013021	+0.025716	0.021484	48828	32552
0	0.000000	— 0.250000	— 0.297526	— 0.024414	0.028971	61849	32552
1	4.596680	— 0.000977	— 1.375651	— 0.209961	0.033854	74870	32552
2	42.453125	+5.076823	— 3.856771	— 0.593424	0.036133	87891	32552
3	200.629883	25.459961	— 8.475260	— 1.237305	0.035807	100911	32552
4	673.968750	80.250000	— 16.090195	— 2.204102	0.032878	113932	32552
5	1829.467773	199.673503	— 27.686849	— 3.556315	0.027311	126953	32552
6	4285.656250	427.582031	— 44.373698	— 5.356445	0.019206	139974	32552
7	9006.969727	823.952148	— 67.385417	— 7.666992	+0.008464	152995	32552
8	17413.125000	1467.385417	— 98.081380	—10.550456	—0.004883	166016	32552
9	31503.495117	2457.608398	—137.945964	—14.069336	—0.020833	179036	32552

$P^{2,4}_{-2,2}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$
—9	—21107.367188	+422.394531	+228.912760	—3.795573	—0.899739	+91146	+130208
—8	— 9369.000000	246.812500	131.361979	—2.884115	—0.675781	91146	130208
—7	— 3651.429688	132.561198	68.756510	—2.097656	—0.483073	91146	130208
—6	— 1186.906250	63.390625	31.596354	—1.436198	—0.321615	91146	130208
—5	— 293.554688	25.550781	11.881510	—0.899740	—0.191406	91146	130208
—4	— 45.375000	7.791667	3.111979	—0.488281	—0.092448	91146	130208
—3	— 2.242188	1.363281	+0.287760	—0.201823	—0.024740	91146	130208
—2	+0.093750	+0.015625	— 0.091146	—0.010365	+0.011719	91146	130208
—1	+0.007812	— 0.001302	— 0.024740	—0.003906	+0.016927	91146	130208
0	0.000000	+0.062500	— 0.013021	—0.002148	—0.009115	91146	130208
1	— 3.304688	1.457031	+0.911010	—0.305990	—0.066406	91146	130208
2	— 49.156250	7.932292	5.346354	—0.611531	—0.151948	91146	130208
3	— 302.679688	25.738281	17.191010	—1.108073	—0.274740	91146	130208
4	— 1204.875000	63.625000	41.986979	—1.696615	—0.425781	91146	130208
5	— 3682.617188	132.842448	86.725260	—2.410156	—0.608073	91146	130208
6	— 9418.656250	247.140625	159.908854	—3.248698	—0.821615	91146	130208
7	—21181.617188	422.769531	271.537760	—4.212240	—1.066406	91146	130208
8	—43216.000000	678.479167	433.111979	—5.300781	—1.342448	91146	130208
9	—81692.179688	1035.519531	657.631510	—6.514323	—1.649739	91146	130208

APPENDIX I. (continued)

$P_{0,0}^{2,4}$

i		D	D^2	D^3	D^4	$10^2 D^5$	$10\ D^6$
—9	—131117.484375	+2037.339844	+1205.847656	—15.121094	—3.730469	+273438	+390625
—8	— 64448.000000	1269.750000	748.773438	—11.933594	—2.933594	273438	390625
—7	— 28774.484375	742.464844	435.472656	— 9.121094	—2.230469	273438	390625
—6	— 11319.750000	399.234375	232.195312	— 6.683594	—1.621094	273438	390625
—5	— 3740.234375	191.308594	109.691406	— 4.621094	—1.105469	273438	390625
—4	— 956.000000	77.437500	43.210938	— 2.933594	—0.683594	273438	390625
—3	— 160.734375	23.871094	12.503906	— 1.621094	—0.355469	273438	390625
—2	— 11.750000	4.359375	+1.820312	— 0.683594	—0.121094	273438	390625
—1	+0.015625	0.152344	— 0.089844	— 0.121094	+0.019531	273438	390625
0	0.000000	0.000000	+0.023438	+0.066406	0.066406	273438	390625
1	+0.015625	0.152344	— 0.089844	— 0.121094	+0.019531	273438	390625
2	— 11.750000	4.359375	+1.820312	— 0.683594	—0.121094	273438	390625
3	— 160.734375	23.871094	12.503906	— 1.621094	—0.355469	273438	390625
4	— 956.000000	77.437500	43.210938	— 2.933594	—0.683594	273438	390625
5	— 3740.234375	191.308594	109.691406	— 4.621094	—1.105469	273438	390625
6	— 11319.750000	399.234375	232.195312	— 6.683594	—1.621094	273438	390625
7	— 28774.484375	742.464844	435.472656	— 9.121094	—2.230469	273438	390625
8	— 64448.000000	1269.750000	748.773438	—11.933594	—2.933594	273438	390625
9	—131117.484375	2037.339844	1205.847656	—15.121094	—3.730469	273438	390625

$P_{0,2}^{2,4}$

i		D	D^2	D^3	D^4	$10^2 D^5$	$10^3 D^6$
—9	+49023.562500	—6930.838542	— 35.882812	+42.570312	—1.203125	—651042	+260417
—8	22208.000000	—3653.416667	— 8.151041	28.346354	—0.976562	—546875	260417
—7	8890.437500	—1748.619792	+5.184896	17.661062	— 0.770833	—442708	260417
—6	2998.500000	— 733.322917	9.250000	10.023437	—0.585938	—338542	260417
—5	782.812500	— 253.401042	8.169271	4.924479	—0.421875	—234375	260417
—4	132.000000	— 63.729167	5.067708	1.867188	—0.278646	— 130208	260417
—3	+7.687500	— 8.182292	2.070312	+0.351562	—0.156250	—026042	260417
—2	— 0.500000	+0.364583	+0.302083	— 0.122396	—0.054688	+078125	260417
—1	+0.062500	+0.036458	— 0.111979	— 0.054688	+0.026042	182292	260417
0	0.000000	— 0.041667	— 0.046875	+0.054688	0.085938	286458	260417
1	2.937500	+0.255208	— 1.377604	— 0.291271	0.125000	390625	260417
2	60.500000	12.052083	— 6.979167	— 1.601562	0.143229	494792	260417
3	427.312500	77.473958	— 20.726562	— 4.367188	0.140625	598958	260417
4	1836.000000	289.645833	— 47.491792	— 9.091146	0.117187	703125	260417
5	5892.187500	812.692708	— 93.158854	—16.273138	0.072917	807292	260417
6	15589.500000	1901.731583	—164.593750	—26.411063	+0.007812	911458	260417
7	35944.562500	3922.911458	—269.674479	—[illegible]	—0.078125	1015625	260417
8	74752.000000	7373.333333	—417.276041	—[illegible]	—0.184896	1119792	260417
9	143459.437500	12901.130208	—617.273437	—[illegible]	—0.312500	1223958	260417

APPENDIX I. (continued)

$P^{2,4}_{0,4}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$
−9	− 6123.304688	+1816.086588	−190.202474	+5.628255	+0.384765	−319010	+65104
−8	− 2456.000000	859.260417	−107.558591	4.125651	0.250651	−266927	65104
−7	− 824.960938	352.823568	− 54.863932	2.852214	0.142578	−214844	65104
−6	− 210.750000	116.776042	− 23.993489	1.807943	0.060547	−162760	65104
−5	− 32.617188	26.367839	− 8.072266	0.992838	+0.004557	−110677	65104
−4	− 1.000000	+ 2.098958	− 1.475260	0.406901	−0.025391	− 58594	65104
−3	− 0.023438	− 0.280599	+ 0.172526	+0.050130	−0.029297	− 6510	65104
−2	0.000000	+ 0.229167	− 0.003906	−0.077474	−0.007161	+ 45573	65104
−1	+ 0.070312	− 0.121745	− 0.129557	+0.024089	+0.041016	97656	65104
0	0.000000	+ 0.166667	+ 0.420573	0.354818	0.115234	149740	65104
1	− 4.085938	− 2.155598	+ 1.021484	0.914714	0.215495	201823	65104
2	− 52.250000	− 25.088542	− 0.201823	1.703776	0.341797	253906	65104
3	− 283.242188	− 111.382162	− 6.374349	2.722005	0.494141	305990	65104
4	− 1027.000000	− 338.536458	− 21.871094	3.969401	0.672526	358073	65104
5	− 2927.148438	− 828.801432	− 52.317057	5.445964	0.876953	410156	65104
6	− 7093.500000	−1759.177083	−104.587239	7.151693	1.107422	462240	65104
7	−15284.554688	−3371.413411	−186.806640	9.086588	1.363932	514323	65104
8	−30120.000000	−5982.010417	−308.350260	11.250651	1.646484	566406	65104
9	−55323.210938	−9992.218099	−479.843098	13.643880	1.955078	618490	65104

$P^{2,4}_{2,0}$

i		D	D^2	D^3	D^4	$10^5 D^5$	$10^6 D^6$
−9	+74664.123047	−8821.037109	−158.484375	+50.326172	−0.980169	−644531	+195312
−8	37259.000000	−4994.156250	− 94.175781	35.728516	−0.810547	−566406	195312
−7	16956.392578	−2626.544922	− 51.386719	24.255859	−0.656250	−488281	195312
−6	6839.015625	−1254.937500	− 24.773438	15.533203	−0.517578	−410156	195312
−5	2337.646484	− 526.318359	− 9.742188	9.185547	−0.391531	−332031	195312
−4	627.375000	− 182.921875	− 2.449219	4.837891	−0.287109	−253906	195312
−3	113.853516	− 47.232422	+ 0.199219	2.115234	−0.195312	−175781	195312
−2	+ 9.546875	− 6.984375	0.546875	0.642578	−0.119141	− 97656	195312
−1	− 0.017578	− 0.162109	+ 0.187500	+ 0.011922	−0.058594	− 19531	195312
0	0.000000	0.000000	− 0.035156	− 0.052734	−0.013672	+ 58594	195312
1	+ 0.001953	0.017578	− 0.027344	− 0.025391	+0.015625	136719	195312
2	2.203125	1.156250	− 0.445312	− 0.248047	0.029297	214844	195312
3	46.880859	14.431641	− 2.695312	− 1.095703	0.027344	292969	195312
4	328.625000	75.609375	− 8.933594	− 2.943359	+0.009766	371094	195312
5	1402.587891	260.205078	− 22.066406	− 6.166016	−0.023438	449219	195312
6	4480.734375	698.484375	− 45.750000	−11.138672	−0.072266	527344	195312
7	11818.091797	1590.462891	− 84.390625	−18.236328	−0.136719	605469	195312
8	27189.000000	3220.906250	−143.144531	−27.833984	−0.216797	683594	195312
9	56453.361328	5974.330078	−227.917969	−40.306641	−0.312500	761719	195312

APPENDIX I. (continued)

$P_{2,2}^{2,4}$

i		D	D^2	D^3	D^4	10^2D^5	10^3D^6
—9	—27916.195312	+ 6811.058594	— 537.493190	+ 3.657552	+ 1.654948	—846354	+ 130208
—8	—12839.000000	3580.104167	—324.575521	2.907552	1.261719	—742188	130208
—7	— 5239.007812	1706.777344	—182.040365	2.240885	0.920573	—638021	130208
—6	— 1811.593750	711.578125	— 92.388021	1.657552	0.631510	—533854	130208
—5	— 489.257812	243.506510	— 40.618490	1.157552	0.394531	—429688	130208
—4	— 86.625000	60.062500	— 14.231771	0.740885	0.209635	—325521	130208
—3	— 5.445312	+ 7.246094	— 3.227865	0.407552	+ 0.076823	—221354	130208
—2	+ 0.406250	— 0.442708	— 0.106771	+ 0.157552	—0.003906	—117188	130208
—1	— 0.070312	— 0.003906	+ 0.131510	—0.009115	—0.032552	— 13021	130208
0	0.000000	+ 0.062500	— 0.013021	—0.092448	—0.009115	+ 91146	130208
1	+ 0.367188	— 0.243490	— 0.540365	—0.092448	+ 0.066406	195313	130208
2	— 11.343750	— 12.421875	— 3.950521	—0.009115	0.194010	299479	130208
3	— 124.632812	— 79.472656	— 15.243490	+ 0.157552	0.373698	403646	130208
4	— 631.125000	— 295.895833	— 41.919271	0.407552	0.605469	507813	130208
5	— 2209.570312	— 827.691406	— 93.977865	0.740885	0.889323	611979	130208
6	— 6170.843750	— 1932.359375	—183.919270	1.157552	1.225266	716146	130208
7	—14762.945312	— 3978.899740	—326.743490	1.657552	1.613281	820313	130208
8	—31536.000000	— 7467.812500	—539.950521	2.240885	2.053386	924479	130208
9	—61767.257812	—13051.097656	—843.540365	2.907552	2.545573	1028646	130208

$P_{2,4}^{2,4}$

i		D	D^2	D^3	D^4	10^2D^5	10^3D^6
—9	+ 3486.881836	—1391.930664	+ 233.526692	—21.030273	+ 1.069011	—289714	+ 32552
—8	1419.875000	— 659.104167	128.689453	—13.487956	0.797201	—250651	32552
—7	486.137695	— 270.672851	63.436198	— 7.976888	0.564453	—211589	32552
—6	127.328125	— 89.425781	26.438802	— 4.184570	0.370768	—172526	32552
—5	20.385742	— 20.027018	8.244141	— 1.798503	0.216146	—133464	32552
—4	0.656250	— 1.515625	+ 1.274088	— 0.506185	0.100586	— 94401	32552
—3	+ 0.016602	+ 0.194336	— 0.174479	+ 0.001883	+ 0.024089	— 55339	32552
—2	0.000000	— 0.186198	+ 0.070312	+ 0.017201	—0.013346	— 16276	32552
—1	— 0.079102	+ 0.178711	+ 0.055339	— 0.066732	—0.011719	+ 22786	32552
0	0.000000	— 0.250000	— 0.297526	— 0.024414	+ 0.028971	+ 61849	32552
1	— 0.510742	+ 0.113607	+ 0.808594	+ 0.486654	0.108724	100911	32552
2	+ 9.796875	13.480469	7.045573	1.778971	0.227539	139974	32552
3	82.612305	70.186523	23.960286	4.165039	0.385415	179036	32552
4	353.031250	226.192708	58.971609	7.957357	0.582357	218099	32552
5	1097.680664	570.581961	121.385417	13.468424	0.818359	257161	32552
6	2807.843750	1233.074219	222.364583	21.010742	1.093425	296224	32552
7	6277.584961	2391.496419	374.958984	30.896810	1.407552	335286	32552
8	12706.875000	4279.312500	594.090495	43.439128	1.760742	374349	32552
9	23819.715820	7193.108398	896.555989	58.950195	2.152995	413411	32552

APPENDIX I. (continued)

$P^{3,2}_{-3,2}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−9	−2338.781250	−151.583333	+17.450521	+1.187500	−0.036458	−26042
−8	−1167.156667	− 87.347917	11.330729	0.895833	−0.031250	−26042
−7	− 519.458333	− 46.223958	6.835938	0.645833	−0.026042	−26042
−6	− 196.812500	− 21.661458	3.716146	0.437500	−0.020833	−26042
−5	− 58.585417	− 8.510417	1.721354	0.270833	−0.015625	−26042
−4	− 11.533333	− 2.520833	0.601563	0.145833	−0.010417	−26042
−3	− 0.812500	− 0.442708	+ 0.106771	0.062500	−0.005208	−26042
−2	+ 0.020833	− 0.026042	− 0.013021	0.020833	0.000000	−26042
−1	0.010417	− 0.020833	− 0.007812	0.020833	+0.005208	−26042
0	0.000000	− 0.010417	− 0.007812	0.010417	0.010417	−26042
1	2.833333	− 1.244792	− 0.622396	0.145833	0.015625	−26042
2	23.354167	− 4.973958	− 1.742188	0.270833	0.020833	−26042
3	98.406250	− 14.114583	− 3.736979	0.437500	0.026042	−26042
4	296.833333	− 32.416667	− 6.856771	0.645833	0.031250	−26042
5	729.479167	− 64.630208	−11.351562	0.895833	0.036458	−26042
6	1559.187500	−116.505208	−17.471354	1.187500	0.041667	−26042
7	3 1.802083	−194.791667	−25.465146	1.520833	0.046875	−26042
8	5281.166667	−307.239583	−35.585938	1.895833	0.052083	−26042
9	9049.125000	−462.598958	−48.080729	2.312500	0.057292	−26042

$P^{3,2}_{-1,2}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−9	+ 9725.625000	−676.453125	−61.726562	+4.390625	+ 0.109375	−78125
−8	5099.500000	−410.718750	−41.101562	3.406250	0.093750	−78125
−7	2427.468750	−232.000000	−25.664062	2.546875	0.078125	−78125
−6	1013.437500	−118.921875	−14.664062	1.812500	0.062500	−78125
−5	350.312500	− 53.109375	− 7.351562	1.203125	0.046875	−78125
−4	90.000000	− 19.187500	− 2.976562	0.718750	0.031250	−78125
−3	13.406250	− 4.781250	− 0.789062	0.359375	+0.015625	−78125
−2	+ 0.437500	− 0.515625	− 0.039062	0.125000	0.000000	−78125
−1	0.000000	− 0.015625	+ 0.023438	0.015625	−0.015625	−78125
0	0.000000	+ 0.093750	0.148438	0.031250	−0.031250	−78125
1	− 2.656250	+ 0.187500	1.085938	0.171875	−0.046875	−78125
2	− 32.062500	− 2.359375	3.585938	0.437500	−0.062500	−78125
3	− 163.312500	− 13.171875	8.398438	0.828125	−0.078125	−78125
4	− 552.500000	− 40.875000	16.273438	1.343750	−0.093750	−78125
5	− 1466.718750	− 97.093750	27.960938	1.984375	−0.109375	−78125
6	− 3514.062500	−196.453125	44.210938	2.750000	−0.125000	−78125
7	− 6673.625000	−356.578125	65.773438	3.640625	−0.140625	−78125
8	−12325.500000	−598.093750	95.398438	4.656250	−0.156250	−78125
9	−21280.781250	−944.625000	127.835938	5.796875	−0.171875	−78125

$P^{3,2}_{1,0}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−9	+33716.250000	−1987.312500	−194.437500	+11.171875	+0.281250	−156250
−8	19608.000000	−1270.250000	−137.578125	8.937500	0.250000	−156250
−7	9947.000000	− 767.375000	− 93.031250	6.953125	0.218750	−156250
−6	4725.000000	− 431.062500	− 59.296875	5.218750	0.187500	−156250
−5	1968.750000	− 219.687500	− 34.875000	3.734375	0.156250	−156250
−4	680.000000	− 97.625000	− 18.265625	2.500000	0.125000	−156250
−3	175.500000	− 35.250000	− 7.968750	1.515625	0.093750	−156250
−2	27.000000	− 8.937500	− 2.484375	0.781250	0.062500	−156250
−1	+ 1.250000	− 1.062500	− 0.312500	0.296875	+0.031250	−156250
0	0.000000	− 0.000000	+ 0.046875	0.062500	0.000000	−156250
1	0.000000	− 0.125000	0.093750	0.078125	−0.031250	−156250
2	− 7.000000	− 1.812500	1.328125	0.343750	−0.062500	−156250
3	− 74.250000	− 11.437500	5.250000	0.859375	−0.093750	−156250
4	− 360.000000	− 41.375000	13.359375	1.625000	−0.125000	−156250
5	− 1187.500000	− 110.000000	27.156250	2.640625	−0.156250	−156250
6	− 3105.000000	− 241.687500	48.140625	3.906250	−0.187500	−156250
7	− 6945.750000	− 466.812500	77.812500	5.421875	−0.218750	−156250
8	−13888.000000	− 821.750000	117.671875	7.187500	−0.250000	−156250
9	−25515.000000	−1348.875000	169.218750	9.203125	−0.281250	−156250

APPENDIX I. (continued)

$P^{3,2}_{1,2}$

i		D	D^2	D^3	D^4	$10^7 D$
−9	−12851.718750	+2330.812500	−105.460938	−3.625000	+0.390625	−78125
−8	− 6979.500000	1441.531250	− 75.226562	−2.718750	0.343750	−78125
−7	− 3476.375000	834.078125	− 51.367188	−1.937500	0.296875	−78125
−6	− 1542.187500	441.703125	− 33.132812	−1.281250	0.250000	−78125
−5	− 580.781250	206.656250	− 19.773438	−0.750000	0.203125	−78125
−4	− 170.000000	80.187500	− 10.539062	−0.343750	0.156250	−78125
−3	− 31.687500	22.546875	− 4.679688	−0.062500	0.109375	−78125
−2	− 1.687500	+2.984375	− 1.445312	+0.093750	0.062500	−78125
−1	+ 0.156250	− 0.250000	− 0.085938	0.125000	+0.015625	−78125
0	0.000000	+0.093750	+0.148438	+0.031250	−0.031250	−78125
1	0.000000	0.265625	0.007812	−0.187500	−0.078125	−78125
2	8.312500	5.515625	0.242188	−0.531250	−0.125000	−78125
3	69.093750	30.093750	1.601562	−1.000000	−0.171875	−78125
4	292.500000	97.250000	4.835938	−1.593750	−0.218750	−78125
5	884.687500	239.234375	10.695312	−2.312500	−0.265625	−78125
6	2177.812500	497.296875	19.929688	−3.156250	−0.312500	−78125
7	4660.031250	921.687500	33.289062	−4.125000	−0.359375	−78125
8	9005.500000	1571.656250	51.523438	−5.218750	−0.406250	−78125
9	16104.375000	2515.453125	75.382812	−6.437500	−0.453125	−78125

$P^{3,2}_{3,0}$

i		D	D^2	D^3	D^4	$10^7 D$
−9	−14337.000000	+2214.375000	−68.760417	−4.682292	+0.322917	−52083
−8	− 8298.666667	1431.083333	−49.046875	−3.833333	0.291667	−52083
−7	− 4487.583333	876.312500	−33.520833	−3.067708	0.260417	−52083
−6	− 2223.000000	500.687500	−21.682292	−2.385417	0.229167	−52083
−5	− 979.166667	260.833333	−13.031250	−1.786458	0.197917	−52083
−4	− 365.333333	119.375000	− 7.067708	−1.270833	0.166667	−52083
−3	− 105.750000	44.937500	− 3.291667	−0.838542	0.135417	−52083
−2	− 19.666667	12.145833	− 1.203125	−0.489583	0.104167	−52083
−1	− 1.333333	− 1.625000	− 0.302083	−0.223958	0.072917	−52083
0	0.000000	0.000000	− 0.088542	−0.041667	0.041667	−52083
1	+0.083333	+0.104167	− 0.062500	+0.057292	+0.010417	−52083
2	− 0.333333	− 0.062500	+0.276042	0.072917	−0.020833	−52083
3	+ 4.500000	+4.750000	1.427083	+0.005208	−0.052083	−52083
4	45.333333	24.958333	3.890625	−0.145833	−0.083333	−52083
5	197.916667	77.187500	8.166667	−0.380208	−0.114583	−52083
6	603.000000	184.062500	14.755208	−0.697917	−0.145833	−52083
7	1486.333333	374.208333	24.156250	−1.098959	−0.177083	−52083
8	3178.666667	682.250000	36.869792	−1.583333	−0.208333	−52083
9	6135.750000	1148.812500	53.565833	2.151042	−0.239583	−52083

$P^{3,2}_{3,2}$

i		D	D^2	D^3	D^4	$10^7 D$
−9	+5464.875000	−1513.067708	+166.466146	−9.078125	+0.244792	−26042
−8	3047.166667	− 951.197917	117.872396	−7.229167	0.218750	−26042
−7	1568.364583	− 561.562500	79.716146	−5.588542	0.192708	−26042
−6	725.562500	− 305.036458	50.747396	−4.156250	0.166667	−26042
−5	288.854167	− 147.494792	29.716146	−2.932292	0.140625	−26042
−4	91.333333	− 59.812500	15.372396	−1.916667	0.114583	−26042
−3	19.093750	− 17.864583	6.466146	−1.109375	0.088542	−26042
−2	+1.229167	− 2.526042	+1.747396	−0.510417	0.062500	−26042
−1	− 0.166667	+0.328125	− 0.033854	−0.119792	0.036458	−26042
0	0.000000	− 0.177083	− 0.127604	+0.062500	+0.010417	−26042
1	− 0.177083	+0.083333	+0.216146	+0.036458	−0.015625	−26042
2	+0.395833	+0.234375	− 0.252604	−0.197917	−0.041667	−26042
3	− 4.187500	− 5.598958	− 2.783854	−0.640625	−0.067708	−26042
4	− 36.833333	− 28.291667	− 8.627604	−1.291667	−0.093750	−26042
5	− 147.447917	− 83.718750	− 19.033854	−2.151042	−0.119792	−26042
6	− 422.937500	− 192.755208	− 35.252604	−3.218750	−0.145833	−26042
7	− 997.208333	− 381.276042	− 58.533854	−4.494792	−0.171875	−26042
8	−2061.166667	− 680.156250	− 90.127604	−5.979167	−0.197917	−26042
9	−3872.718750	−1125.270833	−131.283854	−7.671875	−0.223958	−26042

APPENDIX I. (continued)

P_0^4

i		D	D^2	D^3	D^4
−9	+1628.859375	+0.031250	−10.140625	−0.031250	+0.015625
−8	1015.000000	0.031250	− 8.015625	−0.031250	0.015625
−7	593.359375	0.031250	− 6.140625	−0.031250	0.015625
−6	318.937500	0.031250	− 4.515625	−0.031250	0.015625
−5	152.734375	0.031250	− 3.140625	−0.031250	0.015625
−4	61.750000	0.031250	− 2.015625	−0.031250	0.015625
−3	18.984375	0.031250	− 1.140625	−0.031250	0.015625
−2	3.437500	0.031250	− 0.515625	−0.031250	0.015625
−1	0.109375	0.031250	− 0.140625	−0.031250	0.015625
0	0.000000	0.031250	− 0.015625	−0.031250	0.015625
1	0.109375	0.031250	− 0.140625	−0.031250	0.015625
2	3.437500	0.031250	− 0.515625	−0.031250	0.015625
3	18.984375	0.031250	− 1.140625	−0.031250	0.015625
4	61.750000	0.031250	− 2.015625	−0.031250	0.015625
5	152.734375	0.031250	− 3.140625	−0.031250	0.015625
6	318.937500	0.031250	− 4.515625	−0.031250	0.015625
7	593.359375	0.031250	− 6.140625	−0.031250	0.015625
8	1015.000000	0.031250	− 8.015625	−0.031250	0.015625
9	1628.859375	0.031250	−10.140625	−0.031250	0.015625

P_2^4

i		D	D^2	D^3	D^4
−9	−1605.187500	+166.635417	+0.270833	−0.437500	+0.010417
−8	−1047.166667	121.479167	0.239583	−0.395833	0.010417
−7	− 648.812500	85.322919	0.208333	−0.354167	0.010417
−6	− 376.375000	57.166667	0.177083	−0.312500	0.010417
−5	− 200.104167	36.010417	0.145833	−0.270833	0.010417
−4	− 94.250000	20.854167	0.114583	−0.229167	0.010417
−3	− 37.062500	10.697917	0.083333	−0.187500	0.010417
−2	− 10.791667	4.541667	0.052083	−0.145833	0.010417
−1	− 1.687500	1.385417	+0.020833	−0.104167	0.010417
0	0.000000	0.299167	−0.010417	−0.062500	0.010417
1	+ 0.020833	+0.072917	−0.041667	−0.020833	0.010417
2	+0.125000	− 0.083333	−0.072917	0.020833	0.010417
3	− 1.937500	− 1.239583	−0.104167	0.062500	0.010417
4	− 12.416667	− 4.395833	−0.135417	0.104167	0.010417
5	− 41.562500	− 10.552083	−0.166667	0.145833	0.010417
6	− 103.625000	− 20.708333	−0.197917	0.187500	0.010417
7	− 216.854167	− 35.864583	−0.229167	0.229167	0.010417
8	− 403.500000	− 57.020833	−0.260417	0.270833	0.010417
9	− 689.812500	− 85.177083	−0.291667	0.312500	0.010417

P_4^4

i		D	D^2	D^3	D^4
−9	+565.710938	−109.354167	+7.700521	−0.234375	+0.002604
−8	382.125000	− 81.911458	6.372396	−0.213542	0.002604
−7	247.096354	− 59.468750	5.169271	−0.192708	0.002604
−6	151.250000	− 41.526042	4.091146	−0.171875	0.002604
−5	86.210938	− 27.583333	3.138021	−0.151042	0.002604
−4	44.604167	− 17.140625	2.309895	−0.130208	0.002604
−3	20.054688	− 9.697917	1.606770	−0.109375	0.002604
−2	7.187500	− 4.755208	1.028646	−0.088542	0.002604
−1	+ 1.627604	− 1.812500	0.575521	−0.067708	0.002604
0	0.000000	− 0.369792	0.247396	−0.046875	0.002604
1	− 0.070312	+ 0.072917	+0.044271	−0.026042	0.002604
2	+ 0.041667	+0.015625	−0.033854	−0.005208	0.002604
3	− 0.039062	− 0.041667	+0.013021	+0.015625	0.002604
4	0.312500	0.401042	0.184896	0.036458	0.002604
5	2.721354	1.843750	0.481771	0.057292	0.002604
6	9.812500	4.786458	0.903646	0.078125	0.002604
7	25.210938	9.729167	1.450521	0.098958	0.002604
8	53.541667	17.171875	2.122396	0.119792	0.002604
9	100.429667	27.614583	2.919271	0.140625	0.002604

APPENDIX I. (continued)

$P^{0,\,6}_{0,\,-6}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$
−7	+990.918750	+342.816929	+47.632907	+3.408529	+0.132704	+26693	+2170
−6	498.699243	196.118599	30.779384	2.473850	0.107639	24089	2170
−5	228.378473	104.250348	18.821050	1.726671	0.085178	21484	2170
−4	92.095118	50.287696	10.711024	1.146159	0.065321	18880	2170
−3	30.914931	21.181163	5.527431	0.711480	0.048069	16276	2170
−2	7.705751	7.256272	2.473394	0.401801	0.033420	13672	2170
−1	+ 1.012500	1.713542	0.877940	0.195289	0.021376	11068	2170
0	− 0.067817	+ 0.128494	+ 0.191493	0.074110	0.011936	8464	2170
1	− 0.011111	− 0.048351	− 0.005122	+0.014431	0.005109	5859	2170
2	+ 0.008789	+ 0.008529	− 0.000679	−0.000581	+0.000868	3255	2170
3	− 0.004861	− 0.000347	+ 0.000946	−0.000760	−0.000760	+ 651	2170
4	+ 0.003277	+ 0.000543	− 0.000122	+0.000662	+0.000217	− 1953	2170
5	0.015625	− 0.038281	+ 0.035243	−0.015950	0.003798	− 4557	2170
6	0.816688	0.790647	0.330165	−0.075629	0.009083	− 7161	2170
7	5.868056	− 4.057986	1.207769	−0.197808	0.018772	− 9766	2170

$P^{0,\,6}_{0,\,-4}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$
−7	−2703.770833	−639.846875	−38.310576	+1.649088	+0.268880	+162965	+13021
−6	−1233.153125	−332.055599	−21.673307	1.297526	0.218599	92448	13021
−5	− 497.179167	−155.479167	−11.047656	0.987620	0.172526	82031	13021
−4	− 168.848958	− 63.117577	− 4.808724	0.719401	0.132161	71615	13021
−3	− 44.350000	− 20.720833	− 1.581510	0.492859	0.097005	61198	13021
−2	− 7.557291	− 4.788932	− 0.241016	0.307943	0.067057	50781	13021
−1	− 0.533333	− 0.571875	+ 0.087760	0.164714	0.042318	+ 365	13021
0	− 0.028125	− 0.059661	+ 0.029818	0.063151	0.022786	−29948	13021
1	+ 0.020833	− 0.032292	− 0.039844	+0.003255	+0.008464	19531	13021
2	− 0.011458	+ 0.040234	+ 0.003776	−0.014974	−0.000651	+ 9115	13021
3	+ 0.062500	− 0.102083	+ 0.035677	+0.008464	−0.004557	− 1302	13021
4	− 0.257292	+ 0.540625	− 0.319141	0.075568	−0.003255	− 11719	13021
5	− 8.658333	6.218750	− 1.685677	0.180339	+ 0.003255	− 22135	13021
6	− 52.515625	26.431900	− 4.938932	0.328776	0.014974	− 32552	13021
7	− 197.391667	77.930208	−11.203906	0.518880	0.031901	− 42969	13021

$P^{0,\,6}_{0,\,-2}$

i		D	D^2	D^3	D^4	$10^7 D^5$	$10^8 D^6$
−7	+2666.755208	+277.295573	−30.500000	−3.567383	+0.064779	+139974	+32552
−6	1055.448893	120.156576	−17.875000	−2.174805	0.067057	126953	32552
−5	350.156250	43.401041	− 9.488932	−1.178060	0.066732	113932	32552
−4	89.305012	11.677409	− 4.326172	−0.514648	0.063802	100911	32552
−3	14.729167	1.759115	− 1.496094	−0.122070	0.058268	87891	32552
−2	1.043945	0.044596	− 0.233073	+0.062174	0.050130	74870	32552
−1	+ 0.020833	+ 0.057292	+ 0.103516	0.100586	0.039388	61849	32552
0	− 0.037435	− 0.054362	+ 0.029297	+0.055664	0.026042	48828	32552
1	+ 0.078125	− 0.016927	− 0.065104	−0.010091	+0.010091	35807	32552
2	− 0.142253	+ 0.068034	+ 0.085937	−0.034180	−0.008464	22786	32552
3	+ 3.447917	− 2.776042	0.625047	+0.045898	−0.029622	+ 9766	32552
4	39.026367	− 16.900716	1.561849	0.292643	−0.053385	− 3255	32552
5	193.177083	− 58.532552	2.792969	0.768555	−0.079753	− 16276	32552
6	660.265299	−154.273112	4.082031	1.536133	−0.108724	− 29297	32552
7	1806.812500	−343.598958	5.069661	2.657878	−0.140299	− 42318	32552

APPENDIX I. (continued)

$P^{0,6}_{0,0}$

i		D	D^2	D^3	D^4	$10^5 D^5$	$10^5 D^6$
−7	−1137.437500	+74.536458	+22.290799	−1.402344	−0.150608	+65104	+43403
−6	− 358.767361	34.589844	9.786892	−0.944010	−0.093316	65104	43403
−5	− 83.861112	13.156250	3.306424	−0.569010	−0.046441	65104	43403
−4	− 11.406250	3.485677	+0.599392	−0.277344	−0.009983	65104	43403
−3	− 0.381944	0.328125	− 0.084201	−0.069010	+0.016659	65104	43403
−2	− 0.059028	− 0.066406	+0.005642	+0.055990	0.031684	65104	43403
−1	0.000000	+0.052083	0.118924	0.097656	0.036892	65104	43403
0	− 0.059028	− 0.066406	+0.005642	+0.055990	0.031684	65104	43403
1	− 0.381944	+0.328125	− 0.084201	−0.069010	+0.016059	65104	43403
2	− 11.406250	3.485677	+0.599392	−0.277344	−0.009983	65104	43403
3	− 83.861112	13.156250	3.306424	−0.569010	−0.046441	65104	43403
4	− 358.767361	34.589844	9.786892	−0.944010	−0.093316	65104	43403
5	−1137.437500	74.536458	22.290799	−1.402344	−0.150608	65104	43403
6	−2971.475694	141.246094	43.568142	−1.944010	−0.218316	65104	43403
7	−6772.777778	244.468750	76.868924	−2.569010	−0.296441	65104	43403

$P^{0,6}_{0,2}$

i		D	D^2	D^3	D^4	$10^5 D^5$	$10^5 D^6$
−7	+193.177083	−58.532552	+2.792969	+0.768555	−0.079753	−16276	+32552
−6	39.026367	−16.930716	1.561849	0.292643	−0.053385	− 3255	32552
−5	+3.447917	− 2.776042	0.623047	+0.045898	−0.029622	+9766	32552
−4	− 0.142253	+0.058034	+0.085937	−0.034180	−0.008464	22786	32552
−3	+0.078125	− 0.016927	− 0.065104	−0.010091	+0.010091	35807	32552
−2	− 0.037435	− 0.054362	+0.029297	+0.055664	0.026642	48828	32552
−1	+0.020833	+0.057292	+0.105516	0.160586	0.059388	61849	32552
0	1.043945	0.044596	− 0.233073	+0.062174	0.050130	74870	32552
1	14.729167	1.759115	− 1.496094	−0.122070	0.058268	87891	32552
2	89.305012	11.677409	− 4.326172	−0.514648	0.063802	100911	32552
3	350.156250	43.401041	− 9.488932	−1.178960	0.066732	113932	32552
4	1055.448893	120.156576	−17.875000	−2.174805	0.067057	126953	32552
5	2666.755208	277.295573	−30.560000	−3.567383	0.064779	139974	32552
6	5934.678711	564.794596	−48.50455	−5.418294	0.059896	152995	32552
7	11999.479167	1049.755208	−73.154297	−7.790039	0.052409	166016	32552

$P^{0,6}_{0,4}$

i		D	D^2	D^3	D^4	$10^5 D^5$	$10^5 D^6$
−7	− 8.658333	+6.218750	− 1.685677	+0.185339	+0.003255	−22135	+13021
−6	− 0.257292	+0.540625	− 0.319141	0.073568	−0.003255	−11719	13021
−5	+0.062500	− 0.102083	+0.035677	+0.008464	−0.004557	− 1302	13021
−4	− 0.011458	+0.040234	+0.003776	−0.014974	−0.004651	+9115	13021
−3	+0.020833	− 0.032292	− 0.039844	+0.003255	+0.008464	19531	13021
−2	− 0.028125	− 0.069661	+0.029818	0.063151	0.022786	29948	13021
−1	− 0.533333	− 0.571875	+0.087760	0.164714	0.042318	40365	13021
0	− 7.557291	− 4.788932	− 0.241616	0.307943	0.067057	50781	13021
1	− 44.350000	− 20.720833	− 1.581510	0.492839	0.097005	61198	13021
2	− 168.848958	− 63.117577	− 4.808724	0.719401	0.132161	71615	13021
3	− 497.179167	− 155.479163	−11.047656	0.987630	0.172526	82031	13021
4	− 1233.153125	− 332.055590	−21.673307	1.297526	0.218099	92448	13021
5	− 2703.770833	− 639.846859	−38.310676	1.649088	0.268880	102865	13021
6	− 5400.719792	−1140.602966	−62.834764	2.042318	0.324870	113281	13021
7	−10027.875000	−1912.823958	−97.370573	2.477214	0.386068	123698	13021

APPENDIX I. (continued)

$P^{0,6}_{0,6}$

i		D	D^2	D^3	D^4	$10^2 D^5$	$10^4 D^6$
−7	+0.015625	−0.038281	+0.035243	−0.015950	+0.003798	−4557	+2170
−6	+0.003277	+0.000543	−0.005122	+0.002062	+0.000217	−1953	2170
−5	−0.004861	−0.000347	+0.005916	−0.000760	−0.000760	+651	2170
−4	+0.008789	+0.008529	−0.009679	−0.003581	+0.000868	3255	2170
−3	−0.011111	−0.048351	−0.005122	+0.014431	0.005100	5859	2170
−2	−0.067817	+0.128494	+0.191493	0.074110	0.011936	8464	2170
−1	+1.012500	1.713542	0.877040	0.196289	0.021376	11068	2170
0	7.705751	7.256272	2.473394	0.401801	0.033420	13672	2170
1	30.914931	21.181163	5.527431	0.711480	0.048069	16276	2170
2	92.095117	50.287696	10.711024	1.146159	0.065321	18880	2170
3	228.378472	104.250347	18.821050	1.726671	0.085178	21484	2170
4	498.699240	196.118598	30.779384	2.473850	0.107639	24089	2170
5	990.918750	342.816927	47.632900	3.408529	0.132704	26693	2170
6	1829.950412	565.644813	70.553472	4.551541	0.160373	29297	2170
7	3186.884722	890.776736	100.837977	5.923720	0.190647	31901	2170

$P^{-1,4}_{-1,-4}$

i		D	D^2	D^3	D^4	$10^2 D^5$
−7	−1576.333333	−500.395833	−62.096354	−3.769531	−0.111979	−13021
−6	− 819.601562	−298.803385	−42.408854	−2.933594	−0.098958	−13021
−5	− 385.125000	−165.062500	−27.377604	−2.201823	−0.085937	−13021
−4	− 157.356771	− 81.985677	−16.377604	−1.574219	−0.072917	−13021
−3	− 52.250000	− 34.885417	− 8.783854	−1.050781	−0.059896	−13021
−2	− 12.257812	− 11.574219	− 3.971354	−0.631510	−0.046875	−13021
−1	− 1.333333	− 2.364583	− 1.315104	−0.316406	−0.033854	−13021
0	+0.070312	− 0.069010	− 0.190104	−0.105469	−0.020833	−13021
1	0.000000	0.000000	+0.028646	+0.001302	−0.007812	−13021
2	0.002604	+0.029948	− 0.033854	+0.003906	+0.005208	−13021
3	0.125000	− 0.291667	+0.247396	−0.097656	0.018229	−13021
4	3.914062	− 3.777344	1.497396	−0.303385	0.031250	−13021
5	23.416667	− 15.739583	4.341146	−0.613281	0.044271	−13021
6	84.179688	− 43.990885	9.403646	−1.027344	0.057292	−13021
7	230.250000	− 98.843750	17.309896	−1.545573	0.070312	−13021

$P^{-1,4}_{-1,-2}$

i		D	D^2	D^3	D^4	$10^2 D^5$
−7	+3464.333333	+663.187500	+18.500000	−3.421875	−0.260417	−52083
−6	1649.812500	361.895833	10.286458	−2.697917	−0.229167	−52083
−5	688.500000	176.750000	4.760417	−2.057292	−0.197917	−52083
−4	237.395833	73.375000	+1.421875	−1.500000	−0.166667	−52083
−3	60.500000	23.395833	− 0.229167	−1.026042	−0.135417	−52083
−2	8.812500	4.437500	− 0.692708	−0.635417	−0.104167	−52083
−1	0.333333	0.125000	− 0.468750	−0.328125	−0.072917	−52083
0	+0.062500	+0.083333	− 0.057292	−0.104167	−0.041667	−52083
1	0.000000	− 0.062500	+0.041667	+0.036458	−0.010417	−52083
2	− 0.854167	+1.312500	− 0.671875	0.093750	+0.020833	−52083
3	− 16.500000	11.833333	− 2.697917	+0.067708	0.052083	−52083
4	− 93.937500	45.125000	− 6.536458	−0.041667	0.083333	−52083
5	− 333.166667	120.812500	−12.687500	−0.234375	0.114583	−52083
6	− 907.187500	264.520833	−21.651042	−0.510417	0.145833	−52083
7	−2082.000000	507.875000	−33.927083	−0.869792	0.177083	−52083

APPENDIX I. (continued)

i	$P'^{1,4}_{-1,0}$	D	D^2	D^3	D^4	$10^7 D^5$
—7	—2515.500000	— 85.968750	+39.078125	+1.414062	—0.171875	—78125
—6	—1047.265625	— 31.710938	23.750000	0.820312	—0.156250	—78125
—5	— 358.500000	— 6.437500	12.921875	0.351562	—0.140625	—78125
—4	— 89.296875	+ 1.851562	5.843750	+0.007812	—0.125000	—78125
—3	— 11.750000	2.156250	+ 1.765625	—0.210938	—0.109375	—78125
—2	+ 0.046875	+ 0.476562	— 0.062500	—0.304688	—0.093750	—78125
—1	0.000000	— 0.187500	— 0.390625	—0.273438	—0.078125	—78125
0	0.015625	+ 0.164062	+ 0.031250	—0.117188	—0.062500	—78125
1	0.000000	— 1.468750	0.453125	+0.164062	—0.046875	—78125
2	17.859375	— 11.085938	+ 0.125000	0.570312	—0.031250	—78125
3	119.500000	— 37.687500	— 1.703125	1.101562	—0.015625	—78125
4	448.828125	— 93.273438	— 5.781250	1.757812	0.000000	—78125
5	1257.750000	—192.843750	—12.859375	2.539062	+0.015625	—78125
6	2936.171875	—354.398438	—23.687500	3.445312	0.031250	—78125
7	6042.000000	—598.937500	—39.015625	4.476562	0.046875	—78125

i	$P'^{1,4}_{-1,2}$	D	D^2	D^3	D^4	$10^7 D^5$
—7	+ 666.333333	—116.687500	— 4.312500	+1.515625	—0.010417	—52083
—6	219.187500	— 53.104167	— 1.119792	1.000000	—0.020833	—52083
—5	49.500000	— 19.000000	+ 0.385417	0.567708	—0.031250	—52083
—4	+ 4.270833	— 4.000000	0.703125	+0.218750	—0.041667	—52083
—3	— 0.500000	+ 0.270833	+ 0.333333	—0.046875	—0.052083	—52083
—2	+ 0.187500	0.187500	— 0.223958	—0.229167	—0.062500	—52083
—1	0.333333	0.125000	— 0.468750	—0.328125	—0.072917	—52083
0	+ 2.937500	2.458333	+ 0.098958	—0.343750	—0.083333	—52083
1	0.000000	7.562500	1.979167	—0.276042	—0.093750	—52083
2	— 47.479167	13.812500	5.671875	—0.125000	—0.104167	—52083
3	— 229.500000	17.583333	11.677083	+0.109375	—0.114583	—52083
4	— 707.062500	+ 13.250000	20.494792	0.427083	—0.125000	—52083
5	—1732.166667	— 6.812500	32.625000	0.828125	—0.135417	—52083
6	—3667.812500	— 52.229167	48.567708	1.312500	—0.145833	—52083
7	—7008.000000	—134.625000	68.822917	1.880208	—0.156250	—52083

i	$P'^{1,4}_{-1,4}$	D	D^2	D^3	D^4	$10^7 D^5$
—7	— 46.833333	+ 22.697917	—3.877604	+0.199219	+0.013021	—13021
—6	— 9.132812	6.639325	—1.757812	0.165365	+0.005208	—13021
—5	— 0.375000	+ 0.750000	—0.481771	0.110677	—0.002604	—13021
—4	— 0.013021	— 0.157552	+0.075521	+0.035156	—0.010417	—13021
—3	0.000000	+ 0.229167	+0.039062	—0.061198	—0.018229	—13021
—2	+ 0.210938	— 0.277344	—0.466146	—0.178385	—0.026042	—13021
—1	— 1.333333	— 2.364583	—1.315104	—0.316406	—0.033854	—13021
0	— 4.085938	— 5.220052	—2.382812	—0.475260	—0.041667	—13021
1	0.000000	— 6.531250	—3.544271	—0.654948	—0.049479	—13021
2	+ 31.471354	— 2.485677	—4.674479	—0.855469	—0.057292	—13021
3	128.375000	+ 12.229165	—5.648437	—1.076823	—0.065104	—13021
4	351.257812	44.425781	—6.341146	—1.319010	—0.072917	—13021
5	788.166667	102.416667	—6.627604	—1.582031	—0.080729	—13021
6	1559.648440	196.014323	—6.382812	—1.865885	—0.088542	—13021
7	2823.750000	336.531250	—5.481771	—2.170573	—0.095354	—13021

APPENDIX I. (continued)

$P^{1,4}_{1,-4}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−7	+1576.333333	+303.354167	+11.861980	−0.852862	−0.070312	−13021
−6	819.601562	181.717448	8.085938	−0.673180	−0.062500	−13021
−5	385.125000	100.875000	5.216146	−0.514323	−0.054688	−13021
−4	157.356771	50.514322	3.127605	−0.376300	−0.046875	−13021
−3	52.250000	21.822918	1.695313	−0.259116	−0.039062	−13021
−2	12.257812	7.488281	0.794272	−0.162760	−0.031250	−13021
−1	+ 1.333333	1.697917	0.299480	−0.087239	−0.023438	−13021
0	− 0.070312	+ 0.139323	0.085938	−0.032552	−0.015625	−13021
1	0.000000	0.000000	0.028646	+0.001302	−0.007812	−13021
2	− 0.002604	− 0.032552	+ 0.002604	0.014322	0.000000	−13021
3	− 0.125000	+ 0.229167	− 0.117188	+0.006511	+0.007812	−13021
4	− 3.914062	2.472656	− 0.455730	−0.022136	0.015625	−13021
5	− 23.416667	9.885417	− 1.138022	−0.071616	0.023438	−13021
6	− 84.179688	27.154948	− 2.289064	−0.141925	0.031250	−13021
7	− 230.250000	60.468750	− 4.033855	−0.233073	0.039062	−13021

$P^{1,4}_{1,-2}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−7	−3464.333333	−230.145833	+37.333333	+2.244792	−0.093750	−52083
−6	−1649.812500	−126.208333	24.578125	1.677083	−0.083333	−52083
−5	− 688.500000	− 62.000000	15.135417	1.192708	−0.072917	−52083
−4	− 237.395833	− 25.895833	8.505208	0.791667	−0.062500	−52083
−3	− 60.500000	− 8.270833	4.187500	0.473958	−0.052083	−52083
−2	− 8.812500	− 1.500000	1.682292	0.239583	−0.041667	−52083
−1	− 0.333333	+ 0.041667	0.489583	0.088542	−0.031250	−52083
0	− 0.062500	− 0.020833	0.109375	0.020833	−0.020833	−52083
1	0.000000	− 0.062500	+ 0.041667	0.036458	−0.010417	−52083
2	+ 0.854167	− 0.458333	− 0.213542	0.135417	0.000000	−52083
3	16.500000	− 3.583333	− 1.156250	0.317708	+0.010417	−52083
4	93.937500	− 13.812500	− 3.286458	0.583333	0.020833	−52083
5	333.166667	− 37.520833	− 7.104167	0.932292	0.031250	−52083
6	907.187500	− 83.083333	−13.109375	1.364583	0.041667	−52083
7	2082.000000	−160.875000	−21.802083	1.880208	0.052083	−52083

$P^{1,4}_{1,0}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−7	+2515.500000	−228.468750	−30.171875	+2.914052	+0.078125	−78125
−6	1047.265625	−117.898438	−17.593750	2.132812	0.062500	−78125
−5	358.500000	− 53.312500	− 9.015625	1.476562	0.046875	−78125
−4	89.296875	− 19.710938	− 3.687500	0.945312	0.031250	−78125
−3	+ 11.750000	− 5.093750	− 0.859375	0.539062	+0.015625	−78125
−2	− 0.046875	− 0.460938	+ 0.218750	0.257812	0.000000	−78125
−1	0.000000	+ 0.187500	0.296875	+0.101562	−0.015625	−78125
0	− 0.015625	− 0.148438	0.125000	+0.070312	−0.031250	−78125
1	0.000000	− 1.468750	0.453125	+0.164062	−0.046875	−78125
2	− 17.859375	− 6.773438	2.031250	0.382812	−0.062500	−78125
3	− 119.500000	− 22.062500	5.607375	0.726562	−0.078125	−78125
4	− 448.828125	− 56.335938	11.937500	1.195312	−0.093750	−78125
5	−1257.750000	−121.593750	21.765625	1.789062	−0.109375	−78125
6	−2936.171875	−232.835938	35.843750	2.507812	−0.125000	−78125
7	−6042.000000	−408.062500	54.921875	3.351562	−0.140625	−78125

APPENDIX I. (continued)

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$P^{1,4}_{1,2}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−7	−666.333333	+199.979167	−15.479167	−0.817708	+0.156250	−52083
−6	−219.187500	84.416667	− 8.703125	−0.458333	0.125000	−52083
−5	− 49.500000	27.250000	− 4.239585	−0.182292	0.093750	−52083
−4	− 4.270833	+ 4.854167	− 1.588543	+0.010418	0.062500	−52083
−3	+ 0.500000	− 0.395833	− 0.250000	0.119792	+0.031250	−52083
−2	− 0.187500	− 0.125000	+ 0.276040	0.145833	0.000000	−52083
−1	− 0.333333	+0.041667	0.489582	+0.088542	−0.031250	−52083
0	− 2.937500	0.479167	0.890625	−0.052083	−0.062500	−52083
1	0.000000	7.562500	1.979167	−0.276041	−0.093750	−52083
2	+ 47.479168	33.666667	4.255209	−0.583333	−0.125000	−52083
3	229.500000	97.166667	8.218750	−0.973958	−0.156250	−52083
4	707.062500	222.437500	14.369793	−1.447917	−0.187500	−52083
5	1732.166667	439.854167	23.208333	−2.005208	−0.218750	−52083
6	3667.812500	785.791667	35.234373	−2.645833	−0.250000	−52083
7	7008.000000	1302.625000	50.947917	−3.369792	−0.281250	−52083

$P^{1,4}_{1,4}$

i		D	D^2	D^3	D^4	$10^7 D^5$
−7	+ 46.833333	− 28.552083	+ 7.080730	−0.884112	+0.054686	−13021
−6	9.132812	− 7.944011	2.799480	−0.490886	0.041667	−13021
−5	0.375000	− 0.812500	+ 0.611980	−0.201825	0.028645	−13021
−4	+ 0.013021	+ 0.154947	− 0.106770	−0.016925	0.015625	−13021
−3	0.000000	− 0.229167	+ 0.018230	+0.053802	+0.002604	−13021
−2	− 0.210938	+ 0.347657	0.361980	+0.040364	−0.010416	−13021
−1	+ 1.333333	1.697916	+ 0.299480	−0.087239	−0.023437	−13021
0	+ 4.085938	+ 1.134113	− 0.794270	−0.319010	−0.036458	−13021
1	0.000000	− 6.531250	− 3.544271	−0.654948	−0.049479	−13021
2	− 31.471354	− 28.985677	− 8.575521	−1.095052	−0.062500	−13021
3	− 128.375000	− 76.416650	−16.513021	−1.639323	−0.075520	−13021
4	− 351.257812	−161.511720	−27.981772	−2.287760	−0.088541	−13021
5	− 788.166667	−299.458333	−43.606772	−3.040366	−0.101562	−13021
6	−1559.648440	−507.944009	−64.013022	−3.897136	−0.114582	−13021
7	−2823.750000	−807.156250	−89.825522	−4.858071	−0.127603	−13021

$P^{2,2}_{-2,-2}$

i		D	D^2	D^3	D^4
−7	+681.750000	+189.406250	+19.671875	+0.906250	+0.015625
−6	374.828125	121.093750	14.609375	0.781250	0.015625
−5	185.250000	71.531250	10.296875	0.656250	0.015625
−4	78.515625	37.718750	6.734375	0.531250	0.015625
−3	26.125000	16.656250	3.921875	0.406250	0.015625
−2	5.578125	5.343750	1.859375	0.281250	0.015625
−1	0.375000	+ 0.781250	+ 0.546875	0.156250	0.015625
0	0.015625	− 0.031250	− 0.015625	+0.031250	0.015625
1	0.000000	− 0.093750	+ 0.171875	−0.093750	0.015625
2	1.828125	− 2.406250	1.109375	−0.218750	0.015625
3	13.000000	− 9.968750	2.796875	−0.343750	0.015625
4	47.015625	− 25.781250	5.234375	−0.468750	0.015625
5	123.375000	− 52.843750	8.421875	−0.593750	0.015625
6	267.578125	− 94.156250	12.359375	−0.718750	0.015625
7	511.125000	−152.718750	17.046875	−0.843750	0.015625

APPENDIX I. (continued)

$P^{2,2}_{-2,0}$

i		D	D^2	D^3	D^4
−7	− 972.000000	−123.750000	+ 3.156250	+0.937500	+ 0.031250
−6	− 503.125000	− 73.093750	2.687500	0.812500	0.031250
−5	− 228.000000	− 38.437500	2.218750	0.687500	0.031250
−4	− 84.375000	− 16.781250	1.750000	0.562500	0.031250
−3	− 22.000000	− 5.125000	1.281250	0.437500	0.031250
−2	− 2.625000	− 0.468750	0.812500	0.312500	0.031250
−1	0.000000	+ 0.187500	+0.343750	0.187500	0.031250
0	+ 0.125000	− 0.156250	−0.125000	+0.062500	0.031250
1	0.000000	+ 1.500000	−0.593750	−0.062500	0.031250
2	− 10.125000	8.156250	−1.062500	−0.187500	0.031250
3	− 52.000000	22.812500	−1.531250	−0.312500	0.031250
4	− 159.375000	48.468750	−2.000000	−0.437500	0.031250
5	− 378.000000	88.125000	−2.468750	−0.562500	0.031250
6	− 765.625000	144.781250	−2.937500	−0.687500	0.031250
7	−1392.000000	221.437500	−3.406250	−0.812500	0.031250

$P^{2,2}_{-2\ 2}$

i		D	D^2	D^3	D^4
−7	+ 317.250000	−21.531250	−3.765625	+0.156250	+ 0.015625
−6	148.421875	−14.687500	−2.421875	+0.156250	0.015625
−5	57.000000	− 9.093750	−1.328125	+0.156250	0.015625
−4	15.234375	− 4.750000	−0.484375	+0.156250	0.015625
−3	+ 1.375000	− 1.656250	+ 0.109375	+0.156250	0.015625
−2	− 0.328125	+ 0.187500	0.453125	+0.156250	0.015625
−1	+ 0.375000	0.781250	0.546875	+ 0.156250	0.015625
0	− 0.265625	+ 0.125000	+ 0.390625	+0.156250	0.015625
1	+ 0.000000	− 1.781250	−0.015625	+0.156250	0.015625
2	9.421875	− 4.937500	−0.671875	+0.156250	0.015625
3	42.250000	− 9.343750	−1.578125	+0.156250	0.015625
4	118.734375	−15.000000	−2.734375	+0.156250	0.015625
5	265.125000	−21.906250	−4.140625	+0.156250	0.015625
6	513.671875	−30.062500	−5.796875	+0.156250	0.015625
7	902.625000	−39.468750	−7.703125	+0.156250	0.015625

$P^{2,2}_{0,-2}$

i		D	D^2	D^3	D^4
−7	−1616.000000	−225.687500	−0.781250	+0.937500	+ 0.031250
−6	− 912.625000	−148.468750	−0.687500	0.812500	0.031250
−5	− 468.000000	− 91.250000	−0.593750	0.687500	0.031250
−4	− 209.375000	− 51.031250	−0.500000	0.562500	0.031250
−3	− 76.000000	− 24.812500	−0.406250	0.437500	0.031250
−2	− 19.125000	− 9.593750	−0.312500	0.312500	0.031250
−1	− 2.000000	− 2.375000	−0.218750	0.187500	0.031250
0	+ 0.125000	− 0.156250	−0.125000	+0.062500	0.031250
1	0.000000	+ 0.062500	−0.031250	−0.062500	0.031250
2	− 1.625000	1.281250	+ 0.062500	−0.187500	0.031250
3	− 16.000000	6.500000	0.156250	−0.312500	0.031250
4	− 66.375000	18.718750	0.250000	−0.437500	0.031250
5	− 188.000000	40.937500	0.343750	−0.562500	0.031250
6	− 428.125000	76.156250	0.437500	−0.687500	0.031250
7	− 846.000000	127.375000	0.531250	−0.812500	0.031250

APPENDIX I. (continued)

$P^{2,2}_{0,0}$

i		D	D^2	D^3	D^4
−7	+2304.000000	−25.000000	−24.937500	+0.125000	+0.062500
−6	1225.000000	−18.500000	−18.437500	0.125000	0.062500
−5	576.000000	−13.000000	−12.937500	0.125000	0.062500
−4	225.000000	− 8.500000	− 8.437500	0.125000	0.062500
−3	64.000000	− 5.000000	− 4.937500	0.125000	0.062500
−2	9.000000	− 2.500000	− 2.437500	0.125000	0.062500
−1	0.000000	− 1.000000	− 0.937500	0.125000	0.062500
0	1.000000	− 0.500000	− 0.437500	0.125000	0.062500
1	0.000000	− 1.000000	− 0.937500	0.125000	0.062500
2	9.000000	− 2.500000	− 2.437500	0.125000	0.062500
3	64.000000	− 5.000000	− 4.937500	0.125000	0.062500
4	225.000000	− 8.500000	− 8.437500	0.125000	0.062500
5	576.000000	−13.000000	−12.937500	0.125000	0.062500
6	1225.000000	−18.500000	−18.437500	0.125000	0.062500
7	2304.000000	−25.000000	−24.937500	0.125000	0.062500

$P^{2,2}_{0,2}$

i		D	D^2	D^3	D^4
−7	− 752.000000	+154.937500	−5.656250	−0.562500	+ 0.031250
−6	− 361.375000	93.718750	−4.750000	−0.437500	0.031250
−5	− 144.000000	50.500000	−3.843750	−0.312500	0.031250
−4	− 40.625000	22.281250	−2.937500	−0.187500	0.031250
−3	− 4.000000	+ 6.062500	−2.031250	−0.062500	0.031250
−2	+ 1.125000	− 1.156250	−1.125000	+ 0.062500	0.031250
−1	− 2.000000	− 2.375000	−0.218750	0.187500	0.031250
0	− 2.125000	− 0.593750	+ 0.687500	0.312500	0.031250
1	0.000000	+ 1.187500	1.593750	0.437500	0.031250
2	− 8.375000	− 0.031250	2.500000	0.562500	0.031250
3	− 52.000000	− 7.250000	3.406250	0.687500	0.031250
4	− 167.625000	− 23.468750	4.312500	0.812500	0.031250
5	− 404.000000	− 51.687500	5.218750	0.937500	0.031250
6	− 821.875000	− 94.906250	6.125000	1.062500	0.031250
7	−1494.000000	−156.125000	7.031250	1.187500	0.031250

$P^{2,2}_{2,-2}$

i		D	D^2	D^3	D^4
−7	+934.250000	+23.656250	−8.078125	−0.093750	+0.015625
−6	537.796875	18.062500	−6.171875	−0.093750	0.015625
−5	282.750000	13.218750	−4.515625	−0.093750	0.015625
−4	130.859375	9.125000	−3.109375	−0.093750	0.015625
−3	49.875000	5.781250	−1.953125	−0.093750	0.015625
−2	13.546875	3.187500	−1.046875	−0.093750	0.015625
−1	+ 1.625000	1.343750	−0.390625	−0.093750	0.015625
0	− 0.140625	+ 0.250000	+ 0.015625	−0.093750	0.015625
1	0.000000	− 0.093750	0.171875	−0.093750	0.015625
2	− 0.203125	+ 0.312500	+ 0.078125	−0.093750	0.015625
3	+ 3.000000	1.468750	−0.265625	−0.093750	0.015625
4	19.359375	3.375000	−0.859375	−0.093750	0.015625
5	64.625000	6.031250	−1.703125	−0.093750	0.015625
6	160.546875	9.437500	−2.796875	−0.093750	0.015625
7	334.875000	13.593750	−4.140625	−0.093750	0.015625

APPENDIX I. (continued)

$P'^{2,\ 2}_{2,\ 0}$

i		D	D^2	D^1	D^4
−7	−1332.000000	+166.750000	+ 3.656250	−1.062500	+ 0.031250
−6	− 721.875000	104.093750	3.125000	−0.937500	0.031250
−5	− 348.000000	59.437500	2.593750	−0.812500	0.031250
−4	− 140.625000	29.781250	2.062500	−0.687500	0.031250
−3	− 42.000000	12.125000	1.531250	−0.562500	0.031250
−2	− 6.375000	3.468750	1.000000	−0.437500	0.031250
−1	0.000000	0.812500	+0.468750	−0.312500	0.031250
0	− 1.125000	1.156250	−0.062500	−0.187500	0.031250
1	0.000000	+ 1.500000	−0.593750	−0.062500	0.031250
2	+ 1.125000	− 1.156250	−1.125000	+0.062500	0.031250
3	− 12.000000	+ 9.812500	−1.656250	0.187500	0.031250
4	− 65.625000	− 27.468750	−2.187500	0.312500	0.031250
5	− 198.000000	− 57.125000	−2.718750	0.437500	0.031250
6	− 459.375000	−101.781250	−3.250000	0.562500	0.031250
7	− 912.000000	−164.437500	−3.781250	0.687500	0.031250

$P^{2,\ 2}_{2,\ 2}$

i		D	D^2	D^1	D^4
−7	+434.750000	−139.281250	+16.484375	−0.843750	+0.015625
−6	212.953125	− 82.718750	11.796875	−0.718750	0.015625
−5	87.000000	− 43.406250	7.859375	−0.593750	0.015625
−4	25.390625	− 18.343750	4.671875	−0.468750	0.015625
−3	+ 2.625000	− 4.531250	2.234375	−0.343750	0.015625
−2	− 0.796875	+ 1.031250	+0.546875	−0.218750	0.015625
−1	+ 1.625000	+ 1.343750	− 0.390625	−0.093750	0.015625
0	+ 2.390625	− 0.593750	− 0.578125	+0.031250	0.015625
1	0.000000	− 1.781250	− 0.015625	0.156250	0.015625
2	− 1.046875	+ 0.781250	+ 1.296875	0.281250	0.015625
3	+ 9.750000	10.093750	3.359375	0.406250	0.015625
4	48.890625	29.156250	6.171875	0.531250	0.015625
5	138.875000	60.968750	9.734375	0.656250	0.015625
6	308.203125	108.531250	14.046875	0.781250	0.015625
7	591.375000	174.843750	19.109375	0.906250	0.015625

P'^{3}_{-3}

i		D	D^2	D^1
−7	−49.666667	−10.854167	−0.812500	−0.020833
−6	−30.333333	− 7.791667	−0.687500	−0.020833
−5	−16.750000	− 5.229167	−0.562500	−0.020833
−4	− 7.916667	− 3.166667	−0.437500	−0.020833
−3	− 2.833333	− 1.604167	−0.312500	−0.020833
−2	− 0.500000	− 0.541667	−0.187500	−0.020833
−1	+ 0.083333	+ 0.020833	−0.062500	−0.020833
0	− 0.083333	+ 0.083333	+0.062500	−0.020833
1	0.000000	− 0.354167	0.187500	−0.020833
2	+ 1.333333	− 1.291667	0.312500	−0.020833
3	4.916667	− 2.729167	0.437500	−0.020833
4	11.750000	− 4.666667	0.562500	−0.020833
5	22.833333	− 7.104167	0.687500	−0.020833
6	39.166667	−10.041667	0.812500	−0.020833
7	61.750000	−13.479167	0.937500	

APPENDIX I. (continued)

P_{-1}^{n}

i		D	D^2	D^3
—7	+217.000000	+13.687500	—0.937500	—0.062500
—6	141.750000	10.250000	—0.812500	—0.062500
—5	86.250000	7.312500	—0.687500	—0.062500
—4	47.500000	4.875000	—0.562500	—0.062500
—3	22.500000	2.937500	—0.437500	—0.062500
—2	8.250000	1.500000	—0.312500	—0.062500
—1	+1.750000	0.562500	—0.187500	—0.062500
0	0.000000	0.125000	—0.062500	—0.062500
1	0.000000	0.187500	+0.062500	—0.062500
2	— 1.250000	0.750000	0.187500	—0.062500
3	— 6.750000	1.812500	0.312500	—0.062500
4	— 19.500000	3.375000	0.437500	—0.062500
5	— 42.500000	5.437500	0.562500	—0.062500
6	— 78.750000	8.000000	0.687500	—0.062500
7	—131.250000	11.062500	0.812500	—0.062500

P_{1}^{3}

i		D	D^2	D^3
—7	—297.000000	+18.687500	+1.062500	—0.062500
—6	—203.000000	14.625000	0.937500	—0.062500
—5	—131.250000	11.062500	0.812500	—0.062500
—4	— 78.750000	8.000000	0.687500	—0.062500
—3	— 42.500000	5.437500	0.562500	—0.062500
—2	— 19.500000	3.375000	0.437500	—0.062500
—1	— 6.750000	1.812500	0.312500	—0.062500
0	— 1.250000	0.750000	0.187500	—0.062500
1	0.000000	0.187500	+0.062500	—0.062500
2	0.000000	0.125000	—0.062500	—0.062500
3	+1.750000	0.562500	—0.187500	—0.062500
4	8.250000	1.500000	—0.312500	—0.062500
5	22.500000	2.937500	—0.437500	—0.062500
6	47.500000	4.875000	—0.562500	—0.062500
7	86.250000	7.312500	—0.687500	—0.062500

P_{3}^{3}

i		D	D^2	D^3
—7	+129.666667	—21.854167	+1.187500	—0.020833
—6	91.583333	—17.416667	1.062500	—0.020833
—5	61.750000	—13.479167	0.937500	—0.020833
—4	39.166667	—10.041667	0.812500	—0.020833
—3	22.833333	— 7.104167	0.687500	—0.020833
—2	11.750000	— 4.666667	0.562500	—0.020833
—1	4.916667	— 2.729167	0.437500	—0.020833
0	+ 1.333333	— 1.291667	0.312500	—0.020833
1	0.000000	— 0.354167	0.187500	—0.020833
2	— 0.083333	+ 0.083333	+0.062500	—0.020833
3	+0.083333	+ 0.020833	—0.062500	—0.020833
4	— 0.500000	— 0.541667	—0.187500	—0.020833
5	— 2.833333	— 1.604167	—0.312500	—0.020833
6	— 7.916667	— 3.166667	—0.437500	—0.020833
7	— 16.750000	— 5.229167	—0.562500	—0.020833

APPENDIX I. (continued)

$P^{0,4}_{0,4}$

i		D	D^2	D^3	D^4
−5	+ 31.471355	+ 13.250000	+ 1.950521	+ 0.119792	+ 0.002604
−4	13.662500	7.088542	1.309896	0.098958	0.002604
−3	4.685938	3.177083	0.794271	0.078125	0.002604
−2	+ 0.666667	1.015625	0.403646	0.057292	0.002604
−1	− 0.070312	+ 0.104167	+ 0.138021	0.036458	0.002604
0	0.000000	− 0.057292	− 0.026604	+ 0.015625	0.002604
1	+ 0.002604	+ 0.031250	− 0.018229	− 0.005208	0.002604
2	0.062500	− 0.136208	+ 0.091146	− 0.026042	0.002604
3	1.304688	− 1.041667	0.325521	− 0.046875	0.002604
4	5.854167	− 3.203125	0.684896	− 0.067708	0.002604
5	16.835938	− 7.114583	1.169271	− 0.088542	0.002604

$P^{0,4}_{0,-2}$

i		D	D^2	D^3	D^4
−5	− 47.479167	−9.927083	+ 0.708333	+ 0.229167	+ 0.010417
−4	− 15.125000	−3.958333	0.552083	0.187500	0.010417
−3	− 2.937500	−0.989583	0.395833	0.145833	0.010417
−2	− 0.166667	−0.020833	0.239583	0.104167	0.010417
−1	− 0.062500	−0.052083	+ 0.083333	0.062500	0.010417
0	+ 0.125000	−0.083333	−0.072917	+ 0.020833	0.010417
1	− 0.854167	+ 0.885417	0.229167	−0.020833	0.010417
2	− 8.250000	3.854167	0.285417	−0.062500	0.010417
3	− 31.312500	9.822917	−0.541667	−0.104167	0.010417
4	− 83.291667	19.791667	−0.697917	−0.145833	0.010417
5	−181.437500	34.760417	−0.854167	−0.187500	0.010417

$P^{0,4}_{0,0}$

i		D	D^2	D^3	D^4
−5	+ 17.859375	− 2.156250	−0.953125	+ 0.093750	+ 0.015625
−4	+ 2.937500	− 0.906250	−0.328125	0.093750	0.015625
−3	− 0.015625	− 0.156250	+ 0.046875	0.093750	0.015625
−2	0.000000	+ 0.093750	0.171875	0.093750	0.015625
−1	− 0.015625	− 0.156250	+ 0.046875	0.093750	0.015625
0	+ 2.937500	− 0.906250	0.328125	0.093750	0.015625
1	17.859375	− 2.156250	−0.953125	0.093750	0.015625
2	59.750000	− 3.906250	−1.828125	0.093750	0.015625
3	149.609375	− 6.156250	−2.953125	0.093750	0.015625
4	314.437500	− 8.906250	− 4.328125	0.093750	0.015625
5	587.234375	−12.156250	−5.953125	0.093750	0.015625

$P^{0,4}_{0,2}$

i		D	D^2	D^3	D^4
−5	− 0.854167	+ 0.885417	−0.229167	−0.020833	+ 0.010417
−4	+ 0.125000	− 0.083333	−0.072917	+ 0.020833	0.010417
−3	− 0.062500	− 0.052083	+ 0.083333	0.062500	0.010417
−2	− 0.166667	− 0.020833	0.239583	0.104167	0.010417
−1	− 2.937500	− 0.989583	0.395833	0.145833	0.010417
0	− 15.125000	− 3.958333	0.552083	0.187500	0.010417
1	− 47.479167	− 9.927083	0.708333	0.229167	0.010417
2	−114.750000	−19.895833	0.864583	0.270833	0.010417
3	−235.687500	−34.864583	1.020833	0.312500	0.010417
4	−433.041667	−55.833333	1.177083	0.354167	0.010417
5	−733.562500	−83.802083	1.333333	0.395833	0.010417

APPENDIX I. (continued)

$P_{n0,\,4}^{0,\,4}$

i		D	D^2	D^3	D^4
—5	+ 0.002604	+0.031250	—0.018229	—0.005208	+ 0.002604
—4	0.000000	—0.057292	—0.002604	+0.015625	0.002604
—3	—0.070312	+ 0.104167	+0.138021	0.036458	0.002604
—2	+ 0.666667	1.015625	0.403646	0.057292	0.002604
—1	4.085938	3.177083	0.794271	0.078125	0.002604
0	13.062500	7.088542	1.309896	0.098958	0.002604
1	31.471354	13.250000	1.950521	0.119792	0.002604
2	64.187500	22.161458	2.716146	0.140625	0.002604
3	117.085938	34.322917	3.606771	0.161458	0.002604
4	197.041667	50.234375	4.622396	0.182292	0.002604
5	311.929688	70.395833	5.763021	0.203125	0.002604

$P_{n1,\,2}^{-1,\,-2}$

i		D	D^2	D^3
—5	—58.625000	—19.062500	—1.937500	—0.062500
—4	—28.500000	—12.125000	—1.562500	—0.062500
—3	—10.625000	— 6.687500	—1.187500	—0.062500
—2	— 2.000000	— 2.750000	—0.812500	—0.062500
—1	+ 0.375000	— 0.312500	—0.437500	—0.062500
0	— 0.500000	+ 0.625000	—0.062500	—0.062500
1	— 1.625000	+ 0.062500	+ 0.312500	—0.062500
2	0.000000	— 2.000000	0.687500	—0.062500
3	+ 7.375000	— 5.562500	1.062500	—0.062500
4	23.500000	—10.625000	1.437500	—0.062500
5	51.375000	—17.187500	1.812500	—0.062500

$P_{n1,\,2}^{-1,\,0}$

i		D	D^2	D^3
—5	+ 63.000000	+ 2.750000	—1.875000	—0.125000
—4	24.000000	+ 0.500000	—1.625000	—0.125000
—3	5.000000	—0.750000	—1.375000	—0.125000
—2	0.000000	—1.000000	—1.125000	—0.125000
—1	3.000000	—0.250000	—0.875000	—0.125000
0	8.000000	+ 1.500000	—0.625000	—0.125000
1	+ 9.000000	4.250000	—0.375000	—0.125000
2	0.000000	8.000000	—0.125000	—0.125000
3	— 25.000000	12.750000	+ 0.125000	—0.125000
4	— 72.000000	18.500000	0.375000	—0.125000
5	—147.000000	25.250000	0.625000	—0.125000

$P_{n1,\,2}^{-1,\,2}$

i		D	D^2	D^3
—5	—11.375000	+ 5.312500	—0.437500	—0.062500
—4	— 1.500000	+ 2.125000	—0.562500	—0.062500
—3	+ 0.625000	—0.562500	—0.687500	—0.062500
—2	— 2.000000	—2.750000	—0.812500	—0.062500
—1	— 6.375000	—4.437500	—0.937500	—0.062500
0	— 9.500000	—5.625000	—1.062500	—0.062500
1	— 8.375000	—6.312500	—1.187500	—0.062500
2	0.000000	—6.500000	—1.312500	—0.062500
3	+ 18.625000	—6.187500	—1.437500	—0.062500
4	50.500000	—5.375000	—1.562500	—0.062500
5	98.625000	—4.062500	—1.687500	—0.062500

APPENDIX I. (continued)

$P^{1,\ 2}_{1,\ -2}$

i		D	D^2	D^3
−5	+58.625000	+10.687500	−0.187500	−0.062500
−4	28.500000	7.375000	−0.062500	−0.062500
−3	10.625000	4.562500	+0.062500	−0.062500
−2	+ 2.000000	2.250000	0.187500	−0.062500
−1	− 0.375000	+ 0.437500	0.312500	−0.062500
0	+ 0.500000	− 0.875000	0.437500	−0.062500
1	+1.625000	− 1.687500	0.562500	−0.062500
2	0.000000	− 2.600000	0.687500	−0.062500
3	−7.375000	− 1.812500	0.812500	−0.062500
4	−23.500000	− 1.125000	0.937500	−0.062500
5	−51.375000	+ 0.062500	1.062500	−0.062500

$P^{\prime 1,\ 2}_{1,\ 0}$

i		D	D^2	D^3
−5	−63.000000	+ 6.250000	+1.625000	−0.125000
−4	−24.000000	3.500000	1.375000	−0.125000
−3	− 5.000000	1.750000	1.125000	−0.125000
−2	0.000000	1.000000	0.875000	−0.125000
−1	− 3.000000	1.250000	0.625000	−0.125000
0	− 8.000000	2.500000	0.375000	−0.125000
1	− 9.000000	4.750000	+ 0.125000	−0.125000
2	0.000000	8.000000	−0.125000	−0.125000
3	+ 25.000000	12.250000	−0.375000	−0.125000
4	72.000000	17.500000	−0.625000	−0.125000
5	147.000000	23.750000	−0.875000	−0.125000

$P^{\prime\prime 1,\ 2}_{1,\ 2}$

i		D	D^2	D^3
−5	+ 11.375000	− 6.937500	+1.312500	−0.062500
−4	+ 1.500000	− 2.375000	0.937500	−0.062500
−3	− 0.625000	+ 0.687500	0.562500	−0.062500
−2	+ 2.000000	2.250000	+ 0.187500	−0.062500
−1	6.375000	2.312500	−0.187500	−0.062500
0	9.500000	+ 0.875000	−0.562500	−0.062500
1	+ 8.375000	− 2.062500	−0.937500	−0.062500
2	0.000000	− 6.500000	−1.312500	−0.062500
3	−18.625000	−12.437500	−1.687500	−0.062500
4	−50.500000	−19.875000	−2.062500	−0.062500
5	−98.625000	−28.812500	−2.437500	−0.062500

i	$P^{2,\ 0}_{-2,\ 0}$			$P^{\prime 2,\ 0}_{0,\ 0}$		
		D	D^2		D	D^2
−5	+ 20.125000	+ 3.125000	+0.125000	−49.000000	+0.250000	+ 0.250000
−4	14.250000	2.625000	0.125000	−36.000000	0.250000	0.250000
−3	9.375000	2.125000	0.125000	−25.000000	0.250000	0.250000
−2	5.500000	1.625000	0.125000	−16.000000	0.250000	0.250000
−1	2.625000	1.125000	0.125000	− 9.000000	0.250000	0.250000
0	+ 0.750000	0.625000	0.125000	− 4.000000	0.250000	0.250000
1	− 0.125000	+ 0.125000	0.125000	− 1.000000	0.250000	0.250000
2	0.000000	−0.375000	0.125000	0.000000	0.250000	0.250000
3	+ 1.125000	−0.875000	0.125000	− 1.000000	0.250000	0.250000
4	3.250000	−1.375000	0.125000	− 4.000000	0.250000	0.250000
5	6.375000	−1.875000	0.125000	− 9.000000	0.250000	0.250000

APPENDIX I. (continued)

i	$P''^{2,0}_{2,0}$	D	D^2
−5	+28.875000	−3.875000	+0.125000
−4	21.750000	−3.375000	0.125000
−3	15.625000	−2.875000	0.125000
−2	10.500000	−2.375000	0.125000
−1	6.375000	−1.875000	0.125000
0	3.250000	−1.375000	0.125000
1	+1.125000	−0.875000	0.125000
2	0.000000	−0.375000	0.125000
3	−0.125000	+0.125000	0.125000
4	+0.750000	0.625000	0.125000
5	2.625000	1.125000	0.125000

i	P''^{0}_{0}	$P''^{0,1}_{0,-1}$	D	$P''^{0,1}_{0,1}$	D
−3	+1.000000	+0.500000	+0.500000	+0.500000	+0.500000
−2	1.000000	−0.500000	0.500000	1.500000	0.500000
−1	1.000000	−1.500000	0.500000	2.500000	0.500000
0	1.000000	−2.500000	0.500000	3.500000	0.500000
1	1.000000	−3.500000	0.500000	4.500000	0.500000
2	1.000000	−4.500000	0.500000	5.500000	0.500000
3	1.000000	−5.500000	0.500000	6.500000	0.500000

i	$P''^{0,2}_{0,-2}$	D	D^2	$P''^{0,2}_{0,0}$	D	D^2
−3	+0.500000	+0.625000	+0.125000	0.000000	+0.250000	+0.250000
−2	−0.125000	+0.125000	0.125000	−1.000000	0.250000	0.250000
−1	+0.250000	−0.375000	0.125000	−4.000000	0.250000	0.250000
0	1.625000	−0.875000	0.125000	−9.000000	0.250000	0.250000
1	4.000000	−1.375000	0.125000	−16.000000	0.250000	0.250000
2	7.375000	−1.875000	0.125000	−25.000000	0.250000	0.250000
3	11.750000	−2.375000	0.125000	−36.000000	0.250000	0.250000

i	$P''^{0,2}_{0,2}$	D	D	$P''^{1,0}_{-1,0}$	D
−3	+0.500000	+0.625000	+0.125000	−6.000000	−0.500000
−2	2.125000	1.125000	0.125000	−5.000000	−0.500000
−1	4.750000	1.625000	0.125000	−4.000000	−0.500000
0	8.375000	2.125000	0.125000	−3.000000	−0.500000
1	13.000000	2.625000	0.125000	−2.000000	−0.500000
2	18.625000	3.125000	0.125000	−1.000000	−0.500000
3	25.250000	3.625000	0.125000	0.000000	−0.500000

	$P''^{1,0}_{1,0}$	D
−3	+6.000000	−0.500000
−2	5.000000	−0.500000
−1	4.000000	−0.500000
0	3.000000	−0.500000
1	2.000000	−0.500000
2	1.000000	−0.500000
3	0.000000	−0.500000

APPENDIX II. (Continued)

PARTICULAR VALUES OF OPERATORS FOR CALCULATING CORRECTIONS FOR THE INDIRECT TERM FOR THE PERTURBING FUNCTION. (Operators from -1, +2, -4, +8, -16, +32, -64, +128, -256)

	i			i			i			i	
P^0_0	−1	−1.00000000	$P^{1,6}_{1,0}$	−1	0.05031722	$P^{0,6}_{0,2}$	−1	−0.02411406	$P^{2,2}_{2,2}$	−1	−0.42187500
	+1	−1.00000000		+1	0.00000000		+1	−0.10839844		+1	−0.01687500
P^2_0	−1	+0.50000000	$P^{0,1}_{0,1}$	−1	+1.50000000	$P^{0,8}_{0,2}$	−1	−0.01592500	$P^{2,4}_{2,2}$	−1	−0.11062500
	+1	0.50000000		+1	−0.50000000		+1	+0.02228333		+1	+0.01687500
P^4_0	−1	0.01562500	$P^{0,3}_{0,1}$	−1	0.00000000	$P^{2,2}_{0,2}$	−1	0.06250000	$P^{1,3}_{1,3}$	−1	−0.08333333
	+1	0.01562500		+1	+0.37500000		+1	0.18750000		+1	0.00000000
$P^{0,2}_{0,0}$	−1	0.50000000	$P^{0,5}_{0,1}$	−1	0.00000000	$P^{2,4}_{0,2}$	−1	+0.02083333	$P^{1,5}_{1,3}$	−1	−0.02083333
	+1	0.50000000		+1	−0.05208333		+1	−0.18750000		+1	0.00000000
$P^{0,4}_{0,0}$	−1	0.01562500	$P^{0,7}_{0,1}$	−1	0.00000000	$P^{1,3}_{-1,3}$	−1	0.00000000	$P^{0,4}_{0,4}$	−1	−0.02343750
	+1	0.01562500		+1	+0.01388889		+1	−0.66666667		+1	−0.32552083
$P^{0,6}_{0,0}$	−1	0.02517361	$P^{2,1}_{0,1}$	−1	−0.75000000	$P^{1,5}_{-1,3}$	−1	+0.83333333	$P^{0,6}_{0,4}$	−1	−0.00234375
	+1	0.02517361		+1	+0.25000000		+1	0.00000000		+1	+0.18828125
$P^{0,8}_{0,0}$	1	0.01592009	$P^{2,3}_{0,1}$	−1	0.00000000	$P^{2,4}_{-2,4}$	−1	−0.00292969	$P^{0,8}_{0,4}$	−1	−0.00146800
	+1	+0.01592009		+1	−0.18750000		+1	−1.09863100		+1	−0.27636300
$P^{2,2}_{0,0}$	−1	−0.25000000	$P^{1,2}_{-1,2}$	−1	0.00000000	P^3_3	−1	−5.33333333	$P^{2,4}_{0,4}$	−1	+0.01171875
	+1	−0.25000000		+1	−0.75000000		+1	−0.16666667		+1	+0.16276042
$P^{2,4}_{0,0}$	−1	−0.00781250	$P^{1,4}_{-1,2}$	−1	0.00000000	$P^{3,2}_{3,0}$	−1	+2.66666667	$P^{1,5}_{-1,5}$	−1	0.00000000
	+1	−0.00781250		+1	+0.75000000		+1	0.08333333		+1	−0.67500000
$P^{1,1}_{-1,1}$	−1	0.00000000	$P^{1,6}_{-1,2}$	−1	0.00000000	$P^{2,1}_{2,1}$	−1	+5.06250000	$P^{3,2}_{3,2}$	−1	−0.66666667
	+1	−1.00000000		+1	−0.21679687		+1	−0.06250000		+1	−0.06250000
$P^{1,3}_{-1,1}$	−1	0.00000000	$P^{3,2}_{-1,2}$	−1	0.00000000	$P^{2,3}_{2,1}$	−1	0.00000000	$P^{2,3}_{2,3}$	−1	−0.14062500
	+1	+0.75000000		+1	+0.56250000		+1	+0.01687500		+1	−0.01166667
$P^{1,5}_{-1,1}$	−1	0.00000000	$P^{2,3}_{-2,3}$	−1	−0.00520833	$P^{1,2}_{1,2}$	−1	−0.25000000	$P^{1,4}_{1,4}$	−1	−0.01687500
	+1	−0.10416667		+1	−1.12500000		+1	0.00000000		+1	0.00000000
$P^{3,1}_{-1,1}$	−1	0.00000000	$P^{3,1}_{3,-1}$	−1	−2.66666667	$P^{1,4}_{1,2}$	−1	−0.08333333	$P^{1,6}_{1,4}$	−1	−0.00168750
	+1	+0.75000000		+1	+0.25000000		+1	0.00000000		+1	0.00000000
$P^{2,2}_{-2,2}$	−1	−0.01562500	P^2_2	−1	−3.37500000	$P^{1,6}_{1,2}$	−1	−0.04882810	$P^{0,5}_{0,5}$	−1	−0.01666667
	+1	−1.26562500		+1	−0.12500000		+1	0.00000000		+1	−0.33750000
$P^{2,4}_{-2,2}$	−1	−0.00520833	P^4_2	−1	+3.37500000	$P^{3,2}_{1,2}$	−1	+0.18750000	$P^{0,7}_{0,5}$	−1	+0.00138889
	+1	+1.26562500		+1	−0.04166667		+1	0.00000000		+1	+0.59062500
$P^{3,2}_{3,-2}$	−1	−2.00000000	$P^{2,2}_{2,0}$	−1	+1.68750000	$P^{0,3}_{0,3}$	−1	−0.04166667	$P^{1,6}_{-1,6}$	−1	0.00000000
	+1	−0.02083333		+1	0.06250000		+1	−0.33333333		+1	−0.72947048
$P^{2,1}_{2,-1}$	−1	−1.68750000	$P^{2,4}_{2,0}$	−1	0.05273438	$P^{0,5}_{3,3}$	−1	−0.01041667	$P^{2,4}_{2,4}$	−1	−0.07910156
	+1	+0.18750000		+1	0.00195312		+1	+0.41666667		+1	−0.04069010
$P^{2,3}_{2,-1}$	−1	+1.26562500	$P^{1,1}_{1,1}$	−1	+3.00000000	$P^{0,7}_{0,3}$	−1	−0.00833333	$P^{1,5}_{1,5}$	−1	−0.03333333
	+1	0.00000000		+1	0.00000000		+1	−0.17916667		+1	0.00000000
P^1_1	−1	−2.00000000	$P^{1,3}_{1,1}$	−1	0.00000000	$P^{2,3}_{0,3}$	−1	+0.02083333	$P^{0,6}_{0,6}$	−1	−0.01356337
	+1	0.00000000		+1	0.00000000		+1	+0.16666667		+1	−0.36473524
P^3_1	−1	+1.50000000	$P^{1,5}_{1,1}$	−1	0.00000000	$P^{1,4}_{-1,4}$	−1	0.00000000	$P^{0,8}_{0,6}$	−1	+0.00386800
	+1	0.00000000		+1	0.00000000		+1	−0.65104167		+1	+0.72946100
$P^{3,2}_{1,0}$	−1	−0.75000000	$P^{3,1}_{1,1}$	−1	−2.25000000	$P^{1,6}_{-1,4}$	−1	0.00000000	$P^{1,6}_{1,6}$	−1	−0.02712674
	+1	0.00000000		+1	0.00000000		+1	+0.97656250		+1	0.00000000
$P^{1,2}_{1,0}$	−1	+1.00000000	$P^{0,2}_{0,2}$	−1	−0.12500000	P^4_4	−1	−8.13802083	$P^{0,7}_{0,7}$	−1	−0.01205357
	+1	0.00000000		+1	−0.37500000		+1	−0.21093750		+1	−0.40631921
$P^{1,4}_{1,0}$	−1	0.03125000	$P^{0,4}_{0,2}$	−1	−0.04166667	$P^{3,1}_{3,1}$	−1	+8.00000000	$P^{0,8}_{0,8}$	−1	−0.01139700
	+1	0.00000000		+1	+0.37500000		+1	−0.08333333		+1	−0.46338000

APPENDIX II. (continued)

	i			i			i			i	
$P^{2,2}_{2,-2}$	0	−0.42187500	$P'^{2,1}_{0,1}$	0	+0.25000000	$P'^{0,6}_{0,2}$	0	−0.10839844	$P'^{2,1}_{-2,-1}$	0	+0.18750000
$P^{1,1}_{1,-1}$	0	+3.00000000	$P'^{1,2}_{-1,2}$	0	0.00000000	$P'^{2,2}_{0,2}$	0	+0.18750000	$P'^{1,2}_{-1,-2}$	0	0.00000000
$P^{1,3}_{1,-1}$	0	0.00000000	$P'^{1,4}_{-1,2}$	0	0.00000000	$P''^{1,3}_{-1,3}$	0	0.00000000	$P'^{1,4}_{-1,-2}$	0	0.00000000
P^{0}_{0}	0	−1.00000000	$P'^{2,1}_{-2,1}$	0	−0.06250000	P_{-2}	0	−0.12500000	$P'^{0,3}_{0,-3}$	0	−0.04166667
P^{2}_{0}	0	+0.50000000	P'^{1}_{-1}	0	0.00000000	$P'^{2,2}_{-2,0}$	0	+0.06250000	$P'^{0,5}_{0,-3}$	0	−0.01041667
$P'^{2,2}_{0,0}$	0	−0.25000000	P'^{3}_{-1}	0	0.00000000	$P'^{1,1}_{-1,-1}$	0	0.00000000	$P'^{1,4}_{1,-4}$	0	−0.04687500
$P'^{0,2}_{0,0}$	0	+0.50000000	$P'^{1,2}_{-1,0}$	0	0.00000000	$P'^{1,3}_{-1,-1}$	0	0.00000000	$P'^{2,2}_{2,2}$	0	−1.26562500
$P'^{0,4}_{0,0}$	0	−0.01562500	$P'^{1,4}_{-1,0}$	0	0.00000000	$P'^{0,2}_{0,-2}$	0	−0.12500000	$P'^{1,3}_{1,3}$	0	−0.66666667
$P'^{0,6}_{0,0}$	0	+0.02517361	$P'^{0,1}_{0,-1}$	0	+1.50000000	$P'^{0,4}_{0,-2}$	0	−0.04166667	$P'^{0,4}_{0,4}$	0	−0.32552083
$P'^{1,1}_{-1,1}$	0	0.00000000	$P'^{2,1}_{0,-1}$	0	−0.75000000	$P'^{0,6}_{0,-2}$	0	−0.02441406	$P'^{0,6}_{0,4}$	0	+0.48828125
$P'^{1,3}_{-1,1}$	0	0.00000000	$P'^{0,3}_{0,-1}$	0	0.00000000	$P'^{2,2}_{0,-2}$	0	−0.06250000	$P'^{2,2}_{-2,-2}$	0	−0.01562500
$P'^{2,2}_{-2,2}$	0	−0.04687500	$P'^{0,5}_{0,-1}$	0	0.00000000	$P'^{1,3}_{1,-3}$	0	−0.08333333	$P'^{1,3}_{-1,-3}$	0	0.00000000
$P'^{2,1}_{2,-1}$	0	+5.06250000	$P'^{1,2}_{1,-2}$	0	−0.25000000	P'^{3}_{3}	0	−5.33333333	$P'^{0,4}_{0,-4}$	0	−0.02343750
P'^{1}_{1}	0	−2.00000000	$P'^{1,4}_{1,-2}$	0	−0.08333333	$P'^{2,1}_{2,1}$	0	−1.68750000	$P'^{0,6}_{0,-4}$	0	−0.00234375
P'^{3}_{1}	0	+1.50000000	P'^{2}_{2}	0	−3.37500000	$P'^{1,2}_{1,2}$	0	−0.75000000	$P'^{0,5}_{0,5}$	0	−0.33750000
$P'^{1,2}_{1,0}$	0	1.00000000	$P'^{2,2}_{2,0}$	0	+1.68750000	$P'^{1,4}_{1,2}$	0	+0.75000000	$P'^{1,4}_{1,4}$	0	−0.65104167
$P'^{1,4}_{1,0}$	0	+0.03125000	$P'^{1,1}_{1,1}$	0	−1.00000000	$P'^{0,3}_{0,3}$	0	−0.33333333	$P'^{1,4}_{-1,-4}$	0	0.00000000
$P'^{0,1}_{0,1}$	0	−0.50000000	$P'^{1,3}_{1,1}$	0	+0.75000000	$P'^{0,5}_{0,3}$	0	+0.41666667	$P'^{0,5}_{0,-5}$	0	−0.01666667
$P'^{0,3}_{0,1}$	0	+0.37500000	$P'^{0,2}_{0,2}$	0	−0.37500000	$P'^{1,4}_{-1,4}$	0	0.00000000	$P'^{0,6}_{0,6}$	0	−0.36473524
$P'^{0,5}_{0,1}$	0	−0.05208333	$P'^{0,4}_{0,2}$	0	+0.37500000	P'^{3}_{-3}	0	−0.16666667	$P'^{0,6}_{0,-6}$	0	−0.01356337

PART FOUR

NEW ELEMENTS OF PLUTO

INTRODUCTION

In Part One of the "Theory of Motion of the Planet Pluto", a preliminary improvement was made of Plutonian elements, with consideration of first-order perturbations on the basis of 24 normal places of 1914-1951. /148

After determining second-order perturbations (Part Two of the present work), and recalculating the series of perturbations of the first order (Part Three), the values of the perturbations of Plutonian coordinates were substantially changed and it became necessary to improve the Plutonian elements once again.

During this time new observations of Pluto were obtained, normal positions were compiled from them, and normal positions that participated in the first correction were reexamined.

Part Four of the "Theory of Motion of the Planet Pluto" is devoted to an improvement of Plutonian elements, with consideration of perturbations of the first and second orders on the basis of 1914-1958 observations.

CHAPTER I

COMPARISON WITH OBSERVATIONS

1. Observations of Pluto and Compilation of Normal Positions

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Observations of Pluto, obtained after our improvement of Pluto's elements, were reduced to axes 1950.0, corrected for parallax, and compared with the ephemeris in B. J. Table 1 contains these observations. The distribution of the columns is the same as in Table 21 of Part One.

In compiling the normal positions we used the formulas:

$$\Delta\alpha \cos\delta = a + b(t - t_0),$$
$$\Delta\delta = a' + b'(t - t_0),$$

where

$$t_0 = \frac{t_1 + t_2 + \ldots + t_n}{n}.$$

Corrections for the part of the aberration, depending on the eccentricity of the earth's orbit, were introduced into the normal positions.

The old normal positions (Table 22 of Part One) were reexamined. The observations between 1914 and 1927 were not combined into normal positions, with the exception of four observations from 1919. Two normal positions were formed in each opposition of 1931-1932, 1939-1940, and 1941-1942. An attempt was made to compile normal positions for the oppoisitions 1935-1936, 1942-1943, and 1945-1946. Two separate observations were taken for the 1944-1945 opposition.

Table 2 shows the normal positions.

2. Representation of Normal Positions by Various Theories of Motion

The mean anomalies of Pluto, Jupiter, Saturn, Uranus, and Neptune were calculated 45 times in Table 2. With respect to these arguments, from Tables 4, 8, and 12 of Part Three, and from Tables 19, 19', 19", and 20 of Part Two, we obtained perturbations of the first and second order of the logarithm of the radius vector and longitude, and perturbations of the first order of the longitude of the node and inclination of Pluto, due to the three major planets.

Perturbations due to Neptune were obtained by interpolation from Table 13 of Part Three. Table 3 contains the values of these perturbations on the indicated dates.

The normal positions of Pluto were compared with various theories:

I. The normal positions were compared with the system of Plutonian elements which we obtained in Part One (henceforth we will call this system A), with consideration of perturbations of the first and second order (Table 3);

II. Normal positions were represented by Bauer's XIX system of elements (we will call this system XIX), with consideration of the same perturbations;

III. The normal positions were compared with the ephemeris obtained from Astr. Pap., 12 (Eckert, Brouwer, Clemence, 1951) (henceforth called Astr. Pap., 12);

IV. Normal positions were obtained (beginning with 1932) on the basis of comparing observations with the ephemeris in B. J.* /155

The results of these comparisons are given in columns 3-6 of Table 4.

The authors based the tables of integration of Pluto in Astr. Pap., 12 on system XIX. In determining perturbations and the constants of integration, we used system A. Therefore, it was impossible to expect complete agreement of results II (Table 4, column 4) with results III (Table 4, column 5). Consequently, the difference of II-III did not exceed 0".4, with respect to right ascension, and 0".5 with respect to declination (see Table 4, column 7 or Figure 3). This indicates favorable agreement between the results obtained by our theory and the available results.

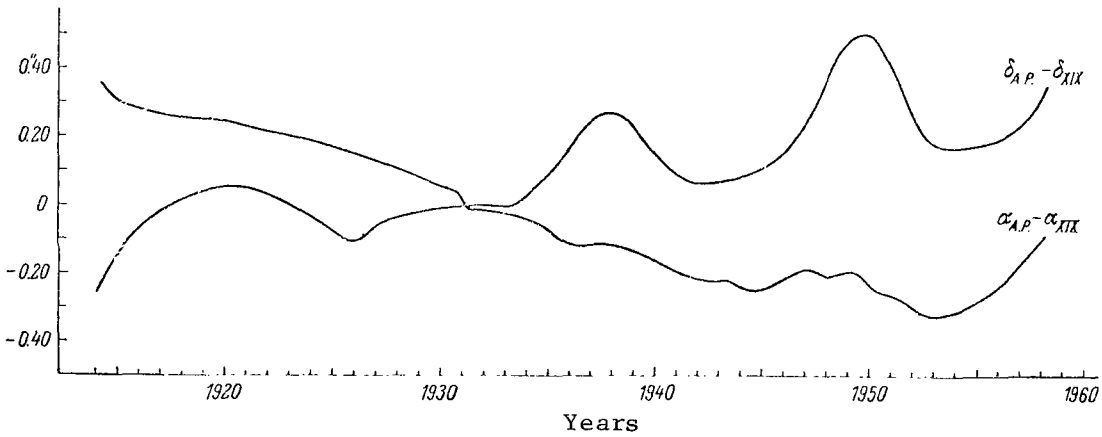

Figure 3

* We note that in the B. J., the ephemeris of Pluto for the end of 1951 and beginning of 1952 experiences a well-defined "jump." This circumstance is not noted in the explanation to B. J.

Table 1

№	Universal Time	$\alpha_{1950.0}$	$\delta_{1950.0}$	Observed-calculated $\Delta\alpha$	Observed-calculated $\Delta\delta$	Observational place	Source
		$h\ m\ s$	$^\circ\ '\ ''$	s	$''$		
1*	1944 Feb. 22.47342	8 44 19.15	+23 55 30.9	+0.79	+ 2.7	Tokyo	T. A. B., (2), 69.
2	1947 Nov. 11.46837	9 18 49.95	07 40.5	+0.28	+ 2.1	Yerkes	A. J., 54, 1176.
3*	23.47383	18 53.95	10 50.9	+1.26	+ 1.7	„	
4	Dec. 9.41154	18 31.50	16 46.2	+0.24	+ 2.0	„	
5*	20.34377	17 56.71	21 45.5	−4.04	+ 2.8	„	
6*	1948 Nov. 29.72639	25 58.11	08 16.9	+1.21	− 9.7	Tokyo	T. A. B., (2), 69.
7*	1949 Feb. 18.45452	20 07.55	48 59.8	+0.16	+ 3.7	„	
8	Dec. 25.67361	32 14.85	14 13.4	+0.18	+ 3.7	„	
9	27.66528	32 07.58	15 17.6	+0.15	+ 4.1	„	
10	1950 Jan. 25.70556	29 48.95	31 38.2	+0.31	+ 2.7	„	
11*	1951 Feb. 1.52708	36 39.40	29 05.7	−1.38	+31.2	„	
12*	28.88269	34 01.09	42 30.2	−2.85	+17.6	Kazan' (Engelgardt Observatory)	Astr. Circ., 132
13*	Mar. 13.83948	32 54.16	47 08.1	−2.92	+17.5	„	
14*	27.81340	31 54.33	50 25.8	−3.03	+18.5	„	
15*	Apr. 1.78844	31 37.01	51 09.2	−3.02	+19.2	„	
16*	3.84691	31 30.48	23 51 23.5	−3.05	+20.3	„	
17*	Oct. 31.80278	47 16.54	22 39 17.5	+0.32	+ 1.0	Tokyo	T. A. B., (2), 150.
18*	Nov. 26.76944	47 56.37	44 49.2	−1.44	+ 2.6	„	
19*	29.79589	47 58.01	22 45 51.9	+0.49	+ 1.5	„	
20*	1952 Feb. 26.54514	41 53.48	23 33 37.0	+0.50	+ 5.0	„	
21	Mar. 17.91095	40 07.79	40 59.8	+0.33	+ 0.2	Pulkovo	Letter
22*	23.58056	39 43.32	42 25.1	+0.72	+ 1.6	Cracow	T. A. B., (2), 150.
23	Apr. 15.87590	38 30.67	44 39.7	+0.35	− 0.1	Pulkovo	Letter
24	19.83596	38 23.85	44 29.6	+0.37	0.0	„	
25	1953 Mar. 11.87265	48 17.93	30 12.9	+0.40	− 0.3	„	
26	13.86264	48 07.82	30 53.9	+0.37	− 0.4	Cracow	A. A., 7, 3.
27	14.87282	48 02.80	31 14.0	+0.38	− 0.4	Pulkovo	Letter
28	14.89170	48 02.72	31 16.2	+0.39	+ 1.5	Carcow	A. A., 7, 3.
29	15.87843	47 57.86	31 33.8	+0.38	+ 0.1	Pulkovo	Letter
30	1954 Feb. 23.93854	57 33.91	12 32.3	+0.35	− 0.4	Algiers	J. O., 39, 2.
31	26.92622	57 16.74	14 02.3	+0.35	− 0.3	„	

Table 1 (continued)

№	Universal Time	$\alpha_{1950.0}$	$\delta_{1950.0}$	Observed-calculated $\Delta\alpha$	Observed-calculated $\Delta\delta$	Observational place	Source
		$h\ m\ s$	$^\circ\ '\ ''$	s	$''$		
32	1954 Mar. 21.80334	9 55 16.27	+23 22 55.4	+0.38	−0.3	Pulkovo	Letter
33	29.90477	54 41.25	24 49.2	+0.42	−0.5	Algiers	J. O., **39**, 2.
34	31.82797	54 33.67	25 10.4	+0.39	−0.2	Pulkovo	Letter
35	Apr. 20.84473	9 53 35.81	23 26 20.8	+0.42	−0.2	„	Letter
36*	1955 Jan. 17.74444	10 08 53.42	22 36 50.7	+0.85	−0.5	Purple Mountain (Nanking)	Acta Astr. Sinica, **4**, 1.
37	20.02080	08 42.26	38 21.8	+0.44	+0.9	Cracow	A. A., **7**, 3.
38*	20.68403	08 39.34	22 38 47.0	+0.72	−0.1	Purple Mountain (Nanking)	Acta Astr. Sinica, **4**, 1.
39	Mar. 23.81845	02 59.12	23 11 34.2	+0.41	0.0	Pulkovo	Letter
40	Jan. 8.80287	01 54.92	14 36.2	+0.44	+0.2	„	Letter
41	24.84926	01 14.46	23 14 43.4	+0.44	+0.2	„	Letter
42	1956 Mar. 16.88356	11 26.79	22 56 02.4	+0.44	+0.4	„	Letter
43*	18.965	11 16.01	22 56 46.8	+0.29	+0.4	Cracow	A. A., **7**, 3.
44	31.80965	10 16.86	23 00 16.8	+0.46	0.0	Pulkovo	Letter
45*	Jan. 8.82843	09 46.46	01 33.7	+0.54	+2.7	Cracow	A. A., **7**, 3.
46	30.88591	08 55.00	23 01 00.1	+0.46	0.0	Pulkovo	Letter
47	1957 Mar. 1.01278	21 00.04	22 33 12.1	+0.41	−0.9	Uccle	B. A. B., **5**, 1.
48	3.98160	20 42.97	34 46.7	+0.52	−0.4	Scalnate-Pleso	B. A. C., **14**, 4.
49	6.85764	20 26.55	36 14.1	+0.54	+0.3	„	B. A. C., **14**, 4.
50	7.96528	20 20.24	36 46.4	+0.52	+0.4	„	B. A. C., **14**, 4.
51	20.89137	19 10.54	42 08.1	+0.48	−0.2	Pulkovo	Letter
52	29.86845	18 27.55	44 48.5	+0.48	−0.4	„	Letter
53	Jan. 1.80908	18 14.70	22 45 28.9	+0.46	−0.4	„	Letter
54	Dec. 1.20655	34 19.16	21 20 58.1	+0.50	+0.5	Uccle	B. A. B., **5**, 1.
55	1958 Mar. 13.85492	27 57.66	22 22 14.9	+0.50	−0.4	Algiers	U. A. I., 1645.
56	17.89260	27 35.62	23 58.4	+0.47	−0.6	Pulkovo	Letter
57	20.87551	27 19.84	25 08.3	+0.48	−0.7	„	Letter
58	21.89153	27 14.57	25 30.9	+0.47	−0.5	Algiers	U. A. I., 1645.
59	24.87956	26 59.46	26 32.9	+0.51	−0.6	„	U. A. I., 1645.
60	Jan. 7.80952	25 56.87	29 59.4	+0.48	−0.2	Pulkovo	Letter

Table 2

№	Universal Time		$\alpha_{1950.0}$	$\delta_{1950.0}$	Number of Observations	
					α	δ
			° ′ ″	° ′ ″		
1	1914 Jan.	23.7904	90 00 11.32	+17 37 26.72	1	1
2	Jan.	23.7905	90 00 15.82	37 25.52	1	1
3	Nov.	12.3260	92 31 55.91	17 51 55.90	1	1
4	1915 Mar.	19.2639	90 39 55.39	18 04 37.05	1	1
5	Apr.	7.1674	90 46 03.43	18 07 49.77	1	1
6	1919 Dec.	29.4285	97 33 11.18	19 21 04.58	4	4
7	1921 Jan.	29.0896	98 08 13.82	42 18.04	1	1
8	Jan.	29.0896	98 08 17.92	19 42 18.53	1	1
9	1925 Dec.	23.2823	105 17 13.03	20 54 20.14	1	1
10	Dec.	23.3441	105 17 06.02	20 54 20.25	1	1
11	1927 Jan.	6.2500	106 18 38.47	21 11 11.78	1	1
12	Oct.	27.8942	105 50 32.16	21 15 39.67	1	1
13	1930 Apr.	24.1151	109 17 46.59	22 07 06.39	103	103
14	Oct.	10.0000	112 50 53.54	21 49 08.98	23	23
15	1931 Apr.	15.0000	110 35 27.04	22 19 41.81	44	44
16	1932 Feb.	4.0000	112 37 48.88	22 36.29	22	22
17	1933 Mar.	15.0000	113 27 27.58	41 34.50	20	20
18	1934 Mar.	8.0000	114 59 05.33	22 51 37.84	7	8
19	1935 Mar.	18.0000	116 21 17.08	23 03 20.38	8	8
20	1936 Feb.	10.8500	118 24 04.54	06 08.93	5	5
21	1937 Mar.	7.0000	119 29 02.13	20 36.80	9	9
22	1938 Mar.	11.0000	120 58 21.02	29 28.12	15	13
23	1939 Feb.	18.0000	122 54 16.68	32 11.55	12	12
24	Dec.	11.0000	125 59 18.94	13 08.78	7	7
25	1940 Mar.	10.0000	124 07 58.58	43 23.09	8	7
26	1941 Mar.	27.0000	125 32 55.14	51 14.54	2	4
27	Dec.	20.0000	129 06 21.10	24 24.73	14	14
28	1942 Apr.	26.0000	127 06 17.50	23 55 29.31	10	10
29	1943 Apr.	2.0000	128 47 32.66	24 00 15.07	6	6
30	1944 Feb.	28.0000	130 57 43.72	23 57 12.87	7	7
31	1945 Feb.	1.0690	133 17 05.40	23 48 37.50	1	1
32	Mar.	10.1193	132 28 47.40	24 01 31.50	1	1
33	1946 Mar.	10.0000	134 13 52.58	24 01 56.13	5	5
34	1947 Jan.	22.0000	137 04 20.19	23 41 44.74	12	12
35	1948 Feb.	18.0000	138 14 11.52	52 00.19	19	21
36	1949 Mar.	12.0000	139 33 48.82	56 47.28	15	15
37	1950 Feb.	2.0000	142 16 53.31	35 40.11	22	22
38	1951 Mar.	22.0000	143 05 09.70	49 01.59	18	18
39	1952 Apr.	16.0000	144 37 36.18	44 39.87	3	3
40	1953 Mar.	14.0000	147 01 46.86	30 56.90	5	4
41	1954 Mar.	21.0000	148 49 59.97	22 42.65	6	6
42	1955 Apr.	9.0000	150 28 33.72	23 14 37.52	3	3
43	1956 Mar.	30.0000	152 36 07.18	22 59 54.17	3	3
44	1957 Mar.	1.0000	155 15 02.22	33 12.58	7	7
45	1958 Mar.	20.0000	156 51 06.33	22 24 48.69	6	6

Table 3

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№	Universal Time of normal position corrected for aberration	$10^5\delta \lg r$ log r'	$\delta\varpi$	$\delta\Omega$	δi
			"	"	"
1	1914 Jan. 23.5373	— 85224	+136.296	—166.543	+256.348
2	Jan. 23.5374	— 85224	136.296	—166.543	256.348
3	Nov. 12.0732	— 61904	125.830	—130.329	225.881
4	1915 Mar. 19.0076	— 50416	122.201	—104.969	203.834
5	Apr. 6.9094	— 48649	121.710	—100.772	+200.148
6	1919 Dec. 29.1831	+ 36994	126.000	+159.145	— 51.925
7	1921 Jan. 28.8447	18611	125.955	116.620	+ 5.010
8	Jan. 28.8447	+ 18611	125.955	+116.620	5.010
9	1925 Dec. 23.0442	— 67218	51.620	— 51.870	298.178
10	Dec. 23.1060	— 67221	51.614	— 51.866	298.174
11	1927 Jan. 6.0133	— 56915	30.311	— 39.970	259.516
12	Jan. 27.6575	— 55990	29.220	— 38.960	255.850
13	1930 Apr. 23.8756	— 1129	+ 0.144	— 0.179	+ 11.879
14	Oct. 9.7615	— 16	— 0.003	— 0.016	— 1.018
15	1931 Apr. 14.7631	— 2205	+ 0.160	— 0.054	— 4.315
16	1932 Feb. 3.7697	— 12939	+ 0.128	+ 1.355	+ 12.026
17	1933 Mar. 14.7687	— 40070	— 2.589	10.009	67.346
18	1934 Mar. 7.7708	— 72047	— 9.363	25.700	133.681
19	1935 Mar. 17.7715	—108059	— 21.729	48.031	204.327
20	1936 Apr. 10.6252	—137221	— 36.980	67.707	254.276
21	1937 Mar. 6.7754	—163124	— 58.974	80.953	282.599
22	1938 Mar. 10.7766	—175091	— 81.124	71.459	266.440
23	1939 Feb. 17.7796	—174379	— 99.887	+ 35.681	211.839
24	Dec. 10.7796	—166646	—112.301	— 14.406	143.256
25	1940 Mar. 9.7798	—163542	—115.179	— 31.922	120.691
26	1941 Mar. 26.7800	—150494	—121.909	—106.894	+ 30.839
27	Dec. 19.7830	—145272	—121.614	—148.857	— 14.942
28	1942 Apr. 25.7789	—144920	—120.254	—162.840	— 29.376
29	1943 Apr. 1.7828	—152721	—113.749	—176.896	— 43.242
30	1944 Feb. 27.7866	—173899	—105.341	—156.266	— 25.056
31	1945 Jan. 31.8578	—208945	— 96.967	—104.375	+ 17.471
32	Mar. 9.9069	—213512	— 96.159	— 97.167	23.138
33	1946 Mar. 9.7893	—264963	— 89.992	— 15.110	84.858
34	1947 Jan. 21.7916	—316620	— 88.141	+ 63.698	139.982
35	1948 Feb. 17.7932	—383219	— 91.114	149.442	195.776
36	1949 Mar. 11.7938	—445315	— 99.376	194.941	223.314
37	1950 Feb. 1.7962	—489827	—108.810	183.263	216.943
38	1951 Mar. 21.7964	—532168	—120.146	+ 93.859	170.531
39	1952 Apr. 15.7962	—559860	—125.659	— 42.425	104.526
40	1953 Mar. 13.8000	—579559	—123.525	—155.296	53.247
41	1954 Mar. 20.8012	—605684	—112.874	—233.466	19.763
42	1955 Apr. 8.8015	—644909	— 94.145	—235.611	18.725
43	1956 Mar. 29.8037	—696456	— 72.364	—168.188	43.733
44	1957 Feb. 28.8064	—759322	— 50.388	— 58.364	82.187
45	1958 Mar. 19.8073	—846120	— 26.432	+ 96.371	133.087

Table 4

№	Universal Time	System A		System XIX		Astr. Pap., 12		B. J.		XIX — Astr. Pap., 12		System A′	
		Δα	Δδ	Δα	Δδ	Δα	Δδ	Δα	Δδ	Δα	Δδ	Δα	Δδ
		″	″	″	″	″	″			″	″	″	″
1	1914 Jan. 23.7904	+ 4.73	+1.05	+2.61	+0.50	+2.25	+0.76	—	—	+0.36	−0.26	−0.79	+0.01
2	Jan. 23.7905	9.23	−0.15	7.11	−0.70	6.75	−0.44	—	—	0.36	−0.26	+3.71*	−1.19*
3	Nov. 12.3260	4.56	+0.47	2.18	0.00	1.87	+0.16	—	—	0.31	−0.16	+0.09	−0.38
4	1915 Mar. 19.2639	5.15	−2.33	2.95	−2.82	2.66	−2.70	—	—	0.29	−0.12	−0.19	−3.12*
5	Apr. 7.1674	6.05	−4.38	3.85	−4.86	3.57	−4.76	—	—	0.28	−0.10	+0.80	−5.14*
6	1919 Dec. 29.4285	+ 4.16	+1.14	+1.40	+1.08	+1.15	+1.03	—	—	0.25	+0.05	+0.73	+1.05
7	1921 Jan. 29.0896	— 2.69	−0.59	−5.45	−0.55	−5.68	−0.60	—	—	0.23	0.05	−6.11*	−0.52
8	Jan. 29.0896	+ 1.41	−0.10	−1.35	−0.06	−1.58	−0.11	—	—	0.23	+0.05	−2.01*	−0.04*
9	1925 Dec. 23.2823	5.07	−0.39	+2.52	+0.26	+2.35	+0.37	—	—	0.17	−0.11	+3.35*	+0.27
10	Dec. 23.3441	2.86	−0.94	+0.33	−0.30	+0.15	−0.18	—	—	0.18	−0.12	+1.11	−0.29
11	1927 Jan. 6.2500	0.40	−4.21	−2.05	−3.42	−2.18	−3.37	—	—	0.13	−0.05	−1.25*	−3.44*
12	Jan. 27.8942	5.44	−0.21	+3.01	+0.57	+2.88	+0.62	—	—	0.13	−0.05	+3.61*	+0.59*
13	1930 Apr. 24.1151	1.16	−1.06	−0.68	+0.15	−0 73	+0.14	—	—	0.05	+0.01	−0.28*	+0.07
14	Oct. 10.0000	1.03	−1.44	−0.49	−0.16	−0.54	−0.15	—	—	+0.05	0.00	+0.88*	−0.40
15	1931 Apr. 15.0000	0.71	−1.89	−0.94	−0.54	−0.93	−0.54	—	—	−0.01	0.00	−0.53	−0.66
16	1932 Feb. 4.0000	0.04	−1.61	−1.32	−0.11	−1.31	−0.11	−1 04	−0 13	−0.01	0.00	−0.64	−0.32
17	1933 Mar. 15.0000	+ 0.43	−1.64	−0.60	+0.01	−0.57	+0.01	−0.47	+0.04	−0.03	0.00	−0.26	−0.22
18	1934 Mar. 8.0000	— 0.55	−1.67	−1.26	0.14	−1.22	0.09	−1.21	0.15	−0.04	0.05	−0.99	−0.17
19	1935 Mar. 18.0000	— 0.66	−1.44	−0.98	+0.51	−0.90	+0.40	−0.80	+0.44	−0.08	0.11	−0.91	+0.14
20	1936 Feb. 10.8500	— 0.45	−3.37	−0.32	−1.28	−0.20	−1.46	+0.10	−1.53	−0.12	0.18	−0.15	−1.74*
21	1937 Mar. 7.0000	+ 0.12	−1.33	+0.66	+0.91	+0.77	+0.64	+0.72	+0.59	−0.11	0.27	+0.46	+0.40
22	1938 Mar. 11.0000	— 1.42	−1.79	−0.41	0.58	−0.29	0.31	−0.36	0.38	−0.12	0.27	−0.83	0.02
23	1939 Feb. 18.0000	— 2.04	−1.32	−0.37	1.17	−0.23	0.94	−0.30	1.00	−0.14	0.23	−0.93	0.52
24	Dec. 11.0000	— 1.59	−1.65	+0.81	0.83	+0.96	0.68	+0.76	0.84	−0.15	0.15	+0.51	+0.04
25	1940 Mar. 10.0000	— 1.13	−2.16	1.04	0.44	1.21	0.31	1.26	0 33	−0.17	0.14	0.13	−0.24
26	1941 Mar. 27.0000	— 1.24	−2.03	1.54	0.67	1.74	0.59	1.66	0.58	−0.20	0.08	0.26	−0.05
27	Dec. 20.0000	— 2.03	−1.59	1.69	1.07	1.90	1.00	1.83	1.00	−0.21	0.07	0.77	+0.20
28	1942 Apr. 26.0000	— 1.42	−2.33	2.00	0.44	2.22	0.37	2.12	0.51	−0.22	0.07	0.36	−0.33
29	1943 Apr. 2.0000	— 1.22	−0.06	2.88	2.81	3.09	2.73	2.82	3.00	−0.21	0.08	0.95	+2.01*
30	1944 Feb. 28.0000	— 2.48	−1.16	2.48	1.77	2.73	1.68	2.50	2.00	−0.25	0.09	9.40	0.92
31	1945 Feb. 1.0690	— 3.06	−0.19	2.81	+2.74	3.02	+2.64	3.00	3.20	−0.21	0.10	+0.60*	+1.82*
32	Mar. 10.1193	— 3.53	−3.25	2.15	−0.25	2.39	−0.37	2.25	0.20	−0.24	0.12	−0.35*	−1.13*
33	1946 Mar. 10.0000	— 3.14	−0.41	3.41	+2.62	3.62	+2.45	3.51	3.12	−0.21	0.17	+0.56	+1.72*
34	1947 Jan. 22.0000	— 4.01	−2.24	3.68	0.68	3.86	0.42	3.61	1.16	−0.18	0.26	0.78	−0.29
35	1948 Feb. 18.0000	— 4.62	−1.05	3.89	1.93	4.10	1.52	4.00	2.46	−0.21	0.41	0.37	+1.00
36	1949 Mar. 12.0000	— 4.72	−1.16	4.65	1.82	4.84	1.32	4.66	2.41	−0.19	0.50	+0.62	0.94
37	1950 Feb. 2.0000	— 6.49	−1.11	4.03	1.70	4.27	1.19	4.10	2.46	−0.24	0.50	−0.02	0.78
38	1951 Mar. 22.0000	— 6.66	−1.58	4.72	1.27	4.98	0.90	4.62	2.30	−0.26	0.37	−0.07	+0.43
39	1952 Apr. 16.0000	— 7.42	−2.44	4.84	+0.30	5.15	+0.06	5.25	+0.03	−0.31	0.24	−0.35	−0.49
40	1953 Mar. 14.0000	— 8.29	−2.68	5.32	−0.14	5.64	−0.31	5.76	−0.32	−0.32	0.17	−0.07	−0.89
41	1954 Mar. 21.0000	— 9.39	−1.77	5.37	+0.60	5.67	+0.42	5.78	+0.32	−0.30	0.18	−0.40	−0.09
42	1955 Apr. 9.0000	— 9.67	−1.86	6.15	0.32	6.43	0.14	6.45	0.13	−0.28	0.18	+0.01	−0.30
43	1956 Mar. 30.0000	−10.57	−1.58	6.62	0.32	6.85	+0.12	6.80	+0.13	−0.23	0.20	0.24	−0.24
44	1957 Mar. 1.0000	−12.01	−1.37	6.68	+0.10	6.83	−0.14	6.86	−0.18	−0.15	0.24	0.32	−0.43
45	1958 Mar. 20.0000	−12.71	−1.36	7.12	−0.14	7.20	−0.49	7.26	−0.50	−0.08	0.35	0.41	−0.57

CHAPTER II

NEW ELEMENTS OF PLUTO

3. Improvement of the Elements of Pluto

In order to correct the ecliptic elements of Pluto, we use a modified Eckert method (Samoylova-Yakhontova, 1945). For the times of the normal positions we calculate by the formulas: /156

	δM_0	δi	$\delta\Omega$	$\delta\omega$	$\frac{\delta a}{a}$	δe
$\delta x = \frac{\dot{x}}{n}$	$N_y z - N_z y$	$-z\sin\varepsilon - y\cos\varepsilon$	$R_y z - R_z x$	$x - \frac{3}{2}t\dot{x}$	$Hx + K\frac{\dot{x}}{n}$
$\delta y = \frac{\dot{y}}{n}$	$N_z x - N_x z$	$x\cos\varepsilon$	$R_z x - R_x z$	$y - \frac{3}{2}t\dot{y}$	$Hy + K\frac{\dot{y}}{n}$
$\delta z = \frac{\dot{z}}{n}$	$N_x y - N_y x$	$x\sin\varepsilon$	$R_x y - R_y x$	$z - \frac{3}{2}t\dot{z}$	$Hz + K\frac{\dot{z}}{n}$

$$\begin{Bmatrix} \rho\cos\delta\Delta\alpha \\ \rho\Delta\delta \end{Bmatrix} = \begin{Bmatrix} \delta x \\ \delta y \\ \delta z \end{Bmatrix} \begin{Bmatrix} -\sin\alpha & -\sin\delta\cos\alpha \\ \cos\alpha & -\sin\delta\sin\alpha \\ 0 & \cos\delta \end{Bmatrix},$$

where

$$N_x = \cos\Omega, \quad R_x = \sin i\sin\Omega, \quad H = -(\cos E + e)\sec^2\varphi,$$
$$N_y = \sin\Omega\cos\varepsilon, \quad R_y = -\sin i\cos\Omega\cos\varepsilon - \cos i\sin\varepsilon, \quad K = \left(1 + \frac{r}{p}\right)\sin E$$
$$N_z = \sin\Omega\sin\varepsilon, \quad R_z = -\sin i\cos\Omega\sin\varepsilon + \cos i\cos\varepsilon,$$

the coefficients of the conditional equations on the basis of system A. We substitute into the second members the results of comparing the normal positions: 1) with system A, taking into account perturbations from Table 3 (we denote the second members by L);* 2) with system XIX, taking into account the same perturbations (we denote the second members by L').

The coefficients of the conditional equations and the two variants of the second members are presented in Table 5. The first 45 numbers of this table pertain to equations for right ascension and the last 45 numbers to declination.

We derived and solved several systems of normal equations with the use of various conditional equations from those presented in Table 5 (for example, all equations with respect to α and δ, and Equations 14-45 with respect to α and δ, etc.). We will not consider these experiments in detail, although they indicate that it would be most advantageous to use the 69 conditional equations for correction (conditional equations marked by an asterisk are not involved in the refinement). From the 69 conditional equations, we compiled a normal system with two variants of the second members (Table 6), and obtained corrections of the elements to system A and to system XIX.

* For the first 45 equations, $L = 10^{-3}\rho\cos\delta\Delta\alpha$ and for the last 45 equations, $L = 10^{-3}\rho\Delta\delta$; the case is the same for L'.

Table 5

№ п./п.	$44\delta M_0$	$23\delta i$	$45\delta\Omega$	$43\delta\omega$	$46\frac{\delta a}{a}$	$80\delta e$	L	L'	v	v'
1	+0.742593	+0.182222	+0.988091	+0.998512	+0.458230	−0.768805	+0.19784	+0.10912	−0.034816	−0.035138
2*	0.742593	0.182222	0.988091	0.998512	0.458230	−0.768805	0.38599	0.29727	+0.153334	+0.153012
3	0.752570	0.165635	0.985044	0.998942	0.416076	−0.790145	0.19011	0.09095	+0.002294	+0.002266
4	0.745032	0.168070	0.983753	0.994540	0.438637	−0.779598	0.21743	0.12444	−0.009034	−0.008924
5	0.745448	0.167209	0.983576	0.994491	0.436767	−0.780572	0.25731	0.16362	+0.032508	+0.032364
6	0.773243	0.101470	0.963931	0.983647	0.299428	−0.847408	+0.16718	+0.05633	+0.031227	+0.032202
7*	0.775627	0.090061	0.958958	0.979237	0.281093	−0.855621	−0.10766	−0.21791	−0.243883	−0.243091
8*	0.775627	0.090061	0.958958	0.979237	0.281093	−0.855621	+0.05621	−0.05401	−0.080013	−0.079191
9*	0.805959	0.036357	0.932976	0.964156	0.133237	−0.912691	0.19535	+0.09729	+0.127291	+0.127159
10	0.805955	0.036357	0.932978	0.964156	0.133252	−0.912686	0.11047	+0.01288	+0.042399	+0.042740
11*	0.810359	0.027413	0.927318	0.960053	0.108433	−0.920808	0.01518	−0.07841	−0.047751	−0.048884
12*	0.808311	0.027396	0.927300	0.959174	0.114559	−0.918688	0.20826	+0.11508	+0.137239	+0.136846
13*	0.822895	+0.002496	0.908493	0.945800	+0.032998	−0.942362	0.04469	−0.02604	−0.008723	−0.009069
14*	0.838066	−0.000522	0.903062	0.946733	−0.023002	−0.957766	0.03949	−0.01867	+0.034867	+0.035041
15	0.828423	−0.003843	0.902391	0.941826	+0.004576	−0.949672	0.02703	−0.03563	−0.018383	−0.019934
16	0.837366	−0.008243	0.896582	0.939565	−0.033150	−0.959262	0.00148	−0.04858	−0.023048	−0.023446
17	0.840714	−0.014170	0.889738	0.933965	−0.056398	−0.963755	+0.01605	−0.02223	−0.010220	−0.009656
18	0.847234	−0.018252	0.882991	0.929777	−0.088724	−0.970252	−0.02028	−0.04602	−0.036350	−0.036462
19	0.852980	−0.021935	0.875947	0.924977	−0.119378	−0.975550	−0.02387	−0.03586	−0.032899	−0.033282
20	0.861736	−0.023304	0.868911	0.921435	−0.158909	−0.982306	−0.01602	−0.01132	−0.006385	−0.007035
21	0.866130	−0.025965	0.861689	0.915888	−0.186065	−0.985381	+0.00422	+0.02363	+0.016165	+0.015191
22	0.872270	−0.026917	0.854338	0.910935	−0.218770	−0.988992	−0.05055	−0.01463	−0.029291	−0.031037
23	0.880286	−0.025778	0.846833	0.906605	−0.257491	−0.992979	−0.07136	−0.01286	−0.033161	−0.032305
24	0.892609	−0.020317	0.838160	0.903095	−0.312002	−0.998055	−0.05587	+0.02850	+0.018173	+0.018558
25	0.885161	−0.025730	0.839340	0.900953	−0.287022	−0.994469	−0.03959	0.03648	0.003821	0.004038
26	0.890750	−0.024361	0.831622	0.895356	−0.319313	−0.995665	−0.04337	0.05376	0.008297	0.009533
27	0.904850	−0.014861	0.822604	0.892179	−0.380398	−0.999526	−0.06996	0.05826	0.025690	0.027046
28	0.896841	−0.021752	0.823442	0.889467	−0.354172	−0.996285	−0.04991	0.07001	0.011570	0.013223
29	0.903484	−0.017539	0.815911	0.884440	−0.389552	−0.996616	−0.04210	0.09903	0.033039	0.033343
30	0.911975	−0.010635	0.807802	0.879565	−0.432054	−0.996651	−0.08396	0.08386	0.012605	0.012954
31*	0.920743	−0.002065	0.799129	0.874242	−0.477109	−0.995505	−0.10260	0.09419	+0.017796	+0.017958
32*	0.917632	−0.005304	0.799740	0.873495	−0.466096	−0.994810	−0.11856	0.07223	−0.011817	−0.013148
33	0.924102	+0.001843	0.791480	0.867521	−0.503430	−0.992345	−0.10465	0.11392	+0.016553	+0.016099
34	0.934398	0.014709	0.782151	0.862130	−0.555817	−0.989239	−0.13271	0.12166	0.022242	0.023194
35	0.938468	0.021126	0.774404	0.855474	−0.585372	−0.984682	−0.15158	0.12745	0.009100	0.009428
36	0.942941	0.028935	0.766422	0.848726	−0.617270	−0.979322	−0.15424	0.15195	+0.015565	+0.016082
37	0.952111	0.044361	0.757107	0.842660	−0.668243	−0.972594	−0.21003	0.13046	−0.005610	−0.007433
38	0.954589	0.051043	0.749807	0.835547	−0.693198	−0.965839	−0.21480	0.15244	−0.008540	−0.008445
39	0.959364	0.062161	0.741629	0.828514	−0.728465	−0.957672	−0.23994	0.15648	−0.018080	−0.019426
40	0.966968	0.078535	0.733073	0.822235	−0.774937	−0.948131	−0.26356	0.16911	−0.008776	−0.010166
41	0.972302	0.092491	0.724960	0.815312	−0.813537	−0.937915	−0.29699	0.16982	−0.019508	−0.020130
42	0.976930	0.106196	0.717024	0.808156	−0.850033	−0.926921	−0.30570	0.19450	−0.007740	−0.008018
43	0.982895	0.123187	0.708816	0.801198	−0.892857	−0.914298	−0.33104	0.20739	−0.002164	−0.000947
44	0.990023	0.143830	0.700062	0.794098	−0.942628	−0.899300	−0.37228	0.20720	−0.001092	+0.000019
45	0.994009	0.158791	0.692644	0.786821	−0.978391	−0.885585	−0.39243	0.21991	+0.002155	0.001981

Table 5 (continued)

№ п./п.	$44\delta M_0$	$23\delta i$	$45\delta\Omega$	$42\delta\omega$	$46\frac{\delta a}{a}$	$80\delta e$	L	L'	v	v'
1	+0.214418	−0.630570	−0.001236	+0.287986	+0.130717	−0.222249	+0.04606	+0.02207	+0.000517	+0.000521
2*	0.214418	−0.630570	−0.001236	0.287986	0.130717	−0.222249	−0.00658	−0.03058	−0.052123	−0.052129
3	0.205050	−0.603770	−0.015944	0.273577	0.119133	−0.214397	+0.02068	0.00000	−0.016765	−0.016469
4*	0.213220	−0.589383	−0.006622	0.283198	0.119335	−0.224042	−0.10363	−0.12504	−0.138688	−0.138581
5*	0.212986	−0.587578	−0.007233	0.282607	0.118150	−0.224015	−0.19377	−0.21515	−0.227587	−0.227490
6	0.188025	−0.415322	−0.050591	0.239586	0.074254	−0.205928	+0.04872	+0.04591	+0.045267	+0.045290
7	0.186543	−0.373991	−0.054902	0.234995	0.065404	−0.205950	−0.02493	−0.02331	−0.021714	−0.021593
8*	0.186543	−0.373991	−0.054902	0.234995	0.065404	−0.205950	−0.00412	−0.00251	−0.000904	−0.000793
9	0.156339	−0.186157	−0.098300	0.187586	0.027909	−0.177021	−0.01622	+0.01073	+0.010591	+0.010729
10	0.156345	−0.186152	−0.098293	0.187593	0.027907	−0.177030	−0.03901	−0.01218	−0.012192	−0.012169
11*	0.152116	−0.145496	−0.104304	0.180358	0.020854	−0.172851	−0.17253	−0.14050	−0.140834	−0.141032
12*	0.154784	−0.143078	−0.101544	0.183153	+0.019943	−0.175909	−0.00879	+0.02337	+0.023912	+0.023773
13	0.140014	−0.014843	−0.120549	0.159221	−0.000720	−0.160014	−0.04396	+0.00619	+0.003272	+0.003263
14	0.119643	+0.003635	−0.141224	0.136881	+0.003111	−0.137108	−0.05919	−0.00639	−0.016206	−0.016374
15	0.133875	0.024096	−0.127438	0.150442	−0.005767	−0.153040	−0.07773	−0.02236	−0.027002	−0.027332
16	0.122834	0.056504	−0.138822	0.137088	−0.007580	−0.140504	−0.06440	−0.00431	−0.012623	−0.012776
17	0.119514	0.100935	−0.142402	0.131165	−0.013920	−0.136446	−0.06580	+0.00044	−0.009074	−0.009306
18	0.111568	0.140252	−0.150142	0.120937	−0.017180	−0.127168	−0.06640	0.00540	−0.006991	−0.006777
19	0.104536	0.181361	−0.156647	0.111709	−0.020641	−0.118808	−0.05695	+0.02036	+0.005499	+0.005543
20*	0.092795	0.217596	−0.167231	0.098281	−0.020515	−0.105308	−0.13143	−0.04994	−0.068235	−0.068147
21	0.087432	0.260122	−0.171484	0.090947	−0.024222	−0.098625	−0.05162	+0.03535	+0.015351	+0.015412
22	0.079184	0.300304	−0.178049	0.081116	−0.025522	−0.088812	−0.06914	0.02230	0.000153	0.000072
23	0.067680	0.337757	−0.187182	0.068547	−0.023889	−0.075586	−0.05039	0.04465	0.019516	0.019673
24	0.047518	0.370091	−0.203060	0.048944	−0.013274	−0.053792	−0.06298	0.03174	+0.001226	+0.001313
25	0.060993	0.379457	−0.191562	0.060505	−0.025404	−0.067388	−0.08260	0.01699	−0.009601	−0.009433
26	0.052852	0.420570	−0.196796	0.051360	−0.025276	−0.057695	−0.07751	0.02558	−0.002289	−0.002458
27	0.028736	0.449626	−0.215147	0.028933	−0.009707	−0.032288	−0.05976	0.04031	+0.007141	+0.006941
28	0.043493	0.462843	−0.202447	0.041402	−0.023424	−0.046846	−0.08943	0.01686	−0.012910	−0.013028
29*	0.033223	0.499087	−0.209036	0.030744	−0.020770	−0.035036	−0.00245	0.10585	+0.075283	+0.075008
30	0.019182	0.534083	−0.218231	0.017144	−0.013980	−0.019659	−0.04294	0.06558	0.033499	0.033238
31*	0.003655	0.569370	−0.227853	0.002774	−0.004352	−0.003265	−0.00707	+0.10032	+0.066285	+0.066211
32*	+0.009798	0.573157	−0.222867	+0.007786	−0.010596	−0.009042	−0.11961	−0.00931	−0.041987	−0.042090
33*	−0.001461	0.610530	−0.228634	−0.002898	−0.004807	+0.003230	−0.01505	+0.09558	+0.062413	+0.062425
34	−0.021384	0.642509	−0.240660	−0.020113	+0.011424	0.023040	−0.08092	0.02446	−0.010697	−0.010724
35	−0.028275	0.681383	−0.242140	−0.026884	0.013570	0.030999	−0.03779	0.06920	+0.035406	+0.035637
36	−0.036582	0.719100	−0.244196	−0.034465	0.018207	0.039770	−0.04152	0.06514	0.032963	0.032966
37	−0.056009	0.749883	−0.254262	−0.050279	0.036778	0.058122	−0.03918	0.05988	0.026928	0.027055
38	−0.060282	0.788617	−0.252484	−0.054402	0.037583	0.063324	−0.05571	0.04484	+0.014834	+0.015144
39	−0.070682	0.824200	−0.254582	−0.062744	0.047157	0.073121	−0.08626	+0.01046	−0.017640	−0.017520
40	−0.087605	0.853157	−0.261727	−0.076016	0.064330	0.088292	−0.09306	−0.00472	−0.031077	−0.030923
41	−0.100050	0.885091	−0.264696	−0.085477	0.077522	0.099135	−0.06087	+0.02061	−0.003057	−0.003205
42	−0.111357	0.917143	−0.266244	−0.093772	0.090238	0.108556	−0.06388	0.01084	−0.010019	−0.010199
43	−0.126352	0.945357	−0.270276	−0.104595	0.108274	0.120491	−0.05376	0.01072	−0.007505	−0.007611
44	−0.145545	0.970330	−0.276718	−0.117991	0.133489	0.134594	−0.04609	+0.00322	−0.013379	−0.013596
45	−0.156418	0.998861	−0.276824	−0.125295	0.147757	0.142361	−0.04553	−0.00471	−0.018061	−0.017659

Table 6

$23\delta i$	$45\delta\Omega$	$43\delta\omega$	$44\delta M_0$	$80\delta e$	$46\frac{\delta a}{a}$	L	L'
+11.317072	— 2.066137	+ 0.169752	+ 0.131965	— 0.103044	+ 0.043612	--0″.991268	+0″.588496
— 2.066137	+25.829705	26.151174	25.607144	—27.310912	— 8.107997	—1.269107	2.142269
+ 0.169752	26.151174	28.648034	28.068908	—29.939533	— 8.946462	—1.956312	2.505516
+ 0.131965	+25.607144	+28.068908	+28.141161	—29.814016	—10.830237	—2.688267	+2.688723
— 0.103044	--27.310912	—29.939533	—29.814016	+31.752030	+10.898355	+2.603758	—2.683043
+ 0.043612	— 8.107997	— 8.946462	—10.830237	10.898355	9.459153	3.107833	—1.532624

Corrections of Elements

	To system A	To system XIX
δi	—2″.138 ± 0″.305	1″.897 ± 0″.308
$\delta\Omega$	4.099 ± 0.378	—0.979 ± 0.382
$\delta\omega$	—193.011 ± 37.674	—259.554 ± 38.088
δM	205.893 ± 41.728	315.898 ± 42.187
δe	—0.00001122 ± 0.00000392	0.00006362 ± 0.00000397
$\frac{\delta a}{a}$	0.00033967 ± 0.00005996	0.00041618 ± 0.00006062
δa	0.013471 ± 0.002378	0.016507 ± 0.002404
δn	—0″.007238 ± 0.001278	—0″.008870 ± 0.001292

The two systems of elements obtained, which we designate as A' and XIX', agree favorably with one another.

	System A'	System XIX'
Epoch and osculation 1930 Sept. 20.0 Ecliptic and equinox 1950.0		
M_0	275°15′27″.03	275°15′25″.20
ω	112 39 29.74	112 39 31.01
Ω	109 37 55.99	109 37 55.98
i	17 6 35.34	17 6 35.34
e	0.24706226	0.24706120
a	39.672599	39.672527
n	14″.199460	14″.199498

The deviations obtained, after substituting the unknown values into the conditional equations, are given in Table 5, columns 10 and 11.

The results of comparing normal positions with the theory, after correcting the element (system A') with consideration of perturbations from Table 3, are given in Table 4, column 8 (the normal positions not involved in improving the elements are noted by an asterisk).

Observations of Pluto still encompass a very small part of its orbit. Thus, the determinant of the system is small, and corrections to the elements are determined with a low accuracy (three significant digits).

As new observations are accumulated, we will have to reexamine the elements of Pluto and recalculate the constants of integration.

We can presently consider that system A', with consideration of perturbations obtained in the present work, most satisfactorily represents available observations.

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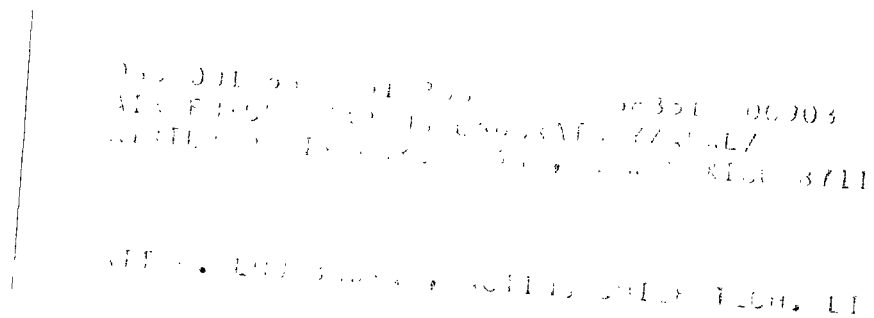

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